UNIT - II

INTEREST

Definition:

There are large number of causes for an individual or a corporate body or a government to borrow. The lender is called the creditor and the borrower is called the debtor. The amount borrowed by the debtor is called the principal. The fees or the charge paid by the borrower for using the money of the lender is called the interest. The total amount to be repaid is called the amount. Hence, amount is principal + plus Interest.

Symbols Used:

P → Principal (or) Present value

A → Amount (or) value at the end

n → Number of units of time

r → Rate of interest

s → Effective rate of interest under compound interest which is the interest per Rs.100 per annum.

I → Simple interest

CI → Compound interest

Simple Interest Formula :-

1) $I = \frac{Pnr}{100}$

2) $A = P + I = P + \frac{Pnr}{100} = P\left(1 + \frac{nr}{100}\right)$

3) $P = \frac{100\,I}{nr}$

4) $n = \frac{100\,I}{Pr}$

5) $r = \frac{100\,I}{Pn}$

Problem :-

1) Find the simple Interest on the sum of Rs. 6000 at 10% per annum. For 3 years.

Solu :-

Given,

$P = Rs\ 6000$

$r = 10\%$

$n = 3$

$SI = \frac{Pnr}{100}$

$= \frac{6000 \times 10 \times 3}{100}$

$= Rs\ 1800$

2) Find the simple interest and total amount on Rs 200 at 8% for 2 years.

Given:

$P = ₹200$

$r = 8\%$

$n = 2$ years

$$S.I = \frac{Pnr}{100}$$

$$= \frac{200 \times 8 \times 2}{100}$$

$= ₹32$

Total amount, $A = P + I$

$= 200 + 32$

$= Rs. 232$

3) Calculate the total amount that will be received from the debtor, when the principal is Rs 10,000 lent to him on interest for 4 years at 9% per annum.

Given,

$P = ₹10,000$

$r = 9\%$

$n = 4$ years

Total amount (required amount) $= A = P\left(1 + \frac{nr}{100}\right)$

$= 10,000\left(1 + \frac{36}{100}\right)$

$= 10,000\left(\frac{136}{100}\right)$

$= 10,000 \times 1.36$

$= Rs.13,600$

4) Rs. 6,000 amongs to Rs. 8940 at 14% per annum interest. Find the number of years for which the amount was lent.

Given,

$P = 6,000$

$A = 8940$

$r = 14\%$

$n = ?$

$A = P + I$

$I = A - P$

$I = 8940 - 6000$

$= Rs. 2940$

$n = \frac{100I}{Pr}$

$= \frac{100 \times 2940}{6000 \times 14}$

$n = 3.5$ years.

5) If a team deposite of Rs. 4000 earns an interest of Rs. 2500 in 5 months. Find the Rate of Interest.

Given,

$P = Rs.\ 4000$

$I = 2500$

$n = \frac{50}{12}$

$r = ?$

$r = \frac{100\,I}{Pn}$

$= \frac{100 \times 2500}{4000 \times \frac{50}{12}}$

$= \frac{100 \times 2500 \times 12}{4000 \times 50}$

$r = 15\%$

6) A certain sum amongs to Rs. 4000 at the end of 5 years at 12% per annum interest. Find the sum.

Given,

$A = 4000$

$n = 5$

$r = 12\%$

$P = ?$

$A = P\left(1 + \frac{nr}{100}\right)$

$P = \frac{A}{1 + \frac{nr}{100}}$

$= \frac{4000}{1 + \frac{5 \times 12}{100}}$

$$= \frac{4000}{1 + \frac{60}{100}} = \frac{4000}{10 + \frac{6}{10}}$$

$$= \frac{4000}{\frac{16}{10}}$$

$$= \frac{4000}{1.6} = \text{Rs. } 2500$$

7) Mr. Ramesh deposited Rs. 25000 on 1.1.94. At the end of 5 months, he withdrew Rs. 5000. Find the Interest due to him on 31.12.94. Rate of Interest = 12% per annum.

Given,

Interest for Rs. 25000 for 5 months $= \frac{Pnr}{100}$

$P = 25,000$

$n = \frac{5}{12}$

$r = 12\%$

$$\frac{Pnr}{100} = \frac{25,000 \times \frac{5}{12} \times 12}{100}$$

$$= 250 \times \frac{5}{12} \times 12$$

$$= \text{Rs. } 1250$$

Interest for Rs 20,000 for 7 months $= \frac{Pnr}{100}$

$$= \frac{20,000 \times \frac{7}{12} \times 12}{100}$$

$$= 200 \times \frac{7}{12} \times 12$$

$$= Rs.1400$$

∴ Interest due on 31.12.94 = Rs. 1250 + 1400

= Rs. 2650

8) A sum of money amounted to Rs. 1071 in 6 months and Rs. 1106 in 16 months. Calculate the Rate of simple interest.

Amount at the end of 6 months = 1071

Amount at the end of 16 months = 1106

Interest = (16 - 6) = 10 months

= 1106 - 1071

= 35

Interest for 6 months $= \frac{35}{10} \times 6$

$= 21$

$P = A - I$

$= 1071 - 21$

$= 1050$

$r = \frac{100 I}{Pn}$

$= \frac{100 \times 21 \times 6}{1050 \times \frac{6}{12}}$

$= \frac{2100}{525}$ $= \frac{100 \times 21 \times 12}{1050 \times 6}$

$= 4\%$. $= \frac{42}{105} = 4\%$

9) A man borrows Rs. 1600 for 3 years at 12% for interest. Calculate the how much he has to repay.

Given,

$P = 1600$

$n = 3$ years

$r = 12\%$.

$A = P\left(1 + \frac{nr}{100}\right)$

$= 1600\left(1 + \frac{3 \times 12}{100}\right)$

$= 1600\left(1 + \frac{36}{100}\right)$

$= 1600 \times 1 + \frac{36}{100}$ $= 1600 (1 + 0.36)$

$= 16 \times 37$

$= 1600\left(100 + \frac{36}{100}\right)$

$= 1600\left(\frac{136}{100}\right)$

$= Rs. 2176$

10) Krishnan deposited Rs 15000 on 1-1-93. At the end of 4 months he withdrew Rs. 5000. Find the interest due to him on 31-12-93. Rate of interest is 11% per annum.

Given,

Interest for the Rs. 15,000 for 4 months $= \frac{Pnr}{100}$

$P = 15,000$

$n = \frac{4}{12}$

$r = 11\%$

$$\frac{Pnr}{100} = \frac{15,000 \times \frac{4}{12} \times 11}{100}$$

$$= 15,000 \times \frac{4}{12} \times 11$$

$$= 550$$

Interest for Rs 10,000 for 8 months $= \frac{Pnr}{100}$

$$= \frac{10,000 \times \frac{8}{12} \times 11}{100}$$

$$= 100 \times \frac{8^{2}}{12_{3}} \times 11$$

$$= 733.3$$

∴ Interest due on 31.12.93 = Rs. 733.3 + 550 = Rs. 1283.3

11) In how many years will Rs. 3610 amount to Rs 5234.50 at 9% simple interest?

Given,

$P = 3610$

$A = 5234.50$

$r = 9$

$n = ?$

$A = P + I$

$I = A - P$

$= 5234.50 - 3610$

$I = Rs. 1624.5$

$$n = \frac{100\,I}{Pr}$$

$$= \frac{100 \times 1624.5}{3610 \times 9}$$

$$= \frac{162450}{32490}$$

$= 5$ years.

12) Find the present value of Rs. 595 due 3 years from now receiving simple interest at 5% per annum. How much is the true discount (interest) given?

$A = 595$

$r = 5\%$

$n = 3$

$$A = P\left(1 + \frac{nr}{100}\right)$$

$$595 = P\left(1 + \frac{5 \times 3}{100}\right)$$

$$595 = P\left(100 + \frac{15}{100}\right)$$

$$595 = P\left(\frac{115}{100}\right)$$

$$595 = P(1.15)$$

$$\frac{595}{1.15} = P$$

$\therefore$ Present value, $P = Rs.517.39$

True discount which is nothing but interest on P,

$$\underline{T} = A - P$$

$$= 595 - 517.39$$

$$= Rs.77.61$$

13) Shiva lents Rs. 1200 to Vinayaga and a certain sum of money to muruga at the same time at 12% per annum. If in 3 years, shiva receives all together Rs. 648 as interest from the two. find the sum borrowed by muruga.

Given

$P = 1200$

$r = 12\%$

$n = 3$ for vinayaga

Interest due from vinayaga $= \frac{Pnr}{100}$

$= \frac{1200 \times 12 \times 3}{100}$

$= Rs. 432$

Interest paid by muruga $= 648 - 432$

$= Rs. 216$

∴ The sum borrowed by muruga

$P = \frac{100I}{nr}$

$= \frac{100 \times 216}{3 \times 12}$

$= 600$

COMPOUND INTEREST :

Formulae :

1) $A = P\left(1 + \frac{r}{100}\right)^n$

2) $CI = A - P$

3) $CI = P\left[\left(1 + \frac{r}{100}\right)^n - 1\right]$

4) $P = \frac{A}{\left(1 + \frac{r}{100}\right)^n}$

5) $r = 100\left[\left(\frac{A}{P}\right)^{1/n} - 1\right]$

$$n = \frac{\log A - \log P}{\log\left(1 + \frac{r}{100}\right)}$$

One Mark :

Note :- Compound interest is more than the simple interest.

1) Find the compound interest on Rs 20,000 for 5 years at 20% per annum. What will be the simple interest in the above case ?

Given,

$P = 20,000$

$n = 5$

$r = 20\%$.

$$CI = P\left[\left(1+\frac{r}{100}\right)^n - 1\right]$$

$$= 20,000\left[\left(1+\frac{20}{100}\right)^5 - 1\right]$$

$$= 20,000\left[\left(\frac{120}{100}\right)^5 - 1\right]$$

$$= 20,000\left[(1.2)^5 - 1\right]$$

$$= 20,000\left[2.48832 - 1\right]$$

$$= 20,000\,(1.488)\,(1.48832)$$

$$= Rs.\ 29,766.4$$

$$I = \frac{Pnr}{100}$$

$$= \frac{20,000 \times 5 \times 20}{100}$$

$$= Rs\ 20,000$$

2) Find the compound interest for Rs. 2500 for 4 years at 8% pa annum.

$P = 2500$

$n = 4$

$r = 8\%$

$$CI = P\left[\left(1+\frac{r}{100}\right)^n - 1\right]$$

$$= 2500\left[\left(1+\frac{8}{100}\right)^4 - 1\right]$$

$$= 2500\left[\left(\frac{108}{100}\right)^4 - 1\right]$$

$$= 2500\left[(1.08)^4 - 1\right]$$

$$= 2500\left[(1.36048 - 1\right]$$

$$= 2500\left[0.36048\right]$$

$$= 901.2$$

Calculate the compound interest in the above case when interest is compounted (A) halfyearly (B) Quarterly

a) Half yearly

$P = 2500$

$n = 8 \rightarrow 4 \times 2$

$r = 4\% \rightarrow 8/2$

$$CI = P\left[\left(1+\frac{r}{100}\right)^{n}-1\right]$$

$$= 2500\left[\left(1+\frac{8}{100}\right)^{4}-1\right]$$

$$= 2500\left[\left(\frac{104}{100}\right)^{8}-1\right]$$

$$= 25000\left[(1.04)^{8}-1\right]$$

$$= 2500\left[(1.04)^{8}-1\right]$$

$$= 2500\left[(1.36856-1)\right]$$

$$= 2500 \times 0.368568$$

$$= 921.42$$

b) Quarterly

$P = 2500$

$n = 16 \longrightarrow 4 \times 4$

$r = 2\% \rightarrow 8/4$

$$CI = 2500\left[\left(1+\frac{2}{100}\right)^{16}-1\right]$$

$$= 2500\left[\left(\frac{102}{100}\right)^{16}-1\right]$$

$$= 2500\left[(1.02)^{16}-1\right]$$

$= 2500\left[(1.372785)-1\right]$

$= 2500\left[(0.372785)\right]$

$= 931.96$

3) Balu borrowed Rs 25000 from Rathnam but could not repay the amount in a period of 5 years. Accordingly, Rathnam demands now Rs. 35,880 from Balu. At what Percentage per annum compound interest did Rathnam lend his money?

Given,

$P = 25,000$

$n = 5$ years

$A = 35,880$

$r = 100\left[\left(\frac{A}{P}\right)^{1/n} - 1\right]$

$= 100\left[\left(\frac{35880}{25,000}\right)^{1/5} - 1\right]$

$= 100\left[(1.4352)^{1/5} - 1\right]$

$= 100\left[(1.4352)^{0.2} - 1\right]$

$= 100\left[1.074935696 - 1\right]$

$= 100\left[0.074935696\right]$

$= 7.4935 = 7.49\%$

4) Mr. Ambikabathi borrows Rs 1,00,000 at 24% compounted monthly. Find the amount he has to repay at the end of 3 years

Given

$P = Rs.\ 1,00,000$

$r = \frac{24\%}{12} = 2$

$n = 3 \times 12$

$= 36$ months

The amount he has to repay

$A = P\left(1 + \frac{r}{100}\right)^n$

$= 1,00,000\left(1 + \frac{2}{100}\right)^{36}$

$= 1,00,000\left(\frac{102}{100}\right)^{36}$

$= 1,00,000\left(1.02\right)^{36}$

$= 1,00,000\left(2.03\right)$

$= Rs.\ 2,03,988.54$

5) The difference between the Compound and the simple interest for 3 years at 5% per annum on a certain sum of money was Rs. 610. Find the sum?

Given,

$CI - S.I = 610$

$n = 3$ years

$r = 5\%$

$$CI - I = P\left[\left(1+\frac{r}{100}\right)^{n}-1\right] - \left(\frac{Pnr}{100}\right)$$

$$610 = P\left[\left(1+\frac{5}{100}\right)^{3}-1\right] - \left(\frac{P\times 3\times 5}{100}\right)$$

$$610 = P\left[\left(\frac{105}{100}\right)^{3}-1\right] - \left(\frac{15P}{100}\right)$$

$$610 = P\left[(1.05)^{3}-1\right] - 0.15P$$

~~$610 = P[1 \cdots P[$~~

~~$610 = 0.15P$~~

$$610 = (1.157625-1) - 0.15P$$

$$610 = (0.157625) - 0.15P$$

$$610 = 0.007625$$

$$\frac{610}{0.007625} = P$$

$\therefore P = Rs, 80,000$

6) The difference between Compound & Simple interest Rs 121. Number of years 5. Rate of interest 8%. Find the principle.

Given,

$CI - SI = 121$

$n = 5$ years

$r = 8\%$

$$CI - I = P\left[\left(1+\frac{r}{100}\right)^n - 1\right] - \left(\frac{Pnr}{100}\right)$$

$$121 = P\left[\left(1+\frac{8}{100}\right)^5 - 1\right] - \left(\frac{P\times 5\times 8}{100}\right)$$

$$121 = P\left[\left(\frac{108}{100}\right)^5 - 1\right] - \left(\frac{40P}{100}\right)$$

$$121 = P\left[\left(1.08\right)^5 - 1\right] - 0.4P$$

$$121 = P\left(1.469328077 - 1\right) - 0.4P$$

$$121 = 0.469328076\ P - 0.4P$$

$$121 = 0.069328076$$

$$\frac{121}{0.219328} = P \qquad \frac{121}{0.069328076} = P$$

$$= 1745$$

7) A certain sum deposited in a bank at 15% per annum compounded monthly amounts to Rs. ~~42,000~~ 42,143.63 at the end of 5 years find the principle.

Given

$r = \frac{15}{12} = 1.25$

$n = 5 \times 12 = 60$ months

$A = 42,143.63$

$$P = \frac{A}{\left(1 + \frac{r}{100}\right)^n}$$

$$= \frac{42,143.63}{\left(1 + \frac{1.25}{100}\right)^{60}}$$

$$= \frac{42,143.63}{\left(\frac{101.25}{100}\right)^{60}}$$

$$= \frac{42,143.63}{(1.0125)^{60}}$$

$$= \frac{42,143.63}{2.107181947}$$

$= 20,000.0014 5$

$= Rs. 20,000$

8) If a term deposit Rs. 12,500 yields an interest of Rs 5,088.75 at 10% compound interest compounded half yearly, find the Period of the term deposit.

Given

$P = 12,500$

$CI = 5,088.75$

$r = \frac{10\%}{2} = 5$

$n = ?$

$$n = \frac{\log A - \log P}{\log\left(1 + \frac{r}{100}\right)}$$

$CI = A - P$

$5,088.75 = A - 12,500$

$5,088.75 + 12,500 = A$

$A = Rs.\ 17,588.75$

$$= \frac{\log(17,588.75) - \log(12,500)}{\log\left(1 + \frac{5}{100}\right)}$$

$$= \frac{4.2452 - 4.0969}{\log(1.05)}$$

$$= \frac{0.1483}{0.0211}$$

$= 7.028436$

$= 7$ months $\therefore n = 3.5$ years

23) Find the sum of money that yields a compound interest of Rs. 432 at 8% per annum during the second year.

Given

$CI = 432$

$r = 8\%$

$P = ?$

$$CI = P\left[\left(1 + \frac{r}{100}\right)\frac{r}{100}\right]$$

$$432 = P\left(1 + \frac{8}{100}\right)\frac{8}{100}$$

$$432 = P\left(\frac{108}{100}\right)\frac{8}{100}$$

$$P = 432 \times \frac{108}{100} \times \frac{8}{100}$$

$$P = 5000$$

$$P = 432 \times \frac{100}{108} \times \frac{100}{8}$$

$$P = 5000$$

What amount lent at 10% pa annum compound interest will fetch Rs 630 as interest in 2 years.

Given $r = 10\%$, $CI = 630$ $n = 2$ years

$P = ?$

$$CI = P\left[1 + \frac{r}{100}\right]^n - 1$$

$$P = \frac{CI}{\left(1 + \frac{r}{100}\right)^n - 1}$$

$$= \frac{630}{\left(1 + \frac{10}{100}\right) - 1}$$

$$= \frac{630}{\left(\frac{110}{100}\right) - 1}$$

$$= \frac{630}{(1.1)^2 - 1}$$

$$= \frac{630}{1.21 - 1} = \frac{630}{0.21}$$

$P = Rs.\ 3000$

Show that the present value of Rs. 500 due in the 4 years at 3% compounded semi annualy in Rs. 444. Approximately

Given,

$A = 500$

$n = 4 \times 2 = 8$

$r = \frac{3}{2} = 1.5$

$$P = \frac{A}{\left(1 + \frac{r}{100}\right)^n}$$

$$= \frac{500}{\left(1 + \frac{1.5}{100}\right)^8}$$

$$= \frac{500}{(1.015)^8}$$

$$= \frac{500}{1.126492587}$$

$$= 443.86$$

$$= 444 \text{ //}$$

A sum of money invested at compound interest amount to Rs. 21632.00 in 2 years and ~~two~~ to Rs. 22497.28 in 3 years find the rate of interest and sum invested.

$P = 21632.00$

$A = 22497.28$

$r = ?$

$r = \frac{100 I}{Pn}$

$I = A - P$

$= 22497.28 - 21632$

$= 865.28$

$r = \frac{100 \times 865.28}{21632 \times 1}$

$= \frac{86528}{21632}$

$= 4\%$

$P = \frac{A}{\left(1 + \frac{r}{100}\right)^n}$ $n = 2$

$= \frac{21632}{\left(1 + \frac{4}{100}\right)^2}$

$$= \frac{21632}{\left(\frac{104}{100}\right)^2}$$

$$= \frac{21632}{(1.04)^2}$$

$$= \frac{21632}{1.0816}$$

$$= ₹20,000$$

B.P. Balaraman deposites Rs. 12000 in Pandurangan associates and gets Rs. 27566.93 at the end of 3 1/2 years. Find the rate of compound interest which the company face per month.

Given,

$P = 12,000$

$A = 27566.93$

$n = 3\ 1/2$ years $= \frac{7}{2} \times 12 = 42$ months

$r = ?$

For finding the monthly rate of interest, consider $r_1 = \frac{r}{12}$

$$r = 100\left[\left(\frac{A}{P}\right)^{1/n} - 1\right]$$

$$\frac{r}{12} = 100\left[\left(\frac{27566.93}{12000.}\right)^{1/42} - 1\right]$$

$$\frac{r}{12} = 100\left[(2.29724\,41)^{1/42} - 1\right]$$

$$\frac{r}{12} = 100(1.019999995 - 1)$$

$$\frac{r}{12} = 100(0.01999995)$$

$$\frac{r}{12} = 1.999995$$

$$\frac{r}{12} = 2$$

$$r = 12 \times 2$$

$$r = 24\% \text{ per annum.}$$

AMOUNT AT THE END OF PERIOD

An industry starts by producing 50,000 units in its first year and the production for every year increases by 8% of the previous year. How many units will it produce in the seventh year? What is the sum total of the whole production in the first 3 years? 8 marks

Although the context is different, the increase in production in every year is similar to that of compound interest.

Number of units likely to be produced in the 7th year = Amount at the end of the 6th year

$= P\left(1+\frac{r}{100}\right)^n$

$= 50,000\left(1+\frac{8}{100}\right)^6 \quad (n=6)$

$= 50,000\left(\frac{108}{100}\right)^6$

$= 50,000\ (1.08)^6$

$= 50,000\ (1.5868)$

$= 79343.71615$

$= Rs.79344$

The total production in the first 3 years =

$P\left[1+\frac{r}{100}\right)^0 + P\left(1+\frac{r}{100}\right)^1 + P\left(1+\frac{r}{100}\right)^2$

$= P + P\left(1+\frac{r}{100}\right) + P\left(1+\frac{r}{100}\right)^2$

$= 50,000 + 50,000\left(1+\frac{8}{100}\right) + 50,000\left(1+\frac{8}{100}\right)^2$

$= 50,000 + 50,000\left(\frac{108}{100}\right) + 50,000\left(\frac{108}{100}\right)^2$

$= 50,000 + 50,000\ (1.08) + 50,000\ (1.08)^2$

$= 50,000 + 54,000 + 50,000\ (1.1664)$

$= 50,000 + 54,000 + 58320$

$= Rs.\ 1,62,320$

2) A person has two daughter A, B aged 13 and 16 years. He has Rs. 40,000 with him now but wants that both of them should get an equal amount when they are 20 years old. How he should divide the money if it have to be

deposited in a bank giving 9% compound interest per annum?

Given, $r = 9\%$.

Let P be the amount deposited for a for 7 years.

$(40,000 - P)$ be the amount deposited for B for ~~four~~ 4 years.

Amount due for $A = P\left(1 + \frac{r}{100}\right)^n$

$= P\left(1 + \frac{9}{100}\right)^7$

$= P(1.09)^7$

$= 1.8280391\ P$ —— (1)

Amount due for $B = P\left(1 + \frac{r}{100}\right)^n$

$= (40,000 - P)\left(1 + \frac{9}{100}\right)^4$

$= (40,000 - P)(1.09)^4$

$= (40,000 - P)(1.4115816)$

$= 56463.264 - 1.4115816\ P$ —— (2).

Equating equation (1) & (2) we get

$\Rightarrow 1.8280391\ P = 56,463.264 - 1.4115816\ P$

$\Rightarrow 1.8280391\ P + 1.4115816\ P = 56,463.264$

$\Rightarrow 3.2396207\ P = 56,463.264$

$P = \frac{56,463.264}{3.2396207}$

$P = Rs.\ 17,428.97371$

$P = Rs.\ 17429$

$\therefore$ A share = 17,429

$\therefore$ B share = (40,000 - 17,429)

= 22571

EMI (Equaled Monthly Instalment)

It is calculated by two methods.

1. ~~Hat~~ flat grate method.
2. Reducing interest method.

Flat rate method

1) $EMI = \frac{P + I}{\text{Period in month}}$

Reducing interest method.

1) $EMI = (L \times I)\left[\frac{(1+I)^n}{(1+I)^n - 1}\right]$

1) Calculate the EMI by the flat rate method given

P = 1,50,000, n = 3yrs, r = 10%.

$EMI = \frac{P+I}{\text{Period of months}}$

$I = \frac{Pnr}{100} = \frac{1,50,000 \times 3 \times 10}{100}$

= Rs. 45000

$EMI = \frac{1,50,000 + 45000}{36}$

= 5,416.66667

= Rs. 5417

Reducing Interest method:

1) Calculate the EMI by Reducing interest method of Rs. 1,00,000, the rate of Interest = 11% at 15 years.

Given

$L = 1,00,000$

$r(I) = 11\% \cdot \frac{11}{100 \times 12} = 9.1666 \times 10^{-3} = 0.000916$

$n = 15 \text{ years} = 15 \times 12 = 180 \text{ months}$

$$EMI = (L \times I)\left[\frac{(I+I)^n}{(L+I)^n - 1}\right]$$

$$= (1,00,000 \times 0.000916)\left[\frac{(1+0.00916)^{180}}{(1+0.00916)^{108} - 1}\right]$$

$$= 91.6\left[\frac{(1.000916)^{180}}{(1.000916)^{180} - 1}\right]$$

$$= 91.6\left(\frac{1.179162606}{0.179162606}\right)$$

$$= 91.6\left(\frac{1.179}{0.179}\right)$$

$$= 91.6\,(6.5865921)$$

$$= 91.6\,(6.586)$$

$$= Rs.\ 603.2776$$

(+) a, d.

Airthematic Progression

a, a+d, a+2d, ...

If the successive terms increase or decrease by a constant the series is called Airthematic Progression.

The standaid form of airthmetic Progression a, a+d, a+2d, ...

The General term of an airthmetic progression is $t_n = a+(n-1)d$

Eg: 1

The 4th and 7th term of an airthmetic progression are 3 and 36 find the airthmetic Progression and its 15th term.

Sol:

4^{th} term $= a+3d = 3$ — ①

7^{th} term $= a+6d = 36$ — ②

2 - 1

$3d = 33$

$d = 33/3$

$d = 11$

by substitute in equation ①

$a + 3 \times 11 = 3$

$a = 3 - 33$

$a = -30$

S.F

$a, a+d, a+2d$

$-30, -30+11, -30+2\times11$

$-30, -19, -8$

$t_n = a + (n-1)\,d$

$t_{15} = -30 + (15-1)\,11$

$t_{15} = -30 + 14 \times 11$

$= 154 - 30$

$= 124$

Eg: 2

In an aithmetic series the 7th and 9th term are respectively 16 and 20. find the n-th term.

sol:-

7th term $= a + (7-1)d = 16$

$= a + 6d = 16$ —— ①

9th term $= a + (9-1)d = 20$

$= a + 8d = 20$ —— ②

② − ①

$a + 6d = 16$

$a + 8d = 20$

(−) (−) (−)

$-2d = -4$

$d = 2$.

Substituting eq ①.

$a + 6d = 16$

$a + 12 = 16$

$a = 16 - 12$

$a = 4$.

$t_n = a + (n-1)d$

$= 4 + (n-1)2$

$= 4 + 2n - 2$

$= 2n + 2$

S.F

$a, a+d, a+2d$

$4, 4+2\times2, 4+2\times2$

$4, 6, 8$

Eg: 3

Find the last term of series

$7+14+21+\ldots$ 20th term.

Sol:-

$t_{50} = t_n = a+(n-1)d$

$t_{20} = 7+(20-1)7$

$t_{20} = 7+19\times7$

$= 7+133$

$= 140$

a = First term
d = difference

Eg

Eg: 4

The sum of 3 numbers in airthmetic progression is 24, and their product is 440 Find the numbers.

Sol:

$(a-d), a, (a+d)$

$(a-d) + a + (a+d) = 24$

$3a = 24$

$a = 24/3$

$a = 8$

$(a-d)\,8\,(a+d) = 440$

$(8-d)\,8\,(8+d) = 440$

$64 - d^2 = \frac{440}{8}$

$64 - d^2 = 55$

$d^2 = 64 - 55$

$d^2 = 9$

$d = 3$.

$= (8-3), 8, (8+3)$

$= 5, 8, 11$

The three numbers are 5, 8, 11

Eg:6

Find the 50th term of the progression.

2, 5, 8, 11, . . .

$$t_n = a + (n-1)d$$

$$t_{50} = (2 + 49)3$$

$$= 2 + 147$$

$$t_{50} = 149.$$

$$\begin{array}{r} 2 \\ 49 \times 3 \\ \hline 147 \end{array}$$

Eg:7

Find the 20th term of the series 4, 9, 14 . . .

$$t_n = a + (n-1)d$$

$$t_{20} = 4 + (20-1)5$$

$$t_{20} = 4 + (19)5$$

$$t_{20} = 4 + 95$$

$$t_{20} = 99$$

Eq: 8

$\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in airthmetic progression. prove that a^2, b^2, c^2 are also in airthmetic progression

Sol:-

If a^2, b^2, c^2 are ~~to~~ be in airthmetic progression ~~$b^2 - a^2 =$~~

$b^2 - a^2 = c^2 - b^2$ is to be true.

given:

$$\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$$

$$\frac{1}{c+a} - \frac{1}{b+c} = \frac{1}{a+b} - \frac{1}{c+a}$$

$$\frac{(b+c)-(c+a)}{(c+a)(b+c)} = \frac{(c+a)-(a+b)}{(a+b)(c+a)}$$

$$\frac{b-a}{(c+a)(b+c)} = \frac{c-b}{(a+b)(c+a)}$$

multiply both sides $(c+a)(b+c)(a+b)$

$$\frac{b-a}{(c+a)(b+c)}(c+a)(b+c)(a+b) = \frac{c-b}{(a+b)(c+a)}(c+a)(b+c)(a+b)$$

$$(b-a)(a+b) = (c-b)(b+c)$$

$$b^2 - a^2 = c^2 - b^2$$

Eg: 9

If $\frac{1}{y+z}, \frac{1}{z+x}, \frac{1}{x+y}$ are in airthmetic progression Prove that x^2, y^2, z^2 are also in airthmetic progression

sol:

$$\frac{1}{z+x} - \frac{1}{y+z} = \frac{1}{x+y} - \frac{1}{z+x}$$

$$\frac{(y+z)-(z+x)}{(z+x)(y+z)} = \frac{(z+x)-(x+y)}{(x+y)(z+x)}$$

$$\frac{(y+z)-(z+x)}{(z+x)(y+z)} = \frac{(z+x)-(x+y)}{(x+y)(z+x)}$$

$$\frac{y-x}{(z+x)(y+z)} = \frac{z-y}{(x+y)(z+x)}$$

multiply both sides by

$$(z+x)(y+z)(x+y)$$

$$\frac{y-x}{(z+x)(y+z)} \cdot (z+x)(y+z)(x+y) =$$

$$\frac{z-y}{(x+y)(z+x)} \cdot (z+x)(y+z)(x+y)$$

$$(y-x)(x+y) = (z-y)(y+z)$$

$$\therefore x^2 - y^2 = z^2 - y^2.$$

~~NSS~~

Geometric Progression

If the successive terms increase or decrease by a constant factor the series is called ~~Geometric Prog~~ Geometric Progression. the common ratio and is denote by ~~a~~ r.

The standard form of a Geometric progression $a, ar, ar^2, \ldots$

When $r > 1$ the series is an increasing Geometric progression

When $r < 1$ the series is decreasing Geometric progression

General term of Geometric progression $t_n = ar^{n-1}$

Eg:

If the 3rd term and 7th term of a Geometric progression are 2 and 1/8. Find the G.P. and it is 10th term

Sol:-

$t_3 = ar^2 = 2 \quad -(1)$

$t_7 = ar^6 = 1/8 \quad -(2)$

$\therefore (2) \div (1)$

$r^4 = 1/16$

$r^4 = 1/2^4$

$r = 1/2$

$r = 1/2$ in (1)

$a(1/2)^2 = 2$

$a(1/4) = 2$

$a = 2 \times 4$

$a = 8$

$t_{10} = ar^9$

$= 8.(1/2)^9$

$= 8\left(\frac{1}{512}\right)$

$= \frac{8}{64}$

$= 64$

Eg:

Find the number of terms in Geometric series $0.03 + 0.06 + 0.12 + \dots + 1.92$

Sol:-

$a = 0.03$

$r = 0.06 \div 0.03$

$r = 2$.

$\therefore t_n = 1.92$

$ar^{n-1} = 1.92$

$(0.03)(2^{n-1}) = 1.92$

$2^{n-1} = \frac{1.92}{0.03}$

$2^{n-1} = 64$

$2^{n-1} = 2^6$

$n - 1 = 6$

$n = 6 + 1$

$n = 7$.