13/10/20

Unit - 5

Transportation Supply

SOURCE\TO	D	E	F	SUPPLY
A	5	8	4	50
B	6	6	3	40
C	3	9	6	60
DEMAND	20	95	35	150/150

North West Corner Rule :-

Transportation Cost = $5 \times 20 + 8 \times 30 + 6 \times 40 + 9 \times 25 + 6 \times 35$

$= 100 + 240 + 240 + 225 + 210$

$= 1015$

By Least Cost Method :-

SOURCE\TO	D	E	F	SUPPLY
A	5	8 [50]	4	50
B	6	6 [5]	3 [35]	40
C	3 [20]	9 [40]	6	60
DEMAND	20	95	35	150/150

Transportation cost = $3 \times 20 + 8 \times 50 + 6 \times 5 + 9 \times 40 + 3 \times 35$

$= 60 + 400 + 30 + 360 + 105$

$= 955$

SOURCE/TO	D	E	F	SUPPLY	
A	5	8 [50]	4	50	(1) (3)
B	6	6 [5]	3 [35]	40	(3) (0)
C	3 [20]	9 [40]	6	60	(3) (6)
DEMAND	20	95	35	150/150	
	(2) (2)	(2) (2)	(1)		

Vogal's appropriation method

Transportation Cost = 3 × 20 + 8 × 50 + 6 × 5 + 9 × 40 + 3 × 35

= 955

Example of Unbalanced Transport Problem

Plant	Warehouse W_1	W_2	W_3	Supply
A	28	17	26	500
B	19	12	16	300
Demand	250	250	500	

Plant	Warehouse W_1	W_2	W_3	Supply
A	28	17	26	500
B	19	12	16	300
Demand	250	250	500	200

North - West Corner Rule

Plant	Warehouse W_1	W_2	W_3	Supply	
A	28 [250]	17 [250]	26	500	250 0
B	19	12	16 [300]	300	0
	0	0	0 [200]	200	0
Demand	250 0	250 0	500 200 0	1000/1000	

i) Balanced

Σ Supply = Σ Demand

500 + 300 + 200 = 250 + 250 + 500

1000 = 1000

(ii) $m \times n - 1$

$3 \times 3 - 1$

$6 - 1 = 5$

(iii) Transportation Cost = $250 \times 28 + 250 \times 17 + 300 \times 16 + 200 \times 0$ =

= 16,050

Scanned with CamScanner

Least Cost Method

Plant	W_1	W_2	W_3	Supply	
A	28 [50]	17	26 [450]	500	50, 0
B	19	12 [250]	16 [50]	300	50, 0
	0 [200]	0	0	200	0
Demand	250	250	500	1000/1000	
	50, 0	0	450, 0		

Transportation cost = 50 × 28 + 200 × 0 + 250 × 12 + 450 × 26 + 50 × 16
= 16,900

Vogel's Approximation Method

VAM – S2S

R.P		Plant	W_1	W_2	W_3	Supply		
9	9	A	28 [250]	17 [250]	26	500	250	0
4	4	B	19	12	16 [300]	300		0
0			0	0	0 [200]	200		0
			250	250	500	1000/1000		
			0	0	300, 0			
					↑			
C.P			9	5	10			
			9	5	10			

= 250 × 28 + 250 × 17 + 300 × 16 + 200 × 0 +

~~250 × 500~~

= ~~1941,050~~

= 7000 + 4250 + 4800 + 0

= 16,050

Vogels Approximation Method

VAM - SLS.

Plant	W_1	W_2	W_3	Supply	R.P
A	28 [50]	17 [250]	26 [200]	~~500~~ ~~250~~ ~~50~~ 0	9 9 9
B	19	12	16 [300]	300	4 4
	0 [200]	0	0	200	0
~~250~~	~~250~~ ~~50~~ 0	~~250~~ 0	~~500~~ ~~200~~ 0	1000/1000	

C.P 19 12 16

9 5 10

= 28 × 50 + 250 × 17 + 200 × 26 + 300 × 16

= 15,650

No. of rows should be equal to no. of columns then the given problem is balanced assignment problem.

No. of rows is not equal to no. of columns is unbalanced assignment problem.

Consider Assignment Problem :-

	J1	J2	J3	J4
A	5	9	5	7
B	3	8	4	4
C	5	5	8	4
D	7	4	5	5

find the assignment schedule & assignment cost.

0	4	0	2
0	5	1	1
0	4	1	1
0	3	1	1

Assignment schedule

A -> J3, B -> J1, C -> J4, D -> J2

Assignment cost = 5 + 3 + 4 + 4

= 16.

11

Assignment Problem

"O.R. produces an integrated solution for the good of entire organisation and not just to effect local improvement"

11:1. INTRODUCTION

The assignment problem is a special case of the transportation problem in which the objective is to **assign** a number of resources to the **equal** number of activities at a minimum cost (or maximum profit).

Assignment problem is completely degenerate form of a transportation problem. The units available at each origin (resource) and units demanded at each destination (activity) are all equal to one. That means exactly one occupied cell in each row and each column of the transportation table, *i.e.*, only n occupied (basic) cells in place of the required $n+n-1$ $(=2n-1)$.

11:2. MATHEMATICAL FORMULATION OF THE PROBLEM

Consider a problem of assignment of n resources (workers) to n activities (jobs) so as to minimize the overall cost or time in such a way that each resource can associate with one and only one job. The cost (or effectiveness) matrix (c_{ij}) is given as under :

		Activity				*Available*
		A_1	A_2	$\cdots$	A_n	
	R_1	c_{11}	c_{12}	$\cdots$	c_{1n}	1
	R_2	c_{21}	c_{22}	$\cdots$	c_{2n}	1
Resource	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	R_n	c_{n1}	c_{n2}	$\cdots$	c_{nn}	1
	Required	1	1	$\cdots$	1	

This cost matrix is same as that of a transportation problem except that availability at each of the resources and the requirement at each of the destinations is unity (due to the fact that assignments are made on a one-to-one basis).

Let x_{ij} denote the assignment of ith resource to jth activity, such that

$$x_{ij} = \begin{cases} 1, & \text{if resource } i \text{ is assigned to activity } j \\ 0, & \text{otherwise} \end{cases}$$

Then, the mathematical formulation of the assignment problem is

$$\text{Minimize } z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij} \text{ subject to the constraints :}$$

$$\sum_{i=1}^{n} x_{ij} = 1 \text{ and } \sum_{j=1}^{n} x_{ij} = 1 ; \quad x_{ij} = 0 \text{ or } 1$$

for all $i = 1, 2, \ldots, n$ and $j = 1, 2, \ldots, n$.

Note. In the above statement c_{ij} is the cost associated with assigning ith resource to jth activity.

Illustration. *Given below is an assignment problem, write it as a transportation problem :*

	A_1	A_2	A_3
R_1	1	2	3
R_2	4	5	1
R_3	2	1	4

Answer. Let x_{ij} denote the assignment of R_i $(i = 1, 2, 3)$ to A_j $(j = 1, 2, 3)$, such that

$$x_{ij} = \begin{cases} 1, & \text{if } R_i \text{ is assigned to } A_j \\ 0, & \text{otherwise} \end{cases}$$

Then the transportation problem is :

$$\text{Minimize } z = 1x_{11} + 2x_{12} + 3x_{13} + 4x_{21} + 5x_{22} + x_{23} + 2x_{31} + x_{32} + 4x_{33}$$

subject to the constraints :

$$\left.\begin{array}{l} x_{11} + x_{12} + x_{13} = 1 \\ x_{21} + x_{22} + x_{23} = 1 \\ x_{31} + x_{32} + x_{33} = 1 \end{array}\right\}, \quad \left.\begin{array}{l} x_{11} + x_{21} + x_{31} = 1 \\ x_{12} + x_{22} + x_{32} = 1 \\ x_{13} + x_{23} + x_{33} = 1 \end{array}\right\},$$

$$x_{ij} = 0 \text{ or } 1 \quad \text{for } i = 1, 2, 3 \text{ and } j = 1, 2, 3.$$

Theorem 11-1 **(Reduction Theorem).** *In an assignment problem, if we add or subtract a constant to every element of any row (or column) of the cost matrix $[c_{ij}]$, then an assignment that minimizes the total cost on one matrix also minimizes the total cost on the other matrix. In other words, if $x_{ij} = x_{ij}^*$ minimizes*

$$z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij} \text{ with } \sum_{i=1}^{n} x_{ij} = 1, \sum_{j=1}^{n} x_{ij} = 1 ; x_{ij} = 0 \text{ or } 1$$

then x_{ij}^* also minimizes $z^* = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij}^* x_{ij}$, where $c_{ij}^* = c_{ij} - u_i - v_j$ for all $i, j = 1, 2, \ldots, n$ and u_i, v_j are some real numbers.

Proof. We write

$$z^* = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij}^* x_{ij} = \sum_{i=1}^{n} \sum_{j=1}^{n} (c_{ij} - u_i - v_j) x_{ij}$$

$$= \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij} - \sum_{i=1}^{n} u_i \sum_{j=1}^{n} x_{ij} - \sum_{i=1}^{n} x_{ij} \sum_{j=1}^{n} v_j$$

$$= z - \sum_{i=1}^{n} u_i - \sum_{j=1}^{n} v_j ;$$

since

$$\sum_{i=1}^{n} x_{ij} = \sum_{j=1}^{n} x_{ij} = 1.$$

This shows that the minimization of the new objective function z^* yields the same solution as the minimization of original objective function z, because Σu_i and Σv_j are independent of x_{ij}.

Theorem 11-2 *If* $c_{ij} \geq 0$ *such that minimum* $\sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} = 0$, *then* x_{ij} *provides an optimum assignment.*

The proof is left as an exercise to the reader.

The above two theorems form the basis of *Assignment Algorithm*. By selecting suitable constants to be added to or subtracted from the elements of the cost matrix we can ensure that each $c_{ij}^* \geq 0$ and can produce at least one $c_{ij}^* = 0$ in each row and each column and try to make assignments from among these 0 positions. The assignment schedule will be optimal if there is exactly one assignment (*i.e.*, exactly one assigned 0) in each row and each column.

Remarks. It may be noted that assignment problem is a variation of transportation problem with two characteristics (*i*) the cost matrix is a square matrix, and (*ii*) the optimum solution for the problem would always be such that there would be only one assignment in a given row or column of the cost matrix.

11:3. SOLUTION METHODS OF ASSIGNMENT PROBLEM

An assignment problem can be solved using the following four methods :

1. Complete Enumeration Method. In this method, a list of all possible assignments among the given resources and activities is prepared. Then an assignment involving the minimum cost, time or distance (or maximum profit) is selected. It represents the optimum solution. In case there are more than one assignment patterns involving the same least cost, then they all represent the optimum solutions — the problem has multiple optima.

In general, if there are n jobs and n workers, there are n ! possible assignments. Thus, the listing and evaluation of all the possible assignments is a simple matter when n is small. When n is large, this method is not very practical. For example, if there are 8 jobs and 8 workers, we have to evaluate a total of 8 ! or 40,320 assignments. The method, therefore, is not suitable for real world situations.

2. Transportation Method. Since an assignment problem is a special case of the transportation problem, it can be solved by transportation methods discussed in the previous chapter. However, every basic feasible solution of a general assignment problem having a square payoff matrix of order n should have $m + n - 1 = n + n - 1 = 2n - 1$ assignments or basic cells. But due to the special structure of this problem, any basic solution cannot have more than n assignments. Thus, the assignment problem is inherently degenerate. In order to remove degeneracy, $(n - 1)$ dummy allocations will be required to proceed with the transportation method. However, because of the large number of dummy allocations in the solution, the transportation method becomes computationally inefficient for solving an assignment problem.

3. Simplex Method. An assignment problem can be formulated as a transportation problem which, in turn, is itself a special case of an LPP. Accordingly, an assignment problem can be formulated as an LPP with integer valued variables and may be solved using a modified simplex method or otherwise. Here, the decision variables take only one of the two values : 1 or 0.

In general let

$$x_{ij} = \begin{cases} 1 & \text{if } i\text{th person is assigned } j\text{th job} \\ 0 & \text{if } i\text{th person is not assigned } j\text{th job} \end{cases}$$

The mathematical formulation of the assignment problem as a 0–1 integer linear programming problem would be :

$$\text{Minimize } z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij} \quad \text{subject to the constraints :}$$

$$x_{i1} + x_{i2} + \ldots + x_{in} = 1 \; ; \qquad i = 1, 2, \ldots, n$$

$$x_{1j} + x_{2j} + \ldots + x_{nj} = 1 \; ; \qquad j = 1, 2, \ldots, n$$

$$x_{ij} = 0 \text{ or } 1 \text{ for all } i \text{ and } j.$$

As can be seen in the general mathematical formulation of the assignment problem, there are $n \times n$ decision variables and $n + n$ or $2n$ equalities/equations. In particular, for a problem involving 5 workers/jobs, there will be 25 decision variables and 10 equalities. That means a simplex table having 25 columns and 10 rows. It is difficult to solve manually and hence this approach to the solution is not considered.

4. Hungarian Assignment Method. An efficient method for solving an assignment problem is the *Hungarian Assignment Method* (also known as reduced matrix method), which is based on the concept of opportunity cost. Opportunity costs show the relative penalties associated with assignment of resource to an activity as opposed to making the best or least cost assignment. If we can reduce the cost matrix to the extent of having at least one zero in each row and each column, then it will be possible to make optimal assignments (opportunity costs are all zero).

The method of solving an assignment problem (minimization case) consists of the following steps :

Step 1. Determine the cost table from the given problem.

(*i*) If the number of sources is equal to the number of destinations, go to *step 3.*

(*ii*) If the number of sources is not equal to the number of destinations, go to *step 2.*

Step 2. Add a dummy source or dummy destination, so that the cost table becomes a square matrix. The cost entries of dummy source/destinations are always zero.

Step 3. Locate the smallest element in each row of the given cost matrix and then subtract the same from each element of that row.

Step 4. In the reduced matrix obtained in *step 3*, locate the smallest element of each column and then subtract the same from each element of that column. Each column and row now have at least one zero.

Step 5. In the modified matrix obtained in *step 4*, search for an optimal assignment as follows :

(*a*) Examine the rows successively until a row with a single zero is found. Enrectangle this zero (□) and cross off (×) all other zeros in its column. Continue in this manner until all the rows have been taken care of.

(*b*) Repeat the procedure for each column of the reduced matrix.

(*c*) If a row and/or column has two or more zeros and one cannot be chosen by inspection, then assign arbitrary any one of these zeros and cross off all other zeros of that row/column.

(*d*) Repeat (*a*) through (*c*) above successively until the chain of assigning (□) or cross (×) ends.

Step 6. If the number of assignments (□) is equal to n (the order of the cost matrix), an optimum solution is reached.

If the number of assignments is less than n (the order of the matrix), go to the next step.

Step 7. Draw the minimum number of horizontal and/or vertical lines to cover all the zeros of the reduced matrix. This can be conveniently done by using a simple procedure :

(*a*) Mark (√) rows that do not have any assigned zero.

(*b*) Mark (√) columns that have zeros in the marked rows.

(*c*) Mark (√) rows that have assigned zeros in the marked columns.

(*d*) Repeat (*b*) and (*c*) above until the chain of marking is completed.

(*e*) Draw lines through all the *unmarked rows* and *marked columns*. This gives us the desired minimum number of lines.

Step 8. Develop the new revised cost matrix as follows :

(*a*) Find the smallest element of the reduced matrix not covered by any of the lines.

(*b*) Subtract this element from all the *uncovered* elements and add the same to all the elements lying at the *intersection* of any two lines.

Step 9. Go to *Step 6* and repeat the procedure until an optimum solution is attained.

SAMPLE PROBLEMS

1101. *A departmental head has four subordinates, and four tasks to be performed. The subordinates differ in efficiency, and the tasks differ in their intrinsic difficulty. His estimate, of the time each man would take to perform each task, is given in the matrix below :*

Tasks	*Men*			
	E	*F*	*G*	*H*
A	18	26	17	11
B	13	28	14	26
C	38	19	18	15
D	19	26	24	10

How should the tasks be allocated, one to a man, so as to minimize the total man-hours?

[Andhra B.E. (Mech. & Ind.) 1996]

Solution.

Step 1. Here, the number of tasks and the number of subordinates each equal 4, therefore the problem is balanced and we move on to *step 3.*

Step 3. Subtracting the smallest element of each row from every element of the corresponding row, we get the reduced matrix :

7	15	6	0
0	15	1	13
23	4	3	0
9	16	14	0

Step 4. Subtracting the smallest element of each column of the reduced matrix from every element of the corresponding column, we get the following reduced matrix :

7	11	5	0
0	11	0	13
23	0	2	0
9	12	13	0

Step 5. Starting with row *1*, we enrectangle (□) (i.e., make assignment) a single zero, if any, and cross (×) all other zeros in the column so marked. Thus, we get

7	11	5	[0]
[0]	11	✕	13
23	[0]	2	✕
9	12	13	✕

In the above matrix, we arbitrarily enrectangled a zero in column *1*, because row *2* had two zeros.

It may be noted that column *3* and row *4* do not have any assignment. So, we move on to the next step.

Step 7. (*i*) Since row *4* does not have any assignment, we mark this row (√).

(*ii*) Now there is a zero in the fourth column of the marked row. So, we mark fourth column (√).

(*iii*) Further there is an assignment in the first row of the marked column. So we mark first row (√).

(*iv*) Draw straight lines through all unmarked rows and marked columns. Thus, we have

7	11	5	[0]	√
[0]	11	~~0~~	13	
23	[0]	2	~~0~~	
9	12	13	~~0~~	√
			√	

Step 8. In *step 7*, we observe that the minimum number of lines so drawn is 3, which is less than the order of the cost matrix, indicating that the current assignment is not optimum.

To increase the minimum number of lines, we generate new zeros in the modified matrix.

The smallest element not covered by the lines is 5. Subtracting this element from all the uncovered elements and adding the same to all the elements lying at the intersection of the lines, we obtain the following new reduced cost matrix :

2	6	0	0
0	11	0	18
23	0	2	5
4	7	8	0

Step 9. Repeating *step 5* on the reduced matrix, we get

2	6	[0]	~~0~~
[0]	11	~~0~~	18
23	[0]	2	5
4	7	8	[0]

Now, since each row and each column has one and only one assignment, an optimal solution is reached. The optimum assignment is :

$$A \to G,\ B \to E,\ C \to F \text{ and } D \to H.$$

The minimum total time for this assignment scheduled is 17 + 13 + 19 + 10 or 59 man-hours.

1102. *A company wishes to assign 3 jobs to 3 machines in such a way that each job is assigned to some machine and no machine works on more than one job. The cost of assigning job i to machine j is given by the matrix below (ijth entry) :*

$$\text{Cost matrix :} \begin{bmatrix} 8 & 7 & 6 \\ 5 & 7 & 8 \\ 6 & 8 & 7 \end{bmatrix}$$

Draw the associated network. Formulate the network LPP and find the minimum cost of making the assignment. **[Madras B.Com. 2007]**

Solution. (*a*) Network formulation of the given problem is given as under :

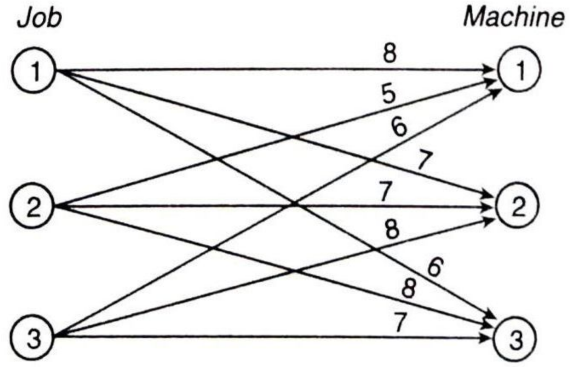

Fig. 11.1

(*b*) Linear programming formulation of the given problem is :

Minimize the total cost involved, *i.e.*,

Minimize $z = (8x_{11} + 7x_{12} + 6x_{13}) + (5x_{21} + 7x_{22} + 8x_{23}) + (6x_{31} + 8x_{32} + 7x_{33})$

subject to the constraints :

$$x_{i1} + x_{i2} + x_{i3} = 1 ; \quad i = 1, 2, 3$$

$$x_{1j} + x_{2j} + x_{3j} = 1 ; \quad j = 1, 2, 3$$

$$x_{ij} = 0 \text{ or } 1, \text{ for all } i \text{ and } j.$$

(*c*) Reduce the cost matrix by subtracting smallest element of each row (column) from the corresponding row (column) elements. In the reduced matrix, make assignments in rows and columns having single zeros and cross off all other zeros in rows and columns in the which the assignment has been made. See table *11.1*. Now, draw the minimum number of lines to cover all the zeros. For this, we proceed as follows:

(*i*) Mark (√) third row since it has no assignment.

(*ii*) Mark (√) first column, since third row has a zero in this column.

(*iii*) Mark (√) second row, since marked column has an assignment in the second row.

(*iv*) Since no other row or column can be marked, draw straight lines through the unmarked rows and marked column as shown in table *11.1* :

2	0	[0]	
[0]	1	3	√
0̸	1	1	√
√			

Table 11.1

3	[0]	0̸
[0]	0̸	2
0̸	0̸	[0]

Table 11.2

Modify the reduced cost matrix (table *11.1*) by selecting the smallest element among all the uncovered elements. Subtract this element from all the uncovered elements including itself and add it to the intersection element (1, 1) which lies at the intersection of two lines. The modified cost matrix so obtained is shown in table *11.2*.

In table *11.2*, we observe that there is no row and column which has single zero. So, we make an assignment arbitrarily at (1, 2) and cross off all zeros of first row and second column. Now, we get a single zero in the second row and therefore an assignment is made at (2, 1). Cross off all zeros in the first column. Finally, we make an assignment at (3, 3) being the single zero in the third row.

Clearly, the number of assignments in table *11.2* is equal to the order of the matrix. Hence, an optimum assignment has been attained, *viz.*,

Job *1* → Machine 2, Job *2* → Machine *1*, Job *3* → Machine *3*.

Total minimum cost will be (7 + 5 + 7), *i.e.*, 19.

1103. *A pharmaceutical company is producing a single product and is selling it through five agencies located in different cities. All of a sudden, there is a demand for the product in another five cities not having any agency of the company. The company is faced with the problem of deciding on how to assign the existing agencies to despatch the product to needy cities in such a way that the travelling distance is minimised. The distance between the surplus and deficit cities (in km) is given in the following table :*

		Deficit cities				
		a	*b*	*c*	*d*	*e*
Surplus cities	*A*	85	75	65	125	75
	B	90	78	66	132	78
	C	75	66	57	114	69
	D	80	72	60	120	72
	E	76	64	56	112	68

[Delhi M.Com. 2009]

Determine the optimum assignment schedule.

Solution. Subtracting the smallest element of each row from every element of that and then subtracting the smallest element of each coloumn from every element of that coloumn, we get the reduced distance table :

	a	*b*	*c*	*d*	*e*
A	2	2	0	4	0
B	6	4	0	10	2
C	0	1	0	1	2
D	2	4	0	4	2
E	2	0	0	0	2

Table 11.3

In the reduced distance table, we make assignments in rows and coloumns having single zeros and cross off all other zeros is those rows and columns, where assignments have been made. Now draw the minimum number of lines to cover all the zeros. This is done in the following steps :

(*i*) Mark (√) row '*D*' since it has no assignment.

(*ii*) Mark (√) coloumn '*C*' since row '*D*' has zero in this column.

(*iii*) mark (√) row *B* since column '*C*' has an assignment in row '*B*'.

(*iv*) Since no other rows or columns can be marked, draw straight lines through the unmarked rows '*A*', '*C*', and '*E*', and marked coloumn '*C*' as shown in Table *11.4.*.

	a	*b*	*c*	*d*	*e*	
A	2	2	~~0~~	4	[0]	
B	6	4	[0]	10	2	√
C	[0]	1	~~0~~	1	2	
D	2	4	~~0~~	4	2	√
E	2	[0]	~~0~~	~~0~~	2	
			√			

Table 11.4

	a	*b*	*c*	*d*	*e*
A	2	2	2	4	0
B	4	2	0	8	0
C	0	1	2	1	2
D	0	2	0	2	0
E	2	0	2	0	2

Table 11.5

Modify the reduced distance table (Table *11.4*) by subtracting the smallest element not covered by lines from all the uncovered elements and add the same at the intersection elements of the lines. The modified distance table so obtain is shown in Table *11.5.*

Repeat the above procedure to find the new assignment in table *11.6.*

	a	*b*	*c*	*d*	*e*	
A	2	2	2	4	[0]	√
B	4	2	[0]	8	~~0~~	√
C	[0]	1	2	1	2	√
D	~~0~~	2	~~0~~	2	~~0~~	√
E	2	[0]	2	~~0~~	2	
	√		√		√	

Table 11.6

	a	*b*	*c*	*d*	*e*
A	2	1	2	3	[0]
B	4	1	[0]	7	~~0~~
C	~~0~~	[0]	2	~~0~~	2
D	[0]	1	~~0~~	1	~~0~~
E	3	~~0~~	3	[0]	3

Table 11.7

Clearly the assignment shown in table *11.6* is also not optimum, since only four assignments are made. To get the next solution, we draw the minimum number of horizontal and vertical lines to cover all the zeros in table *11.6.* Subtracting the smallest unconvered element (*viz.*, 1) from all uncovered elements and adding the same to the intersection element of two lines gives us table *11.7.*

The new assignment schedule is shown in table *11.7*. Since, both the rows '*C*' and '*E*' have two zeros, the arbitrary selection of a cell in any of these two rows will give us an alternative solution having the same total distance.

Since the number of assignments is equal to the order of the given matrix, an optimum solution is attained, *viz.*,

$A \to e,\ B \to c,\ C \to b,\ D \to a,\ E \to d$; or $A \to e,\ B \to c,\ C \to d,\ D \to a,\ E \to b.$

Total minimum distance in both the cases will be 399 kilometres.

1104. *A department head has four tasks to be performed and three subordinates, the subordinates differ in efficiency. The estimates of the time, each subordinate would take to perform, is given below in the matrix. How should he allocate the tasks one to each man, so as to minimize the total man-hours?*

Task	*Men* *1*	*2*	*3*
I	9	26	15
II	13	27	6
III	35	20	15
IV	18	30	20

Solution. Here we have three subordinates who have to perform four tasks. So, the given problem is unbalanced and therefore we add a dummy subordinate (column) with all its entries as zero. The resulting balanced problem is :

Subordinate		*1*	*2*	*3*	*Dummy*
Task	*I*	9	26	15	0
	II	13	27	6	0
	III	35	20	15	0
	IV	18	30	20	0

Now, reduce the balanced time-matrix by subtracting the smallest element of each column from all the elements of that column. In the reduced matrix, make assignments in rows and columns having single zeros and cross off all other zeros of the rows and columns, where assignment have been made. We get the following assignment solution :

	1	*2*	*3*	*Dummy*
I	[0]	6	9	✗
II	4	7	[0]	✗
III	26	[0]	9	✗
IV	9	10	14	[0]

Table 11.8

The optimum assignment is

$I \to 1,\ II \to 3$ and $III \to 2$; while task *IV* should be assigned to a dummy man, *i.e.*, it remains to be done. The minimum time is 35 hours.

PROBLEMS

1105. Four professors are each capable of teaching any one of four different courses. Class preparation time in hours for different topics varies from professor to professor and is given in the table below. Each professor is assigned only one course. Determine an assignment schedule so as to minimize the total course preparation time for all courses :