Definition of a Matrix:-

A Matrix is an arrangement of elements (or) numbers in rows and columns. The numbers are enclosed by parenthesis (or) brackets (or) Double bars.

For example:-

$$\begin{bmatrix} 1 & 5 & 9 \\ 3 & 7 & 6 \\ 4 & 14 & 19 \end{bmatrix} \quad \begin{Vmatrix} 13 & 24 \\ 23 & 21 \end{Vmatrix} \quad \begin{bmatrix} 0 & 15 \\ -3 & 32 \\ 7 & 41 \end{bmatrix}$$

General Form of a matrix

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1j} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2j} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots & & \vdots \\ a_{i1} & a_{i2} & & a_{ij} & & a_{in} \\ \vdots & \vdots & & \vdots & & \\ a_{m1} & a_{m2} & & a_{mj} & & a_{mn} \end{bmatrix}$$

Types of matrices:- (Square matrix)

The number of rows and the number of columns of a matrix are equal the matrix is called square matrix.

Example:-

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \quad \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

(ii) Row matrix :-

If there is only one row in a matrix it is called a row matrix or row vector.

Example :-

$$A = \begin{bmatrix} 10 & 32 & 50 \end{bmatrix}$$

(iii) Column matrix :-

If there is only one column in a matrix it is called a column matrix or column vector.

Example :-

$$A = \begin{bmatrix} 5 \\ 10 \end{bmatrix}$$

(iv) Zero (or) Null matrix :-

If all the elements of a matrix are zero it is called a zero matrix and it is denoted by zero. (or) null matrix

Example :-

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(v) Equal Matrices :-

Two matrices $A = (a_{ij})$ and $B = (b_{ij})$ are equal $A = B$.

1. For equal $A = B$, we have same order $A = (a_{ij})\ m/n$ and $B = (b_{ij})\ m/n$

2. If elements at the Corresponding places are equal that is $(a_{ij}) = (b_{ij})$

For every i and j

$$A = \begin{bmatrix} 2 & 4 & 6 \\ 5 & 7 & 9 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 4 & 6 \\ 5 & 7 & 9 \end{bmatrix}$$

(vi) Equivalent matrix :-

Two matrices A and B of the same order are said to be equivalent if one of them can be obtained from the other by elementary transformation.

A equivalent to B.

$A \sim B$ → tild symbol denotes equivalent matrix.

Example 1:-

If $A = \begin{bmatrix} 4 & 6 & 9 \\ 3 & 5 & 10 \end{bmatrix} \quad B = \begin{bmatrix} 5 & 0 & 1 \\ 4 & -7 & -3 \end{bmatrix}$

$$A + B \begin{bmatrix} 4+5 & 6+0 & 9+1 \\ 3+4 & 5+(-7) & 10+(-3) \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 6 & 10 \\ 7 & -7+5 & 10-3 \end{bmatrix}$$

$$A+B = \begin{bmatrix} 9 & 6 & 10 \\ 7 & -2 & 7 \end{bmatrix}$$

$$B + A \begin{bmatrix} 5+4 & 0+6 & 1+9 \\ 4+3 & -7+5 & -3+10 \end{bmatrix}$$

$$B+A = \begin{bmatrix} 9 & 6 & 10 \\ 7 & -2 & 7 \end{bmatrix}$$

Therefore

$$A+B = B+A$$

$$\begin{bmatrix} 9 & 6 & 10 \\ 7 & -2 & 7 \end{bmatrix} = \begin{bmatrix} 9 & 6 & 10 \\ 7 & -2 & 7 \end{bmatrix}$$

Hence proved.

Example - 2 :-

If $A = \begin{bmatrix} 4 & 6 & 9 \\ 3 & 5 & 10 \end{bmatrix}$ $B = \begin{bmatrix} 5 & 0 & 1 \\ 4 & -7 & -3 \end{bmatrix}$

$$A - B \begin{bmatrix} 4+(-5) & 6-0 & 9-1 \\ 3-4 & 5(-7) & 10-(-3) \end{bmatrix}$$

$$\begin{bmatrix} -1 & 6 & 8 \\ -1 & 12 & 13 \end{bmatrix}$$

$$B - A \begin{bmatrix} 5-4 & 0-6 & 1-9 \\ 4-3 & -7-5 & -3-10 \end{bmatrix} = \begin{bmatrix} 1 & -6 & -8 \\ 1 & -12 & -13 \end{bmatrix}$$

$A - B \neq B - A.$

Example 3:-

$$A = \begin{bmatrix} 4 \\ -1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 4 & -1 & 0 \\ -3 & 5 & -6 \\ 2 & -7 & 8 \end{bmatrix} \quad B = \begin{bmatrix} -1 & 0 & 1 \\ 5 & -2 & 2 \\ 3 & 4 & 3 \end{bmatrix}$$

$$A + B \begin{bmatrix} 4+(-1) & -1+0 & 0+1 \\ -3+5 & 5+(-2) & -6+2 \\ 2+3 & -7+4 & 8+3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -1 & 1 \\ 2 & 3 & -4 \\ 5 & -3 & 11 \end{bmatrix}$$

$$B + A \begin{bmatrix} -1+4 & 0+(-1) & 1+0 \\ 5+(-3) & -2+5 & 2+(-6) \\ 3+2 & 4+(-7) & 3+8 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -1 & 1 \\ 2 & 3 & -4 \\ 5 & -3 & 11 \end{bmatrix}$$

$$A + B = B + A$$

Doubt

$$A - B \begin{bmatrix} 4-(-1) & -1-0 & 0-1 \\ -3-5 & 5-(-2) & -6-2 \\ 2-3 & -7-4 & 8-3 \end{bmatrix}$$

$$\begin{bmatrix} 5 & -1 & -1 \\ -8 & 7 & -8 \\ -1 & -11 & 5 \end{bmatrix}$$

$$B - A \begin{bmatrix} -1-4 & 0-(-1) & 1-0 \\ 5-(-3) & -2-5 & 2-(-6) \\ 3-2 & 4-(-7) & 3-8 \end{bmatrix}$$

$$\begin{bmatrix} -5 & 1 & 1 \\ 8 & -7 & 8 \\ 1 & 11 & -5 \end{bmatrix}$$

$$A - B \neq B - A$$

(vii) Scalar Multiplication :-

$$A = \begin{bmatrix} 0 & 4 \\ 2 & 5 \\ -7 & 9 \end{bmatrix} \quad 3 \begin{bmatrix} 0 & 4 \\ 2 & 5 \\ -7 & 9 \end{bmatrix} = \begin{bmatrix} 0 & 12 \\ 6 & 15 \\ -21 & 27 \end{bmatrix}$$

Find $3a$, $-4a$

$$-4 \begin{bmatrix} 0 & 4 \\ 2 & 5 \\ -7 & 9 \end{bmatrix} = \begin{bmatrix} 0 & -16 \\ -8 & -20 \\ 28 & -36 \end{bmatrix}$$

Home Work :-

1) If $A = \begin{bmatrix} 2 & 3 \\ 10 & 15 \end{bmatrix}$ $B = \begin{bmatrix} 4 & 7 \\ 8 & 24 \end{bmatrix}$

find $A + B$, $A - B$, $2A + 3B$

2) If $A = \begin{bmatrix} 3 & 1 & 5 \\ 2 & 6 & 4 \\ 8 & 7 & 9 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 2 & 4 \\ 1 & 5 & 3 \\ 2 & 4 & 0 \end{bmatrix}$

Find the value of $5A$, $-2A + 3B$

1) Example - 1 :-

If $A = \begin{bmatrix} 2 & 3 \\ 10 & 15 \end{bmatrix}$ $B = \begin{bmatrix} 4 & 7 \\ 8 & 24 \end{bmatrix}$

Soln :-

(i) $A = \begin{bmatrix} 2 & 3 \\ 10 & 15 \end{bmatrix}$ $B = \begin{bmatrix} 4 & 7 \\ 8 & 24 \end{bmatrix}$

$$A + B = \begin{bmatrix} 2+4 & 3+7 \\ 10+8 & 15+24 \end{bmatrix}$$

$$A+B \Rightarrow \begin{bmatrix} 6 & 10 \\ 18 & 39 \end{bmatrix}$$

(ii) $A - B$

$A = \begin{bmatrix} 2 & 3 \\ 10 & 15 \end{bmatrix}$ $B = \begin{bmatrix} 4 & 7 \\ 8 & 24 \end{bmatrix}$

$$A - B = \begin{bmatrix} 2-4 & 3-7 \\ 10-8 & 15-24 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -4 \\ 2 & -9 \end{bmatrix}$$

(iii) $2A + 3B$

$A = \begin{bmatrix} 2 & 3 \\ 10 & 15 \end{bmatrix}$

$$= 2 \begin{bmatrix} 2 & 3 \\ 10 & 15 \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ 20 & 30 \end{bmatrix}$$

$$B = \begin{bmatrix} 4 & 7 \\ 8 & 24 \end{bmatrix}$$

$$= 3\begin{bmatrix} 4 & 7 \\ 8 & 24 \end{bmatrix} = \begin{bmatrix} 12 & 21 \\ 24 & 72 \end{bmatrix}$$

$2A + 3B =$

$$2A \begin{bmatrix} 4 & 6 \\ 20 & 30 \end{bmatrix} + 3B \begin{bmatrix} 12 & 21 \\ 24 & 72 \end{bmatrix} = \begin{bmatrix} 4+12 & 6+21 \\ 20+24 & 30+72 \end{bmatrix}$$

$$= \begin{bmatrix} 16 & 27 \\ 44 & 102 \end{bmatrix}$$

Example - 2 :-

2) If $A = \begin{bmatrix} 3 & 1 & 5 \\ 2 & 6 & 4 \\ 8 & 7 & 9 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 2 & 4 \\ 1 & 5 & 3 \\ 2 & 4 & 0 \end{bmatrix}$

(i) $5A$

$$A = \begin{bmatrix} 3 & 1 & 5 \\ 2 & 6 & 4 \\ 8 & 7 & 9 \end{bmatrix}$$

$$= 5\begin{bmatrix} 3 & 1 & 5 \\ 2 & 6 & 4 \\ 8 & 7 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 15 & 5 & 25 \\ 10 & 30 & 20 \\ 40 & 35 & 45 \end{bmatrix}$$

(ii) $-2A + 3B$

$$A = \begin{bmatrix} 3 & 1 & 5 \\ 2 & 6 & 4 \\ 8 & 7 & 9 \end{bmatrix} \Rightarrow -2\begin{bmatrix} 3 & 1 & 5 \\ 2 & 6 & 4 \\ 8 & 7 & 9 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -6 & -2 & -10 \\ -4 & -12 & -8 \\ -16 & -14 & -18 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 2 & 4 \\ 1 & 5 & 3 \\ 2 & 4 & 0 \end{bmatrix}$$

$$3 \begin{bmatrix} 1 & 2 & 4 \\ 1 & 5 & 3 \\ 2 & 4 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 6 & 12 \\ 3 & 15 & 9 \\ 6 & 12 & 0 \end{bmatrix}$$

$-2A + 3B.$

$$\Rightarrow \begin{bmatrix} -6 & -2 & -10 \\ -4 & -12 & -8 \\ -16 & -14 & -18 \end{bmatrix} + \begin{bmatrix} 3 & 6 & 12 \\ 3 & 15 & 9 \\ 6 & 12 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -6+3 & -2+6 & -10+12 \\ -4+3 & -12+15 & -8+9 \\ -16+6 & -14+12 & -18+0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -3 & 4 & 2 \\ -1 & 3 & 1 \\ -10 & -2 & -18 \end{bmatrix}$$

7) Diagonal Matrix:-

A square matrix of all the elements except a diagonals are zero is called diagonal matrix.

For ex :-

$$\begin{bmatrix} 4 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 8 \end{bmatrix}$$

8) Scalar Matrix :-

A diagonal matrix in which all the elements in the diagonal are called a scalar matrix.

The matrix $A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$

9) Unit Matrix or Identity Matrix :-

A square matrix whose diagonal elements are & each non - diagonal elements are 0 is called a Unit matrix (or) Identity matrix

For ex :-

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

10) Symmetric Matrix :-

A square matrix such that $a_{ij} = a_{ji}$ for all i and j is called symmetric matrix.

$$A' = A$$

where A' is a transpose of A and it has defined the matrix Operation I.

11) Skew Symmetric Matrix :-

A square matrix such that $a_{ij} = - a_{ij}$ for all i and j is called a skew symmetric matrix (ie) $A' = -A$.

12) Sub matrix :-

A matrix obtained by delete one or more rows and one or more columns is called a sub matrix.

For ex :-

$$\begin{bmatrix} 5 & 4 & 9 \\ 6 & 10 & 8 \end{bmatrix} \begin{bmatrix} 5 & 4 \\ 6 & 10 \end{bmatrix} \begin{bmatrix} 4 & 9 \\ 10 & 8 \end{bmatrix} \begin{bmatrix} 5 & 4 \end{bmatrix}$$

13) Orthogonal matrix :-

A square matrix A is said to be an orthogonal matrix.

$$A'A = AA' = I$$

14) Non - Singular matrix :-

A square matrix A is said to be non - singular if $|A| \neq 0$. $|A|$ denotes a singular matrix is $|A| = 0$.

Example - 1 :-

If $A = \begin{bmatrix} 7 & 8 \\ 6 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 5 & 8 \\ 9 & 4 \end{bmatrix}$ find $A+B$, $A-B$.

$A = \begin{bmatrix} 7 & 8 \\ 6 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 5 & 8 \\ 9 & 4 \end{bmatrix}$

$2 \times \boxed{2 \quad 2} \times 2$ matrix

$$A+B = \begin{bmatrix} 7+5 & 8+8 \\ 6+9 & 4+4 \end{bmatrix}$$

$$= \begin{bmatrix} 12 & 16 \\ 15 & 8 \end{bmatrix}$$

$$A - B = \begin{bmatrix} 7-5 & 8-8 \\ 6-9 & 4-4 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 \\ -3 & 0 \end{bmatrix}$$

Scalar Matrix :-

Ex 1 :- If $A = \begin{bmatrix} 0 & 4 \\ 2 & 5 \\ -7 & 9 \end{bmatrix}$

$$3A = \begin{bmatrix} 0 & 12 \\ 6 & 15 \\ -21 & 27 \end{bmatrix}$$

Ex 2 :-

If $A = \begin{bmatrix} 0 & 4 \\ 2 & 5 \\ -7 & 9 \end{bmatrix}$ find $-4a$

$$-4A \begin{bmatrix} 0 & 4 \\ 2 & 5 \\ -7 & 9 \end{bmatrix}$$

$$-4 \begin{bmatrix} 0 & -16 \\ -8 & -20 \\ 28 & -36 \end{bmatrix}$$

Ex 3 :-

If $A = \begin{bmatrix} 3 & 5 \\ 2 & a \end{bmatrix}$ $B = \begin{bmatrix} 4 & b \\ 2 & 9 \end{bmatrix}$ $C = \begin{bmatrix} 26 & a \\ 14 & 45 \end{bmatrix}$

find a and b $2A + 5B = C$

$$2A \begin{bmatrix} 3 & 5 \\ 2 & a \end{bmatrix} = \begin{bmatrix} 6 & 10 \\ 4 & 2a \end{bmatrix}$$

$$5B \begin{bmatrix} 4 & b \\ 2 & 9 \end{bmatrix} = \begin{bmatrix} 20 & 5b \\ 10 & 45 \end{bmatrix}$$

$$C = \begin{bmatrix} 26 & 10+5b \\ 14 & 45+2a \end{bmatrix}$$

$b = -2$

$10 + 5b = a$ ① $45 + 2a = 45 \rightarrow$ ②

$10 + 5(-2) = a$

$-10 + 10 = a$

$a = 0$

$45 + 2a = 45$

$2a = 45 - 45$

$2a = 0$

$a = \frac{0}{2}$

$a = 0$

$45 + 2(0) = 45$

$45 - 45 = 0$

$10 + 5b = 0$

$5b = -10$

$b = \frac{-10}{5}$

$b = -2$

<u>Ex-4:-</u>

If $A = \begin{bmatrix} 9 & 1 \\ 4 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 5 \\ 7 & 12 \end{bmatrix}$

Find the matrix x such that $3A + 5B + 2x = 0$

$3A \begin{bmatrix} 9 & 1 \\ 4 & 3 \end{bmatrix} = 3A \begin{bmatrix} 27 & 3 \\ 12 & 9 \end{bmatrix}$

$5B \begin{bmatrix} 1 & 5 \\ 7 & 12 \end{bmatrix} = 5B \begin{bmatrix} 5 & 25 \\ 35 & 60 \end{bmatrix}$

Let $x = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $2x = \begin{bmatrix} 2a & 2b \\ 2c & 2d \end{bmatrix}$

$3A + 5B + 2x = 0$

$\begin{bmatrix} 32 + 2a & 28 + 2b \\ 47 + 2c & 69 + 2d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$32 + 2a$

$2a = -32$

$a = \frac{-32}{2}$, $a = -16$

$28 + 2b = 0$ $\quad$ $47 + 2c = 0$ $\quad$ $69 + 2d = 0$

$2b = -28$ $\quad$ $2c = -47$ $\quad$ $2d = -69$

$b = \frac{-28}{2}$ $\quad$ $c = \frac{-47}{2}$ $\quad$ $d = \frac{-69}{2}$

$b = -14$ $\quad$ $c = -23.5$ $\quad$ $d = -34.5$

$$\therefore x = \begin{bmatrix} -16 & -14 \\ -23.5 & -34.5 \end{bmatrix}$$

Ex - 5 :-

If $A = \begin{bmatrix} 2 & 3 & 5 \\ 4 & 7 & 9 \\ 1 & 6 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 1 & 2 \\ 4 & 2 & 5 \\ 6 & -2 & 7 \end{bmatrix}$

find $5(a+b) = 5a + 5b$

L.H.S

$$A = \begin{bmatrix} 2 & 3 & 5 \\ 4 & 7 & 9 \\ 1 & 6 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 3 & 1 & 2 \\ 4 & 2 & 5 \\ 6 & -2 & 7 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 2+3 & 3+1 & 5+2 \\ 4+4 & 7+2 & 9+5 \\ 1+6 & 6+(-2) & 4+7 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 4 & 7 \\ 8 & 9 & 14 \\ 7 & 4 & 11 \end{bmatrix}$$

$$5(A+B) = \begin{bmatrix} 25 & 20 & 35 \\ 40 & 45 & 70 \\ 35 & 20 & 55 \end{bmatrix}$$

R.H.S

$$A = \begin{bmatrix} 2 & 3 & 5 \\ 4 & 7 & 9 \\ 1 & 6 & 4 \end{bmatrix} = 5A \begin{bmatrix} 10 & 15 & 25 \\ 20 & 35 & 45 \\ 5 & 30 & 20 \end{bmatrix}$$

$$B = \begin{bmatrix} 3 & 1 & 2 \\ 4 & 2 & 5 \\ 6 & -2 & 7 \end{bmatrix}$$

$$5B \begin{bmatrix} 15 & 5 & 10 \\ 20 & 10 & 25 \\ 30 & -10 & 35 \end{bmatrix}$$

$$5A + 5B = \begin{bmatrix} 10+15 & 15+5 & 25+10 \\ 20+20 & 35+10 & 45+25 \\ 5+30 & 30+(-10) & 20+35 \end{bmatrix}$$

$$= \begin{bmatrix} 25 & 20 & 35 \\ 40 & 45 & 70 \\ 35 & 20 & 55 \end{bmatrix}$$

$\therefore$ LHS = RHS

$5(a+b) = 5A + 5B$

Multiplication of matrixes :-

If $A = (a_{ij})_{m \times p}$ and $B = (b_{ij})_{n \times p}$ their product AB is a matrix $C = C_{ij \, m \times n}$ that is c_{ij} of c is the sum of the products of the pairs of elements of i^{th} row and j^{th} column of b. The elements of every row of A and the elements of every column of B are to be multiplied in the pairs of and the sum is written the product of matrix

ex-1 :-

If $A = \begin{bmatrix} 2 & 3 & 5 \end{bmatrix}$ $B = \begin{bmatrix} 4 & 7 \\ 0 & 1 \\ -6 & 9 \end{bmatrix}$ find AB

$A = \begin{bmatrix} 2 & 3 & 5 \end{bmatrix} \quad B = \begin{bmatrix} 4 & 7 \\ 0 & 1 \\ -6 & 9 \end{bmatrix}$

$1 \times \boxed{3 \qquad 3} \times 2$ matrix

$AB = \begin{bmatrix} 2\times4 + 3\times0 + 5\times(-6) & 2\times7 + 3\times1 + 5\times9 \end{bmatrix}$

$AB = \begin{bmatrix} 8 - 30 & 14 + 3 + 45 \end{bmatrix}$

$= \begin{bmatrix} -22 & 62 \end{bmatrix}$

Ex - 2 :-

If $A = \begin{bmatrix} 3 & 5 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$ find AB

$A = \begin{bmatrix} 3 & 5 & 6 \end{bmatrix} \quad B = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$

$1 \times \boxed{3 \qquad 3} \times 1$

$AB = \begin{bmatrix} 3\times4 + 5\times1 + 6\times2 \end{bmatrix}$

$= \begin{bmatrix} 12 + 5 + 12 \end{bmatrix}$

$= \begin{bmatrix} 29 \end{bmatrix}$

Ex - 3 :-

If $A = \begin{bmatrix} 4 & -2 \\ 3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix}$

find $AB = B \quad BA = A$

$A = \begin{bmatrix} 4 & -2 \\ 3 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix}$

$2 \times \boxed{2 \qquad 2} \times 2$ matrix

$AB = \begin{bmatrix} 4\times2 + (-2)\times3 & 4\times4 + (-2)\times6 \\ 3\times2 + (-1)\times3 & 3\times4 + (-1)\times6 \end{bmatrix}$

$= \begin{bmatrix} 8 + (-6) & 16 + (-12) \\ 6 + -3 & 12 + (-6) \end{bmatrix}$

$$\begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix}$$

$AB = B$

$$B = \begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix} \quad A = \begin{bmatrix} 4 & -2 \\ 3 & -1 \end{bmatrix}$$

$2 \times \boxed{2 \quad 2} \times 2$ matrix

$$BA = \begin{bmatrix} 2\times4 + 4\times3 & 2\times(-2) + 4\times(-1) \\ 3\times4 + 6\times3 & 3\times(-2) + 6\times(-1) \end{bmatrix}$$

$$= \begin{bmatrix} 8+12 & -4-4 \\ 12+18 & -6-6 \end{bmatrix}$$

$$= \begin{bmatrix} 20 & -18 \\ 30 & -12 \end{bmatrix}$$

$BA \neq A$

<u>Ex-4 :-</u>

If $A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 3 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 5 \\ 7 & 3 \\ 5 & -2 \end{bmatrix}$

find AB

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 3 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 5 \\ 7 & 3 \\ 5 & -2 \end{bmatrix}$$

$2 \times \boxed{3 \quad 3} \times 2$ matrix

$$= \begin{bmatrix} 2\times1 + 1\times7 + 0\times5 & 2\times5 + 1\times3 + 0\times(\\ 1\times1 + 3\times7 + (-2)\times5 & 1\times5 + 3\times3 + (-2)\times(-2) \end{bmatrix}$$

$$= \begin{bmatrix} 2+7+0 & 10+3+0 \\ 1+21+(-10) & 5+9+4 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 13 \\ 12 & 18 \end{bmatrix}$$

Ex - 5 :

If $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & -2 & -4 \\ -1 & -2 & -4 \\ 1 & 2 & 4 \end{bmatrix}$

Find AB & BA

$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$ $B = \begin{bmatrix} -1 & -2 & -4 \\ -1 & -2 & -4 \\ 1 & 2 & 4 \end{bmatrix}$

$3 \times \boxed{3 \qquad 3} \times 3$ matrix

$$\begin{bmatrix} 1\times(-1)+2\times(-1)+3\times1 & 1\times(-2)+2\times(-2)+3\times2 & 1\times(-4)+2\times(-4)+3\times(4) \\ 2\times(-1)+4\times(-1)+6\times1 & 2\times(-2)+4\times(-2)+6\times2 & 2\times(-4)+4\times(-4)+6\times4 \\ 3\times(-1)+6\times(-1)+9\times1 & 3\times(-2)+6\times(-2)+9\times2 & 3\times(-4)+6\times(-4)+9\times4 \end{bmatrix}$$

$$= \begin{bmatrix} -1+(-2)+3 & -2+(-4)+6 & -4+(-8)+12 \\ -2+(-4)+6 & -4+(-8)+12 & -8+(-16)+24 \\ -3+(-6)+9 & -6+(-12)+18 & -12+(-24)+36 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} -1 & -2 & -4 \\ -1 & -2 & -4 \\ 1 & 2 & 4 \end{bmatrix} \quad A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$$

$3 \times \boxed{3 \qquad\qquad 3} \times 3$ matrix

$$= \begin{bmatrix} -1\times1+(-2)\times2+(-4)\times3 & -1\times2+(-2)\times4+(-4)\times6 & -1\times3+(-2)\times6+(-4)\times9 \\ -1\times1+(-2)\times2+(-4)\times3 & -1\times2+(-2)\times4+(-4)\times6 & -1\times3+(-2)\times6+(-4)\times9 \\ 1\times1+2\times2+4\times3 & 1\times2+2\times4+4\times6 & 1\times3+2\times6+4\times9 \end{bmatrix}$$

$$= \begin{bmatrix} -1+(-4)+(-12) & -2+(-8)+(-24) & -3+(-12)+(-36) \\ -1+(-4)+(-12) & -2+(-8)+(-24) & -3+(-12)+(-36) \\ 1+4+12 & 2+8+24 & 3+12+36 \end{bmatrix}$$

$$= \begin{bmatrix} -17 & -34 & -51 \\ -17 & -34 & -51 \\ 17 & 34 & 51 \end{bmatrix}$$

<u>Ex - 6</u> :-

If $A = \begin{bmatrix} 2 & -1 & 4 \\ 6 & 2 & 8 \\ 0 & 2 & -1 \\ 3 & -4 & 5 \end{bmatrix}$ $B = \begin{bmatrix} 4 & 1 & 0 & 8 & -5 \\ -1 & 6 & 2 & 1 & 6 \\ 2 & -7 & -1 & 4 & -7 \end{bmatrix}$

find AB.

$$A = \begin{bmatrix} 2 & -1 & 4 \\ 6 & 2 & 8 \\ 0 & 2 & -1 \\ 3 & -4 & 5 \end{bmatrix} \quad B = \begin{bmatrix} 4 & 1 & 0 & 8 & -5 \\ -1 & 6 & 2 & 1 & 6 \\ 2 & -7 & -1 & 4 & -7 \end{bmatrix}$$

$4 \times \boxed{3 \qquad\qquad 3} \times 5$

$$2\times4+(-1)\times(-1)+4\times2 \quad 2\times1+(-1)\times6+4\times(-7)$$
$$2\times0+(-1)\times2+4\times(-1) \quad 2\times8+(-1)\times1+4\times4$$
$$2\times(-5)+(-1)\times6+4\times(-7)$$

$$6\times4+2\times(-1)+8\times2 \quad 6\times1+2\times6+8\times(-7)$$
$$6\times0+2\times2+8\times(-1) \quad 6\times8+2\times1+8\times4$$
$$6\times(-5)+2\times6+8\times(-7)$$

$$0\times4+2\times(-1)+(-1)\times2 \quad 0\times1+2\times6+\overset{(-1)}{0}\times(-7)$$
$$0\times0+2\times2+(-1)\times(-1) \quad 0\times8+2\times1+(-1)\times4$$
$$0\times(-5)+2\times6+(-1)\times(-7)$$

$$3\times4+(-4)\times(-1)+5\times2 \quad 3\times1+(-4)\times6+5\times(-7)$$
$$3\times0+(-4)\times2+5\times(-1) \quad 3\times8+(-4)\times1+5\times4$$
$$3\times(-5)+(-4)\times6+5\times(-7)$$

$$= \quad 8+1+8 \quad 2+(-6)+(-28) \quad 0+(-2)+(-4)$$
$$16+(-1)+16 \quad -10+(-6)+(-28)$$

$$24+(-2)+16 \quad 6+12+(-56) \quad 0+4+(-8)$$
$$48+2+32 \quad -30+12+(-56)$$

$$0+(-2)+(-2) \quad 0+0+0 \quad 0+\overset{12}{4}+7$$
$$\cancel{0+(-4)+20} \quad 0+2+(-4) \quad 0+12+7$$

$$12+4+10 \quad 3+(-24)+(-35)$$
$$0+(-8)+(-5) \quad 24+(-4)+20$$
$$-15+(-24)+(-35)$$

$$= \begin{bmatrix} 17 & -32 & -6 & 31 & -44 \\ 38 & -38 & -4 & 82 & -74 \\ -4 & 19 & 5 & -2 & 19 \\ 26 & -56 & -13 & 40 & -74 \end{bmatrix}$$

Transpose of Matrix :-

Let AB a matrix of order $m \times n$. The transpose of A is denoted by A^T or A'. It is of order $n \times m$ it is obtained by writing the first row of a as the first column. A as the first column the second row of A as the second column etc., the last row of A and the last column

Ex - 1 :-

If $A = \begin{bmatrix} 1 & 5 & 9 \\ 10 & 14 & 19 \end{bmatrix}$

$$A^T = \begin{bmatrix} 1 & 10 \\ 5 & 14 \\ 9 & 19 \end{bmatrix}$$

$(A')' = A \Leftarrow$ Notes

Verify that $B^T \times A^T = (AB)^T$

$A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 2 \\ 2 & 0 \\ -1 & 1 \end{bmatrix}$

$B^T = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 0 & 1 \end{bmatrix}$ $A^T = \begin{bmatrix} 1 & 2 \\ 1 & 1 \\ 2 & 0 \end{bmatrix}$

L.H.S

$$B^T A^T = \begin{bmatrix} 1\times1+2\times1+(-1)\times2 & 1\times2+2\times1+(-1)\times0 \\ 2\times1+0\times1+1\times2 & 2\times2+0\times1+1\times0 \end{bmatrix}$$

$$= \begin{bmatrix} 1+2+(-2) & 2+2+0 \\ 2+0+2 & 4+0+0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 4 \\ 4 & 4 \end{bmatrix}$$

RHS

$$AB \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 0 \\ -1 & 1 \end{bmatrix}$$

$$2 \times \boxed{3 \quad 3} \times 2$$

$$= \begin{bmatrix} 1\times1+1\times2+2\times(-1) & 1\times2+1\times0+2\times1 \\ 2\times1+1\times2+0\times(-1) & 2\times2+1\times0+0\times1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+2+(-2) & 2+0+2 \\ 2+2+0 & 4+0+0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 4 \\ 4 & 4 \end{bmatrix}$$

$$B^T A^T = (AB)^T$$

$\therefore$ L.H.S = R.H.S

Properties of Matrixes :-

1. Addition of Matrixes :-

(i) Commutative property :-

If A and B are the matrixes of the same order .

$$A+B = B+A$$

(ii) Associative property :-

If A, B and C are matrixes of the same order

$(A+B)+C = A+(B+C)$

(iii) Scalar Matrix :-

$$k(a+b) = ka + kb$$

(iv) Existence of identity :-

The null matrix O of the order of A is the additive identity

$$A+O = A, \quad O+A = A$$

(v) Existence of inverse :-

$-A$ is the additive inverse of

$$A+(-A) = O$$
$$(-A)+A = O$$

2. Multiplication of Matrices :-

(i) Not Commutative :-

For two matrices A and B, AB and BA need not be equal

That is $AB \neq BA$

(ii) Associative Property :-

If ABC is defined

$$ABC = (AB)C = A(BC)$$

(iii) Existence of identity :-

If A is a square matrix of order n. I_n is the identity matrix

$$AI_n = A$$
$$I_n A = A$$

(iv) Existence of inverse :-

If A is a square matrix of order n and non - singular there exist a square matrix P of order n such that

$AB = I_n = BA$

3. Transpose of Matrix :-

The transpose of the transpose of a matrix is the original matrix That is $(A')' = A$

(ii) The transpose of the sum of matrixes is the sum of the transpose of the individual series. That is

$(A+B)' = A' + B'$

(iii) $(KA)' = KA'$

where k is a scalar.

Adjoint Matrix :-

The adjoint matrix of a square matrix A is denoted by adjoint A. It is obtained from A after writing the transpose of A and then replacing every element in a A^T by its co-factor

$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ Adj. of $A = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

Ex-1 :-

Find the $A = \begin{bmatrix} 1 & 5 \\ -3 & 0 \end{bmatrix}$

$$A^T = \begin{bmatrix} 1 & -3 \\ 5 & 0 \end{bmatrix}$$

$$|A| = \begin{bmatrix} 1 & 5 \\ -3 & 0 \end{bmatrix}$$

$$= 0 + 15$$

$$= 15$$

$$(\text{adj} A) \cdot A = A \cdot \text{Adj}(A)$$

$$|A| I = \begin{bmatrix} 15 & 0 \\ 0 & 15 \end{bmatrix}$$

$$= 15 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Inverse of a Matrix :-

The Inverse of a square matrix A is denoted by A^{-1}. A inverse exist if and only if A is a non - singular matrix

$$A^{-1} = \frac{1}{|A|} \text{adj} \cdot A$$

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$A^T = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$$

$$|A| = ad - bc$$

$$\text{Adj of } A = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$A^{-1} = \frac{1}{ad-bc} \times \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

Ex-1:-

Find the inverse of $A = \begin{bmatrix} 2 & 2 \\ 3 & 5 \end{bmatrix}$

$$A^T = \begin{bmatrix} 2 & 3 \\ 2 & 5 \end{bmatrix}$$

$$|A| = |10 - 6|$$
$$= |4|$$

$$\text{Adj of } A = \begin{bmatrix} 5 & -2 \\ -3 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|4|} \begin{bmatrix} 5 & -2 \\ -3 & 2 \end{bmatrix}.$$

$$= \begin{bmatrix} 1.25 & -0.5 \\ -0.75 & 0.5 \end{bmatrix}$$

If $10A - 50I = 0$ and $A = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

find the A^{-1}

Note:-

$$A \cdot A^{-1} = I$$
$$I \cdot A^{-1} = A^{-1}$$

$$10A - 50I = 0$$
$$10A \cdot A^{-1} - 50IA^{-1} = 0$$
$$10I - 50A^{-1} = 0$$
$$-50A^{-1} = -10I$$
$$A^{-1} = \frac{-10}{50}$$
$$= 0.2$$

$$A^{-1} = \begin{bmatrix} 0.2 & 0 & 0 \\ 0 & 0.2 & 0 \\ 0 & 0 & 0.2 \end{bmatrix}$$

Find $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ the equation

$$A^3 - 6A^2 + 9A - 4I = 0$$

<u>Soln :-</u>

$$A^3 = a \times (a^2)$$

$$A^3 - 6A^2 + 9A - 4I = 0$$

$$A^2 = A \times A$$

$$= \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$3 \times \boxed{3 \quad 3} \times 3$$

$$= \begin{bmatrix} 2\times2+(-1)\times(-1)+1\times1 & 2\times(-1)+(-1)\times2+1\times(-1) & 2\times1+-1\times(-1)\times1\times2 \\ -1\times2+2\times(-1)+(-1)\times1 & -1\times-1+2\times(2)+(-1)\times(-1) & -1\times1+2\times(-1)+(-1)\times2 \\ 1\times2+(-1)\times(-1)+2\times1 & 1\times(-1)+(-1)\times2+(-2)\times(-1) & 1\times1+(-1)\times(-1)+2\times \end{bmatrix}$$

$$= \begin{bmatrix} 4+1+1 & -2+(-2)+(-1) & 2+1+2 \\ -2+(-2)+(-1) & +1+4+1 & -1+(-2)+(-2) \\ 2+1+2 & -1+(-2)+(-2) & 1+1+4 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}$$

$$A^3 = A^2 \cdot A$$

$$= \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 6\times2+(-5)\times(-1)+5\times1 & 6\times(-1)+(-5)\times2+5\times(-1) & 6\times1+(-5)\times(-1)+5\times2 \\ -5\times2+6\times(-1)+(-5)\times1 & -5\times(-1)+6\times2+(-5)\times(-1) & -5\times1+6\times(-1)+(-5)\times2 \\ 5\times2+(-5)\times(-1)+6\times1 & 5\times(-1)+(-5)\times2+6\times(-1) & 5\times1+(-5)\times(-1)+6\times2 \end{bmatrix}$$

$$= \begin{bmatrix} 12+5+5 & -6+(-10)+(-5) & 6+5+10 \\ -10+(-6)+(-5) & 5+12+5 & -5+(-6)+(-10) \\ 10+5+6 & -5+(-10)+(-6) & 5+5+12 \end{bmatrix}$$

$$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix}$$

$$6A^2 = 6 \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} = \begin{bmatrix} 36 & -30 & 30 \\ -30 & 36 & -30 \\ 30 & -30 & 36 \end{bmatrix}$$

$$9A = 9 \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 18 & -9 & 9 \\ -9 & 18 & -9 \\ 9 & -9 & 18 \end{bmatrix}$$

$$4I = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$A^3 - 6A^2 + 9A - 4I = 0$$

$$\begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix} - \begin{bmatrix} 36 & -30 & 30 \\ -30 & 36 & -30 \\ 30 & -30 & 36 \end{bmatrix} +$$

$$\begin{bmatrix} 18 & -9 & 9 \\ -9 & 18 & -9 \\ 9 & -9 & 18 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} = 0$$

<u>L.H.S</u>

$$\begin{bmatrix} 22-36+18-4 & -21+30-9-0 & 21-30+9-0 \\ -21+30-9-0 & 22-36+18-4 & -21+30-9-0 \\ 21-30+9-0 & -21+30-9-0 & 22-36+18-4 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$$

$\therefore$ L.H.S = R.H.S

$$A^3 - 6A^2 + 9A - 4I = 0$$

$$A^3 \cdot A^{-1} - 6A^2 \cdot A^{-1} + 9A \cdot A^{-1} - 4I \cdot A^{-1} = 0 \cdot A^{-1}$$

$$A^2 - 6A + 9A - 4A^{-1} = 0$$

$$-4A^{-1} = (A^2 - 6A + 9I)$$

$$A^{-1} = \frac{1}{4}(A^2 - 6A + 9I)$$

$$A^{-1} = \frac{1}{4}\begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} - \begin{bmatrix} 12 & -6 & 6 \\ -6 & 12 & -6 \\ 6 & -6 & 12 \end{bmatrix} + \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0.75 & 0.25 & -0.25 \\ 0.25 & 0.75 & 0.25 \\ -0.25 & 0.25 & 0.75 \end{bmatrix}$$

Determinants with each square Matrix :-

A we can associate a determinant which is denoted by $|A|$ or Δ when A is a square matrix of order n is a square matrix of order n the corresponding determinant $|A|$ is said to be a determinant of order n

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} \quad |ad - bc|$$

$\begin{bmatrix} 1 & 5 \\ 6 & 9 \end{bmatrix}$, $\begin{bmatrix} 9 & 5 \\ 6 & 1 \end{bmatrix}$, $\begin{bmatrix} 9 & 6 \\ 5 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 6 \\ 5 & 9 \end{bmatrix}$

are different matrices

$|9-30|$, $|9-30|$ $|9-30|$ $|9-30|$ but the corresponding are determinant the same value -21

Eg:-

The value of a first order determinant is the single element $|5| = 5$, $|0| = 0$
$|-15| = -15$.

The value of a second other determinant

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = |ad - bc|.$$

$$\begin{vmatrix} 5 & 4 \\ 3 & 2 \end{vmatrix} = |10 - 12| = |-2|$$

Eg:-

$$\begin{vmatrix} 6 & 1 \\ 4 & 3 \end{vmatrix} = |18 - 4| = |14|$$

Eg:-

$$\begin{vmatrix} 8 & 2 \\ -3 & -1 \end{vmatrix} = |-8 + 6| = |-2|$$

Find the value of determinant

$(+, -, +)$ we taken

$$\begin{vmatrix} 3 & -2 & 1 \\ 2 & 3 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 3 \begin{vmatrix} 3 & -1 \\ 1 & 1 \end{vmatrix} - $$
$$-2 \begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix} + 1 \begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix}$$

$= 3(4) - (-2)(3) + 1(2-3)$

$= 12 + 6 - 1 = 17$

Find the value of determinant

$$\begin{bmatrix} 3 & 4 & 7 \\ 2 & 1 & 3 \\ 7 & 2 & 1 \end{bmatrix}$$

$$3 \begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix} - 4 \begin{bmatrix} 2 & 3 \\ 7 & 1 \end{bmatrix} + 7 \begin{bmatrix} 2 & 1 \\ 7 & 2 \end{bmatrix}$$

$3|1-6| - 4|2-21| + 7|4-7|$

$3|-5| - 4|-19| + 7|-3|$

$|-15 + 76 - 21|$

$= |76 - 36| = |40|$

Find the value of determinant

$$|A| = \begin{vmatrix} 3 & -1 & 2 \\ 5 & 3 & 0 \\ 1 & 4 & -6 \end{vmatrix}$$

$$2 \begin{vmatrix} 5 & 3 \\ 1 & 4 \end{vmatrix} - 0 \begin{vmatrix} 3 & -1 \\ 1 & 4 \end{vmatrix} + (-6) \begin{vmatrix} 3 & -1 \\ 5 & 3 \end{vmatrix}$$

$2|20-3| - 0|12+1| - 6|9+5|$

$2|17| - 0|13| - 6|14|$

$34 - 0 \quad 84$

$= +50$

Difference between Matrix & Determinant

Matrixes	Determinant
* No. of rows and no. of columns can be equal or unequal	* No. of rows and No. of columns are equal
* Elements are enclosed by brackets or parenthesis or double bar	* Elements are enclosed by a pair of vertical line.
* A matrix has no numerical value	* A determinant has a numerical value
* Matrixes are arrangement by interchanging two elements in a matrix a new matrix is obtained	* Even after interchanging two elements in a determinant the value may remind the same

Grammar rule or determinant methods.

Let the system of linear equations to be solved

$$a_1 x + b_1 y + c_1 z = d_1$$

$$a_2 x + b_2 y + c_2 z = d_2$$

$$a_3 x + b_3 y + c_3 z = d_3$$

In matrix form It is $Ax = c$ where

$$A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}, \quad x = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and}$$

$$C = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

$$A_x = \begin{bmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{bmatrix}, \quad A_y = \begin{bmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{bmatrix}$$

$$A_z = \begin{bmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{bmatrix}$$

$$x = \frac{|Ax|}{|A|}, \quad y = \frac{|Ay|}{|A|} \text{ and}$$

$$z = \frac{|Az|}{|A|}$$

$$A = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}, \quad x = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } c = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

$$A_x = \begin{bmatrix} c_1 & b_1 \\ c_2 & b_2 \end{bmatrix} \text{ and } Ay = \begin{bmatrix} a_1 & c_1 \\ a_2 & c_2 \end{bmatrix}$$

$$x = \frac{|Ax|}{|A|} \text{ and } \frac{|Ay|}{|A|}$$

Solve the following equations by grammer rule method

$$3x + 2y = 8$$

$$5x - 3y = 7$$

$$A = \begin{bmatrix} 3 & 2 \\ 5 & -3 \end{bmatrix} \quad x = \begin{bmatrix} x \\ y \end{bmatrix} \quad c = \begin{bmatrix} 8 \\ 7 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 3 & 2 \\ 5 & -3 \end{vmatrix} = -9-10 = -19$$

$$|Ax| = \begin{vmatrix} 8 & 2 \\ 7 & -3 \end{vmatrix} = -24-14 = -38$$

$$|Ay| = \begin{vmatrix} 3 & 8 \\ 5 & 7 \end{vmatrix} = 21-40 = -19$$

$$C.R\ x = \frac{|Ax|}{|A|} = \frac{-38}{-19} = 2$$

$$y = \frac{|Ay|}{|A|} = \frac{-19}{-19} = 1$$

$$x = 2, y = 1$$

Eg:- Solve the following equations by Grammar Rule method

$$x - 2y = 16$$
$$3x + y = -1$$

$$A = \begin{bmatrix} 1 & -2 \\ 3 & 1 \end{bmatrix} \quad x = \begin{bmatrix} x \\ y \end{bmatrix} \quad C = \begin{bmatrix} 16 \\ -1 \end{bmatrix}$$

$$Ax = C$$

$$|A| = \begin{vmatrix} 1 & -2 \\ 3 & 1 \end{vmatrix} = 1+6 = 7$$

$$|Ax| = \begin{vmatrix} 16 & -2 \\ -1 & 1 \end{vmatrix} = (16 \times 1) - (2) = 14$$

$$|Ay| = \begin{vmatrix} 1 & 16 \\ 3 & -1 \end{vmatrix} = -1-48 = -49$$

$$C.R\ x = \frac{|Ax|}{|A|} = \frac{14}{7} = 2$$

$$y = \frac{|Ay|}{|A|} = \frac{-49}{7} = -7$$

$x = 2, y = -7$

Eg: Solve the following by Grammar rule method.

$$\frac{1}{a} + \frac{2}{b} = 4$$

$$\frac{3}{a} - \frac{1}{b} = 5$$

Let $x_1 = \frac{1}{a}$ and $x_2 = \frac{1}{b}$

$$x_1 + 2x_2 = 4$$

$$3x_1 - x_2 = 5$$

$$Ax = C$$

$$A = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix} \quad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad C = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

$$|A| = -1 - 6 = -7$$

$$|Ax_1| = \begin{vmatrix} 4 & 2 \\ 5 & -1 \end{vmatrix} = -4 - 10 = -14$$

$$|Ax_2| = \begin{vmatrix} 1 & 4 \\ 3 & 5 \end{vmatrix} = 5 - 12 = -7$$

C.R.

$$x_1 = \frac{|Ax|}{|A|} = \frac{-14}{-7} = 2$$

$$x_2 = \frac{|Ay|}{|A|} = \frac{-7}{-7} = 1$$

$$x_1 = 2, \; x_2 = 1$$

$\frac{1}{a} = 2$ or $a = \frac{1}{2}$ and $\frac{1}{b} = 1$ or $b = 1$

Eg:- Solve the following system of simultaneous by Grammer Rule method

$$2x + 3y + 3z = 22$$
$$x - y + z = 4$$
$$4x + 2y - z = 9$$

$$Ax = c$$

$$A = \begin{bmatrix} 2 & 3 & 3 \\ 1 & -1 & 1 \\ 4 & 2 & -1 \end{bmatrix} \quad x = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad c = \begin{bmatrix} 22 \\ 4 \\ 9 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & 3 & 3 \\ 1 & -1 & 1 \\ 4 & 2 & -1 \end{vmatrix}$$

$$= 2(1-2) - 3(-1-4) + 3(2+4)$$
$$= -2 + 15 + 18$$
$$= 31$$

$$|Ax| = \begin{vmatrix} 22 & 3 & 3 \\ 4 & -1 & 1 \\ 9 & 2 & 1 \end{vmatrix}$$

$$= 22|-1-2| - 3|-4-9| + 3|8+9|$$
$$= 22(-3) - 3(-13) + 3(17)$$
$$= -66 + 39 + 51$$
$$= 68$$

$$|Ay| = \begin{bmatrix} 2 & 22 & 3 \\ 1 & 4 & 1 \\ 4 & 9 & -1 \end{bmatrix}$$

$= 2(-4-9) - 22(-1-4) + 3(9-16)$

$= 2(-13) - 22(-5) + 3(-7)$

$= -26 + 110 - 21$

$= 110 - 47$

$= 63$

$$|Az| = \begin{vmatrix} 2 & 3 & 22 \\ 1 & -1 & 4 \\ 4 & 2 & 9 \end{vmatrix}$$

$= 2(-9-8) - 3(9-16) + 22(2+4)$

$= 2(-17) + 21 + 132$

$= -34 + 153$

$= 119$

$$x = \frac{|Ax|}{|A|} = \frac{68}{31} = 2.19$$

$$y = \frac{|Ay|}{|A|} = \frac{63}{31} = 2.032$$

$$z = \frac{|Az|}{|A|} = \frac{119}{31} = 3.838$$

Eg:-

$3x - y + 2z = 8$

$x + y + z = 2$

$2x + y - z = -1$

$$A = \begin{bmatrix} 3 & -1 & 2 \\ 1 & 1 & 1 \\ 2 & 1 & -1 \end{bmatrix} \quad x = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad c = \begin{bmatrix} 8 \\ 2 \\ -1 \end{bmatrix}$$

$$Ax = c$$

$|A| = 3\,|-1-1| - (-1)\,|-1-2| + 2\,|1-2|$

$= -6 - 3 - 2 = -11$

$$|Ax| = \begin{vmatrix} 8 & -1 & 2 \\ 2 & 1 & 1 \\ -1 & 1 & -1 \end{vmatrix}$$

$$= 8|-1-1| - (-1)|-2+1| + 2|2+1|$$

$$= -16 - 1 + \cancel{1}6$$

$$= -11$$

$$|Ay| = \begin{vmatrix} 3 & 8 & 2 \\ 1 & 2 & 1 \\ 2 & -1 & -1 \end{vmatrix}$$

$$= 3|-2+1| - 8|-1-2| + 2|-1-4|$$

$$= 3|-1| - 8|3| + 2|-5|$$

$$= 3 + 24 - 10$$

$$= 11$$

$$Az = \begin{vmatrix} 3 & -1 & 8 \\ 1 & 1 & 2 \\ 2 & 1 & -1 \end{vmatrix}$$

$$= 3|-1-2| + 1|-1-4| + 8|1-2|$$

$$= 3(-31+1) - 5(1+8) - 1$$

$$= -9 - 5 - 8$$

$$= -22$$

C.R

$$x = \frac{|Ax|}{|A|} = \frac{-11}{-11} = 1$$

$$y = \frac{|Ay|}{|A|} = \frac{11}{-11} = -1$$

$$z = \frac{|Az|}{|A|} = \frac{-22}{-11} = 2$$

$x = 1,\ y = -1,\ z = 2$

Evaluate $\begin{vmatrix} 2 & 3 \\ 1 & 4 \end{vmatrix} \times \begin{vmatrix} 2 & 4 \\ 3 & 1 \end{vmatrix}$

$= |8-3| \times |2-12|$

$= |5| \times |-10|$

$= |-50|$

Product of $\begin{vmatrix} 2 & 3 \\ 1 & 4 \end{vmatrix} \times \begin{vmatrix} 2 & 4 \\ 3 & 1 \end{vmatrix}$

$= \begin{vmatrix} 2\times 2 + 3\times 4 & 2\times 3 + 3\times 1 \\ 1\times 2 + 4\times 4 & 1\times 3 + 4\times 1 \end{vmatrix}$

$= \begin{vmatrix} 16 & 9 \\ 18 & 7 \end{vmatrix}$

$= (16\times 7) - (18\times 9)$

$= 112 - 162$

$= -50$

Find the product $\begin{vmatrix} 4 & 1 & 3 \\ 2 & 0 & -6 \\ 5 & -7 & 9 \end{vmatrix} \times \begin{vmatrix} 5 & -2 & 0 \\ 1 & 6 & 8 \\ 3 & 4 & 7 \end{vmatrix}$

$= \begin{vmatrix} 4 & 1 & 3 \\ 2 & 0 & -6 \\ 5 & -7 & 9 \end{vmatrix}$

$= |4(0-42) - 1(18+30) + 3(-14-0)|$

$= |-168 - 48 - 42|$

$= |-258|$

$\begin{vmatrix} 5 & -2 & 0 \\ 1 & 6 & 8 \\ 3 & 4 & 7 \end{vmatrix}$

$= 5(42-32) + 2(7-24) + 0(4-18)$

$= 50 - 34$

$= 16$

$= |-258| \times |16|$

$= -4128$

Product method :-

$$\begin{vmatrix} 4 & 1 & 3 \\ 2 & 0 & -6 \\ 5 & -7 & 9 \end{vmatrix} \times \begin{vmatrix} 5 & -2 & 0 \\ 1 & 6 & 8 \\ 3 & 4 & 7 \end{vmatrix}$$

$$= \begin{vmatrix} 20-2+0 & 4+6+24 & 12+4+21 \\ 10-0+0 & 2+0-48 & 6+0-42 \\ 25+14+0 & 5-42+72 & 15-28+63 \end{vmatrix}$$

$$= \begin{vmatrix} 18 & 34 & 37 \\ 10 & -46 & -36 \\ 39 & 35 & 50 \end{vmatrix}$$

$$= 18(-2300+1260) - 34(500+1404) + 37(350+1794)$$

$$= 18(-1040) - 34(1904) + 37(2144)$$

$$= -18720 - 64736 + 79328$$

$$= -4128$$

Rank of matrix

The rank of a matrix is the order of the largest square sub matrix whose determinant is not zero. Rank of a matrix is denoted by $e(A)$. Rank can be obtained either by searching for the largest non-singular submatrix or by transforming to an equivalent matrix whose rank is readily available.

Eg:-

Find the rank of $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$

$$\begin{vmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{vmatrix} = 1(4\times 6 - 5\times 5) - 2(2\times 6 - 5\times 3) + 3(2\times 5 - 4\times 3)$$

$$= -1 + 6 - 6$$

$$= -1$$

$-1 \neq 0.$

$\therefore$ The rank of given matrix is 3.

Find the rank of $\begin{bmatrix} 3 & 2 & -1 \\ 7 & 8 & 0 \\ 4 & 6 & 1 \end{bmatrix}$

$$\begin{vmatrix} 3 & 2 & -1 \\ 7 & 8 & 0 \\ 4 & 6 & 1 \end{vmatrix} = 3(8\times 1 - 0\times 6) - 2(7\times 1 - 0\times 4) - 1(7\times 6 - 8\times 4)$$

$$= 3(8-0) - 2(7-0) - 1(42-32)$$

$$= 24 - 14 - 10$$

$$= 0$$

$\therefore$ The rank is not 3, consider a sub matrix $\begin{bmatrix} 3 & 2 \\ 7 & 8 \end{bmatrix}$

$$\begin{vmatrix} 3 & 2 \\ 7 & 8 \end{vmatrix} = 24 - 14 = 10$$

$10 \neq 0$

$\therefore$ The rank of given matrix is 2

Eg:- Find the rank of $\begin{bmatrix} -2 & 1 & 3 & 4 \\ 0 & 1 & 1 & 2 \\ 1 & 3 & 4 & 7 \end{bmatrix}$

$$\begin{vmatrix} 1 & 3 & 4 \\ 1 & 1 & 2 \\ 3 & 4 & 7 \end{vmatrix} = 1(1\times 7 - 2\times 4) - 3(1\times 7 - 2\times 3) + 4(1\times 4 - 1\times 3)$$

$$= 1(7-8) - 3(7-6) + 4(4-3)$$

$$= -1 - 3 + 4$$

$$= 0$$

The rank is not 3, consider sub matrix $\begin{vmatrix} 1 & 3 \\ 1 & 1 \end{vmatrix}$

$$\begin{vmatrix} 1 & 3 \\ 1 & 1 \end{vmatrix} = 1 - 3$$
$$= -2$$

$-2 \neq 0$

The rank of sub matrix is 2

$$\begin{vmatrix} -2 & 3 & 4 \\ 0 & 1 & 2 \\ 1 & 4 & 7 \end{vmatrix}$$

$$= -2(1\times 7 - 2\times 4) - 3(0\times 7 - 2\times 1) + 4(0\times 4 - 1\times 1)$$

$$= -2(7-8) - 3(0-2) + 4(0-1)$$

$$= 2 - 6 - 4$$

$$= -8$$

$-8 \neq 0$. The rank of sub matrix is 3

$$\begin{vmatrix} -2 & 1 & 4 \\ 0 & 1 & 2 \\ 1 & 4 & 7 \end{vmatrix}$$

$$= -2(7-8) - 1(0-2) + 4(0-1)$$

$$= -3 - 3 - 4$$

$$= -10$$

$-10 \neq 0$ The rank of matrix is 3

$$\begin{bmatrix} -2 & 1 & 4 \\ 0 & 1 & 2 \\ 1 & 3 & 7 \end{bmatrix}$$

$$\begin{vmatrix} -2 & 1 & 4 \\ 0 & 1 & 2 \\ 1 & 3 & 7 \end{vmatrix} = -2(1\times 7 - 2\times 3) - 1(0\times 7 - 2\times 1) + 4(0\times 3 - 1\times 1)$$

$$= -2(7-6) - 1(0-2) + 4(0-1)$$

$$= -2(1) - 1(-2) + 4(-1)$$

$$= -2 + 2 - 4$$

$$= -4$$

$$-4 \neq 0.$$

The rank of sub matrix is 3.

Rank from an equivalent matrix

$$\begin{bmatrix} 1 & -2 & 0 & 1 \\ 2 & -1 & 1 & 0 \\ 3 & -3 & 1 & 1 \\ -1 & -1 & -1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & 0 & 1 \\ 0 & 3 & 1 & -2 \\ 0 & 3 & 1 & -2 \\ 0 & 3 & -1 & 2 \end{bmatrix}$$

$R_2 = R_2 + (-2)R_1$

$R_3 = R_3 + (-3)R_1$

$$\begin{bmatrix} 1 & -2 & 0 & 1 \\ 0 & 3 & 1 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_4 = R_4 + (1)R_1$

$R_3 = R_3 + (-1)R_2$

$R_4 = R_4 + (1)R_2$