

UNIT-III / BUSINESS ECONOMICS

PRODUCTION FUNCTION

A production function expresses the technological or engineering relationship between the output of a commodity and its factor inputs. Traditionally, economic theory considers four factors of production, namely, land, labour, capital and organisation or management. Now, technology is also considered as an important determinant, as it contributes to output growth. Therefore, output is a positive function of the quantities of land, labour, capital, the quality of management, and the level of technology employed in its production (Mote, et. al, 1997). This relationship may be expressed as follows:-

$$X = f(A, L, K, M, T)$$

Where, $f_1, f_2, f_3, f_4, f_5 > 0$

X = output of commodity X,

A = land employed in the production of X,

L = labour employed,

K = capital employed,

M = management employed,

T = technology used,

f = unspecified function, and

f_1 = partial derivative of f with respect to the ith independent variable.

This function describes a general production function. For the production of different commodities, one or all the factor inputs may not be equally important for all commodities. The importance of a factor of production varies from product to product. For instance, while land is the most important factor in the case of an agricultural product, its importance is relatively lower in the case of a manufacturing product. Meanwhile, the significance of management and technology may be greater in the case of an industrial product, rather than for an agricultural product. Therefore, researchers modify the production function according to the product and the specific objectives analysed.

Generally for the analysis of production decision problems, labour and capital are the only two factor inputs considered for convenience. Then, the production function reduces to:-

$$X = f(L, K)$$

For a given level of output of commodity X, various combinations of L and K may be used, which is known as production process or technology. Further, these combinations would also vary with variations in the level of X. Usually for production, both labour and capital are necessary and they substitute each other. When an entrepreneur employs more of labour than capital, then the production process is known as labour intensive production technique. Whereas, if more of capital is used in relation to labour, the production technique becomes capital intensive.

Producer's equilibrium

A producer is in equilibrium when he or she maximizes output for the given total outlay. In other words, a producer is in equilibrium when the highest isoquant is reached, given a particular isocost or price line. An isoquant (IQ) represents different combinations of labour and capital which yields the same level of output of a commodity. An isocost/price line represents the different combinations of labour and capital that an entrepreneur can purchase, given the prices of the two factor inputs and the total outlay available to the him/her at a point of time. Producer's equilibrium occurs when an isoquant is tangent to the isocost line. At the point of tangency, the absolute slope of the isoquant is equal to the absolute slope of the isocost line. That is, at equilibrium, $MRTS_{L,K} = P_L/P_K$. Since $MRTS_{L,K} = MP_L/MP_K$, at equilibrium,

$$\frac{MP_L}{MP_K} = \frac{P_L}{P_K} \quad \text{or} \quad \frac{MP_L}{P_L} = \frac{MP_K}{P_K}$$

Here, MP = marginal product; and $MRTS_{L,K}$ = marginal rate of technical substitution of labour(L) for capital (K). $MRTS_{L,K}$ is defined as the number of units of K given up to employ one more unit of L. P_L and P_K represent the prices of labour and capital respectively. The slope of the isocost line represents the price ratios of the two factor inputs, L and K, given the total outlay. Marginal product (MP) is the addition of output made to total output by employing one more unit of the factor input. The slope of the isoquant represents the MP ratios of L and K. At equilibrium, the MP of the last unit spent on labour is the same as the MP of the unit spent on capital. The same applies to the other factors, if the firm's production function is expressed in terms of more than two factors of production. Diagram-4 shows a producer's equilibrium.

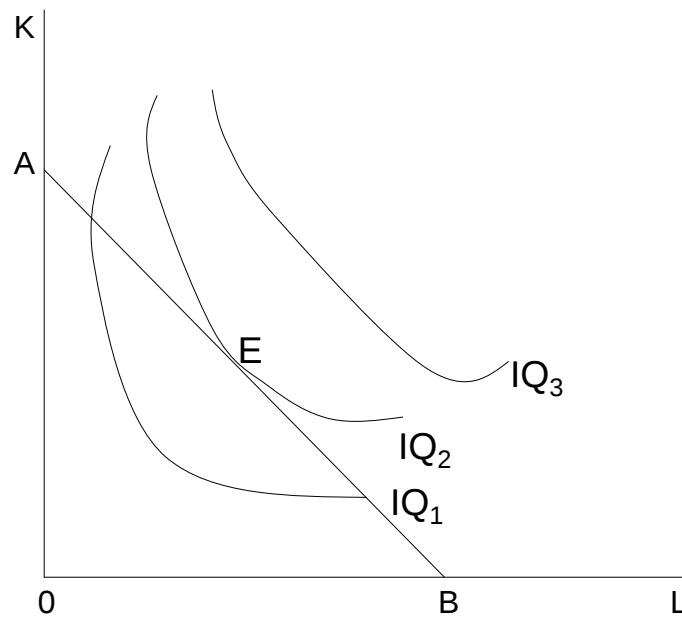


Diagram 4: Producer's equilibrium

Given the total outlay constraint AB, IQ_2 is the highest isoquant the firm can reach. IQ_3 is desirable, but not attainable with the given isocost line. Further, at IQ_1 the firm would not be maximizing output. Therefore, a rational producer who aims at maximum output with the given total outlay, would be at equilibrium at point E, where $MRTS_{L,K} = P_L/P_K$

Law of variable proportions

The production function shows the maximum quantity of the output that can be produced per unit of time for each set of alternative inputs, given the best available production technology available. In the short-run, atleast one factor of production remains fixed. For instance, in the case of an agricultural production function, various alternative commodities of labour or capital per unit of time may be used in relation a fixed amount of land. The total product curve increases at an increasing rate first until the point of inflection, after which it starts increasing at a decreasing rate, reaches its maximum and then starts declining. The average product of labour (AP_L/AP_K) is then obtained from total product (TP) divided by the number of units of labour/capital used. The marginal product of labour (MP_L/MP_K) represents the change in TP per unit change in the quantity of labour/capital used. The shapes of curve determine the shape of the AP_L and MP_L curves. The AP_L , at any point on the TP_L curve is given by the slope of the straight line from the origin to that point on the TP curve. The AP curve usually first rises,

reaches a maximum, and then falls, but remains positive as long as the TP is positive. The MP_L is equal to the slope of the TP curve, reflecting the change in output due to a unit change in input. between the two points. The MP curve also rises first, reaches a maximum (before the AP curve reaches its maximum), and then declines. The MP becomes zero when the TP is maximum. This is the law of diminishing returns. If labour is factor input considered, the relationship between the AP_L and MP_L curves can be used to define the three stages of production. Stage I starts from the origin to the point where the AP_L is maximum. Stage II starts from the point where the AP_L is maximum to the point where the MP_L is zero. Stage III covers the area over which the MP_L is negative. A rational producer will not operate in stage III, even with free labour, because it is possible to increase total output by using less labour on the given land. Likewise, a rational producer will not operate in stage I because it corresponds to the area where full TP is still increasing with an additional unit of labour employed. Therefore, a rational producer would only operate in stage II. In stage I; $MP_L > 0$, when TP is maximum $MP_L = 0$, and when TP falls $MP_L < 0$. This is shown by diagram-5.

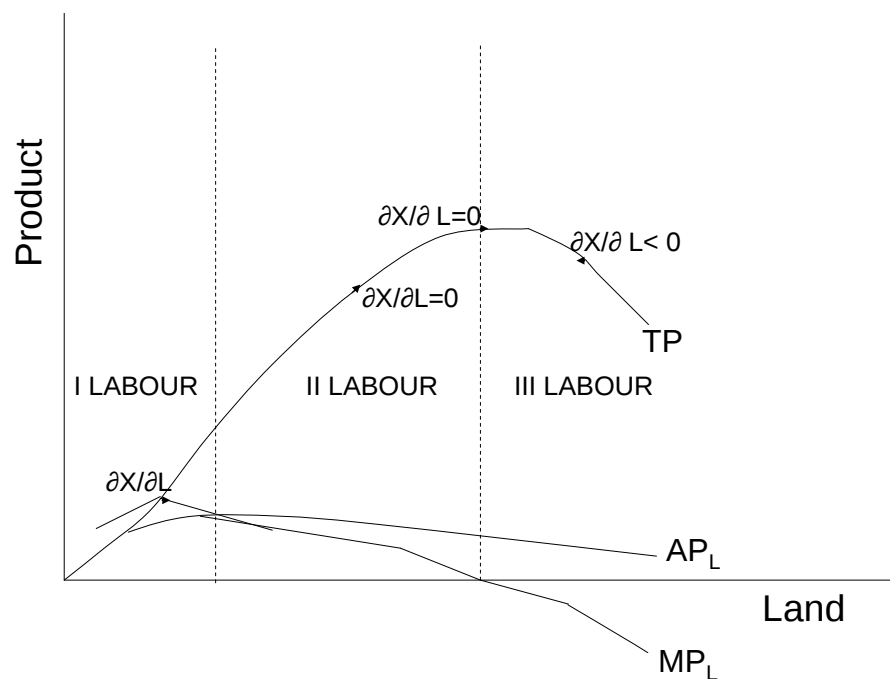


Diagram 4 : Law of Variable Proportions

Returns to scale

Law of returns to scale represents the long-term perspective of production analysis, when all factors of production are variable. There are three types of returns to scale.

(a) Constant returns to scale: This indicates that if all factors of production are increased in a given proportion then the output produced would also increase in exactly the same proportion. That is, if the quantities of labour or capital or both are increased by 10%, output would also increase by 10%. This is illustrated by diagram-6.

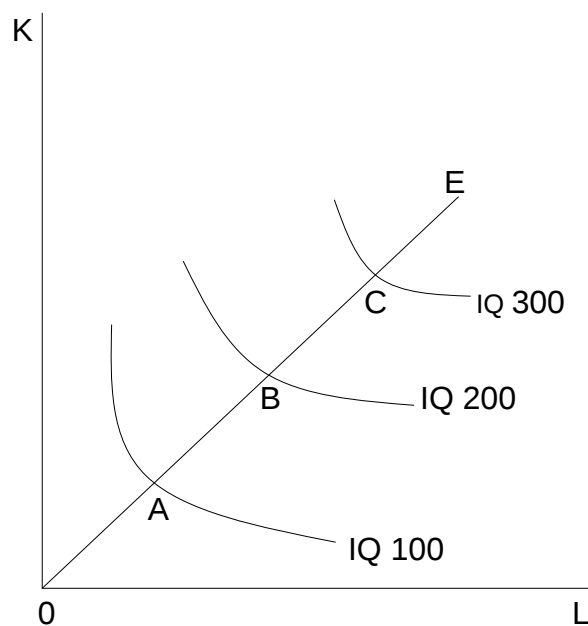


Diagram 6: Constant Returns to Scale

The equi-distance between successive isoquants along the product line reflects the phenomenon. Here, the distance $OA = AB = OC$ on the ray from the origin which is known as product line. It represents alternative paths of increasing output by increasing combinations of factor inputs L and K, regardless of their prices.

(b) Increasing returns to scale: It indicates that when all factors are increased in a given proportion, output increases in a greater proportion. That is, if labour and capital are increased by 10%, output increases by more than 10%. Increasing returns to scale may occur because of expansion in the scale of operation, and greater productive efficiency of managers and labour due to greater specialization. This is known as economies of scale. Diagram-7 illustrates this situation.

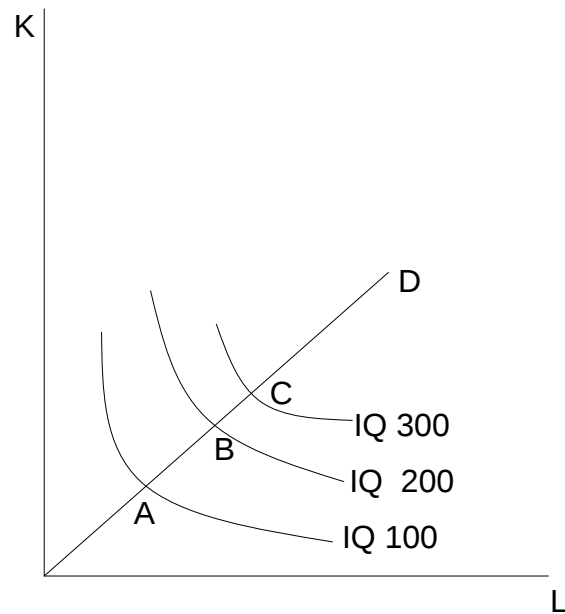


Diagram 7: Increasing Returns to Scale

In the diagram, the distance between successive isoquants go on decreasing, indicating that larger quantities of output may be produced with smaller quantities of factor inputs. This is shown along the product line, where $OA > AB > BC$.

(c) Decreasing returns to scale: It indicates that output increases in less than proportion to the increase in factor inputs. This may be due to the scale of operation beyond the optimum plant capacity, over-utilization of machineries resulting in wear and tear and break-down leading to increased maintenance cost, overworking labour, managerial constraints in over-seeing expanded business, wastage of raw materials, etc. This is known as diseconomies of scale. Diagram-8 shows this.

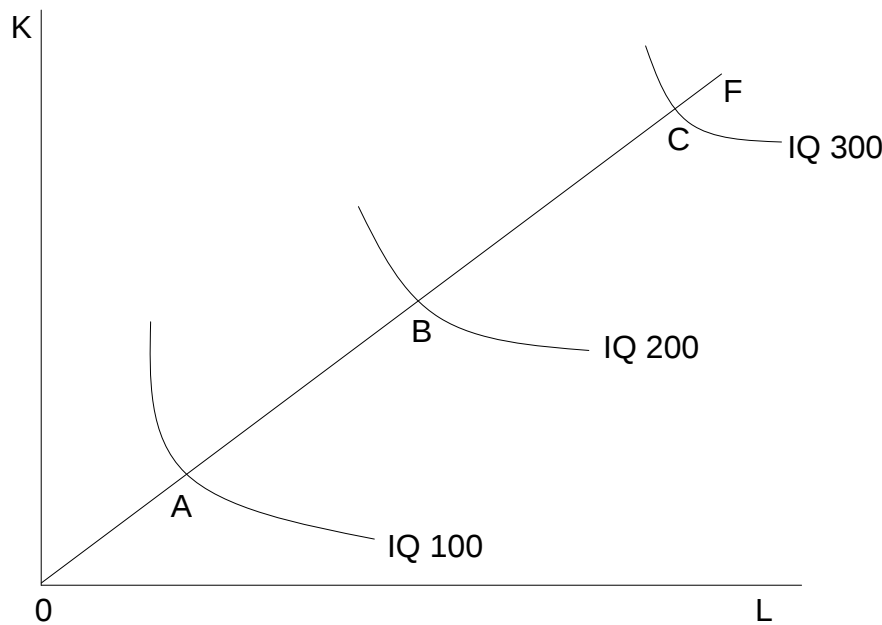


Diagram 8: Decreasing Returns to Scale

The diagram shows that the distance between the successive isoquants along the product line goes on increasing, due to increased input requirements resulting from diseconomies of scale. That is, $OA < AB < BC$.

COST CONCEPTS

A cost function expresses the relationship between the cost of production and levels of output. The various cost concepts are:-

- (1) **Social and private costs:** Social cost of producing a commodity refers to the opportunities of producing other commodities foregone, given the scarce resources. In simple terms, it is the cost of alternative good sacrificed by a community in producing a certain amount of one good. Private costs, on the other hand, include the costs incurred by an individual firm to obtain the resources used for the production of commodity. The reduction in private cost of a product would result in the reduction in of social cost, due to the emergence of a divergence between the social and private motives.
- (2) **Explicit and implicit costs:** Production of a commodity generally requires different kinds of labour and capital in many forms. Modern economists call the direct production expenses as the explicit costs of production. It includes the expenses incurred by a producer on buying the

productive services owned by others. Whereas, implicit costs include the evaluation of a producer's efforts and sacrifices incurred in production process. In other words, it refers to the reward a producer would like to pay self for self-owned and self-employed resources. They include a normal return on own investment, and the opportunity cost (alternative earnings) of own labour.

(3) **Economic versus accounting costs:** Accounting cost includes the expenses incurred on production process, in addition to the wear and tear of machines and equipments, which can be translated into monetary terms. The accountant records all the explicit costs in the account book, so as to compare them with the sale proceeds in order to compute profits. Whereas, economic cost includes all the implicit and explicit costs of production. It involves the estimation of opportunity cost, which is the price a factor of production can receive in any alternative use, including the implicit costs of the factors owned by the entrepreneur.

(4) **Sunk costs:** They are the costs which cannot be recovered, and therefore, are not included in the decision making process. They include the costs of highly specialized resources or inputs, which once installed, cannot be put to any alternative use. For e.g., a big plant or machine installed by a firm which has become obsolete or inoperative due to non-availability of some parts, then the money spent on it is known as a sunk cost. It is sunk because neither can the firm use it, nor sell it, or put it to any alternative use. Hence, sunk costs have no relevance in decision making.

(5) **Fixed and variable costs:** In production process, some factors are constant in the short run, while others are variable. Fixed costs are costs which do not vary with a change in output. The examples are interest on capital, rent on building, salaries to the staff, etc., which must be incurred, regardless of the level of output. On the other hand, variable costs change with the variations in the level of output. They include the payments made to the variable factors, such as wages paid to workers, raw materials, electricity, transportation cost, etc.,

Total cost is the sum total of fixed and variable costs.

$$\text{Total Cost} = \text{Fixed Costs} + \text{Variable Costs}$$

$$\text{or } \text{TC} = \text{FC} + \text{VC}$$

$$\text{ATC or Average Total Cost} = \frac{\text{TC}}{Q} = \frac{\text{FC}}{Q} + \frac{\text{VC}}{Q}$$

where, Q = quantity of output produced.

Thus, average cost of production is the sum total of average fixed cost and average variable cost.

In ordinary accounting statements generally only explicit or money costs incurred in production process are considered. However, for a realistic computation of costs, two additional variables must be included, viz., normal profit and implicit costs or opportunity costs (already seen). Normal profit refers to the returns which the owners expect to receive from the business done by the firm, in the absence of which they would prefer to quit. When total revenue (TR) is equal to total cost (TC), the firm earns only the expected minimum return on the capital invested, i.e. normal profit. Hence, normal profit is a part of the cost of production. When TR exceeds TC, the firm gets super normal profit, which is more than what a firm needs to remain in business. But when $TR = TC$, the firm earns only normal profit.

Theory of cost in the short run

Short-run is the duration of time in which some factors of production remain fixed, while other factors are variable. It is the time period during which a firm cannot vary its production capacity. This capacity is determined by the amount of fixed inputs or the size of the plant, and the costs associated with it, which must be paid by a firm regardless of the level of output. Meanwhile, the level of variable costs vary directly with the level of output. When returns to variable inputs increase at an increasing rate, variable costs would increase at a decreasing rate, and when returns to variable inputs increase at a decreasing rate, variable costs increase at an increasing rate

Thus, while the variable cost increases directly with the level of output, but total fixed cost remains unchanged.

Diagram shows the fixed, variable and total costs.

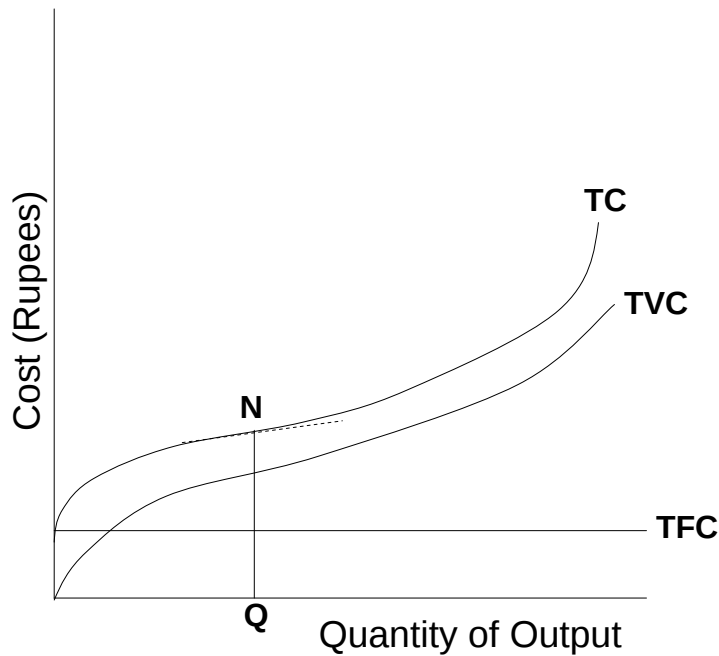


Diagram - 9 Fixed, Variable and Total Cost Curves

The diagram shows that both total variable cost (TVC) and total cost (TC) curves are increasing functions of the level of output. TVC initially increases at a decreasing rate and then at an increasing rate. However, the total fixed cost curve remains constant at all output levels. The shape of total cost curve (TC) reflects that at each level of output, total cost exceeds the variable cost. The vertical distance between TC and TVC represents total fixed cost. Initially, the distance between TC and TVC is relatively high when the proportion of fixed to total cost is high. But, as the level of output increases, fixed cost constitutes a small fraction of total cost and hence the TC converges towards TVC. This is because, with the rise in the level of output produced, the fixed cost gets distributed across larger units of output. This reduces the fixed cost per unit of output.

Total product and total variable cost: Logically, TVC is expected to be a strictly increasing function of output. Further, larger levels of output generally require greater outlays. This relationship may be expressed as follows: -

$$\begin{array}{ll} \text{But,} & \text{TVC} = f(Q) \\ \text{Therefore,} & Q = g(X), \\ & \text{TVC} = h(X) \end{array} \quad \text{---- (1)}$$

Equation (1) states that since total variable cost is a function of the level of output (Q), the output itself is a function of the level of variable input (X). Therefore, TVC is a function of variable input. However, as larger levels of Q require higher amounts of variable input, with constant price of X; this relationship can be written as follows: -

$$G(X_1) > f(X_0) \text{ if and only if } X_1 < X_0$$

Hence,

$$H(X_1) > h(X_0) \quad \text{---- (2)}$$

Equation (2) indicates that TVC will increase as the level of input use increases from X_0 to X_1 , which in turn results in an increase in output from Q to Q_1 , thus affecting TVC (Barla , 2000).

Average and marginal cost: Unit cost and the incremental cost of production can also be computed. It has been seen that total cost of production is the sum total of total fixed cost and total variable cost, i.e.,

$$TC = TFC + TVC \quad \text{----- (3)}$$

Dividing both sides of equation (3) by the quantity of output (Q) would give average cost or unit cost of production. Therefore, average cost is the sum total of average fixed cost and average variable cost. i.e.,

$$\frac{TC}{Q} = \frac{TFC}{Q} + \frac{TVC}{Q}$$

$$\text{or} \quad AC = AFC + AVC \quad \text{----- (4)}$$

As output increases, average fixed cost registers a decline. Average variable cost, average total cost and marginal cost, decline initially and then show an upward trend.

Total fixed cost divided by the quantity of output (Q) is average fixed cost (AFC). Section A of diagram - 10 shows that average fixed cost has an inverse relationship with the quantity of output, such that as output increases, AFC decreases. This makes AFC a rectangular hyperbola, because total fixed cost is divided by different levels of output. Thus,

$$Q.AFC = \bar{C}, \text{ a constant.}$$

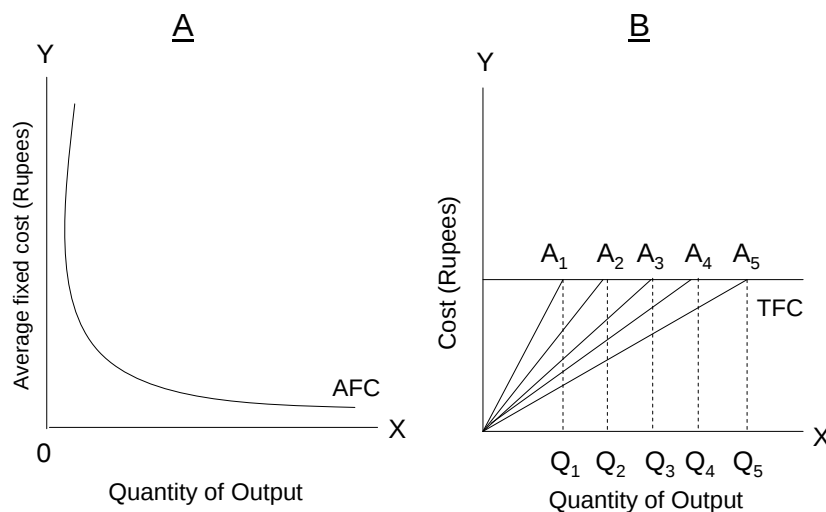
That is, the area under AFC is always equal to $TFC = C$, —

The AFC curve is a rectangular hyperbola ($Q.AFC = C$), which never touches the axes even if extended indefinitely upward or downward to the right. It only approaches the axes asymptotically. That is, as output increases AFC curve will approach closer to the X axis, but

will never touch it because the numerator is still a positive constant. The two axes in section A of diagram - 10 represent the asymptotes of the AFC function.

Section B of diagram-10 shows the derivation of AFC from in section A. Choice of certain points on TFC and dividing the vertical distance by the corresponding quantity of output would give the slopes of different rays starting from origin and extending to the TFC. The slopes of OA₁, OA₂, OA₃, OA₄ and OA₅ indicate average fixed cost at different output levels. Thus:

$$OA_1 = \frac{A_1X_1}{OQ_1} = \frac{TFC}{OQ_1}$$



Diagram–10: Derivation of Average Fixed Cost

It clearly shows that as OQ₁ increases, the slope of OA₁ declines. This illustrates the inverse relationship between AFC and the level of output (Barla 2000).

It is also possible to derive average variable cost (AVC) by measuring the slopes of different rays from the origin to the corresponding points on the TVC curve. However, analyzing the pattern of variation in AVC is more complicated as compared to the AFC, because both of these elements determine its change. Since $AVC = \frac{TVC}{Q}$, both the numerator (TVC) and the denominator (output) increase together, but not necessarily in the same proportion. The traditional AVC curve is 'U' shaped due to the operation of law of variable proportions.

Decreasing costs arise due to economies of scale reaped by a firm, whereas increasing costs occur due to diseconomies of scale. Section B in diagram-11 shows TVC at different levels of output. The change in slope indicates that TVC increases at different rates at different quantities of output. For instance, for producing OQ_1 units of output, the AVC is OC_1 , which is nothing but the slope of the ray OC at A on the TVC curve.

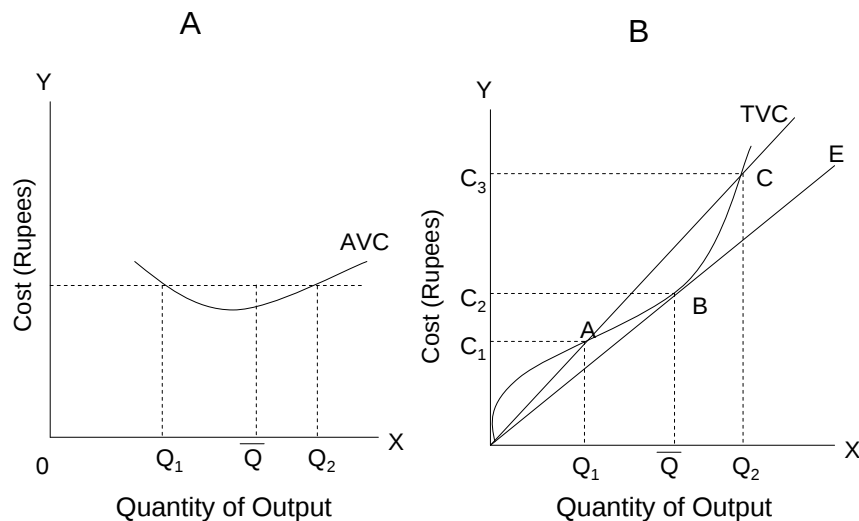


Diagram-11: Derivation of Average Variable Cost Curve

Marginal cost (MC) is the change in total cost due to an additional unit of output produced, i.e., $MC = \Delta C / \Delta Q$. Since total fixed cost remains fixed at all levels of output, the MC reflects the change in TVC only. The slope of TVC represents the marginal cost, i.e., $\Delta C / \Delta Q = MC$. The MC significantly influences the behaviour of a rational profit maximizing firm. It falls faster than the fall in AVC, becomes constant and cuts the AC and AVC at minimum and rises faster than the AVC. This is because it reflects the immediate changes in per unit cost in response to the increased in the level of output. This is the consequence of the law of variable proportions, the AC has its minimum to the right of AVC, because it combines AFC. Diagram - 12 illustrates this behaviour of the MC curve.

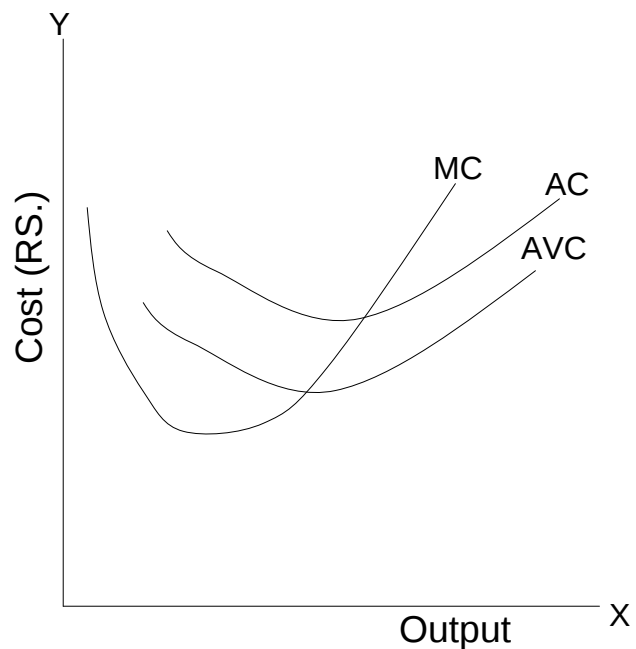


Diagram-12: Cost Curves

Long-run costs

Long-run is defined as the period in which all factors of production are variable. While, in the short-run some costs are fixed and others vary (variable costs), in the long-run all the costs are variable. Hence, the long run cost reflects the returns to scale. When a manager decides to increase all the factors of production, it is known as a change in the scale of a firm's operation. In response to the change in the scale, the firm may experience increasing, constant and/or diminishing returns to scale. These changes in returns may be expressed in terms of cost conditions as decreasing costs, and constant costs and/or increasing costs. This is shown in the below diagram .

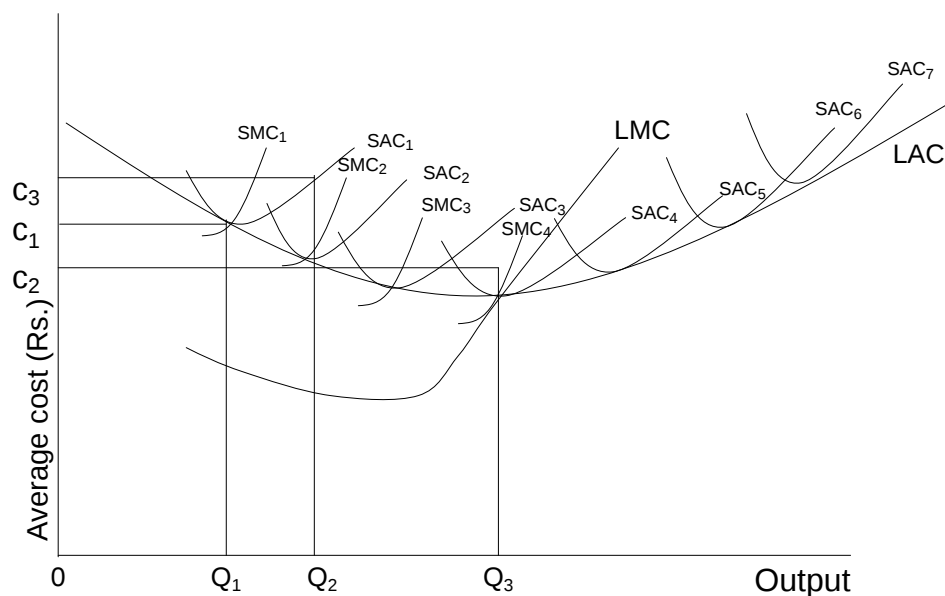


Diagram-13: Long Run Cost Curves

At the initial short-run average cost SAC_1 , the firm produces OQ_1 units of output at the per unit cost OC_1 . When the manager plans to increase output to OQ_2 units, the average cost would be OC_3 on the rising part of the SAC_1 cost curve if the same plant is used. On the other hand, if an additional plant is installed, the cost would fall to OC_2 ($OC_2 < OC_1$). Thus, the installation of a new plant decreases the cost per unit of output. The diagram shows that average cost will successively fall till the installation of the fourth plant. The lowest AC level is reached at output level OQ_3 . This level is known as the optimum level of output, at which the long run average cost (LAC) is minimum and the LMC cuts it from below. Here, the long run equilibrium condition of $LAC = LMC$ and LMC cutting LAC from below have been reached. If output increases beyond OQ_3 , the LAC would rise for every additional plants installed. No rational manager would install new plant beyond it, as they wish to make atleast normal profits in the long run. The long run average cost curve (LAC) is also known as envelope curve as it envelopes several average cost curves corresponding to different plant size. Further, it is also known as a planning curve, as it guides the manager in planning the future expansion of plant and output.

ECONOMIES OF SCALE

What are Economies of Scale?

Economies of scale refer to the cost advantage experienced by a firm when it increases its level of output. The advantage arises due to the inverse relationship between per-unit fixed cost and the quantity produced. The greater the quantity of output produced, the lower the per-unit fixed cost.

Economies of scale also result in a fall in average variable costs (average non-fixed costs) with an increase in output. This is brought about by operational efficiencies and synergies as a result of an increase in the scale of production.

Economies of scale can be realized by a firm at any stage of the production process. In this case, production refers to the economic concept of production and involves all activities related to the commodity, not involving the final buyer. Thus, a business can decide to implement economies of scale in its marketing division by hiring a large number of marketing professionals. A business can also adopt the same in its input sourcing division by moving from human labor to machine labor.

Effects of Economies of Scale on Production Costs

1. It reduces the per-unit fixed cost. As a result of increased production, the fixed cost gets spread over more output than before.
2. It reduces per-unit variable costs. This occurs as the expanded scale of production increases the efficiency of the production process.

Types of Economies of Scale

1. Internal Economies of Scale

This refers to economies that are unique to a firm. For instance, a firm may hold a patent over a mass production machine, which allows it to lower its average cost of production more than other firms in the industry.

2. External Economies of Scale

These refer to economies of scale enjoyed by an entire industry. For instance, suppose the government wants to increase steel production. In order to do so, the government announces that all steel producers who employ more than 10,000 workers will be given a 20% tax break. Thus, firms employing less than 10,000 workers can potentially lower their average cost of production by employing more workers. This is an example of an external economy of scale – one that affects an entire industry or sector of the economy.

Sources of Economies of Scale

1. Purchasing

Firms might be able to lower average costs by buying the inputs required for the production process in bulk or from special wholesalers.

2. Managerial

Firms might be able to lower average costs by improving the management structure within the firm. The firm might hire better skilled or more experienced managers.

3. Technological

A technological advancement might drastically change the production process. For instance, fracking completely changed the oil industry a few years ago. However, only large oil firms that could afford to invest in expensive fracking equipment could take advantage of the new technology.

Diseconomies of Scale

As firms get larger, they grow in complexity. Such firms need to balance the economies of scale against the diseconomies of scale. For instance, a firm might be able to implement certain economies of scale in its marketing division if it increased output. However, increasing output might result in diseconomies of scale in the firm's management division.

Frederick Herzberg, a distinguished professor of management, suggested a reason why companies should not blindly target economies of scale:

“Numbers numb our feelings for what is being counted and lead to adoration of the economies of scale. Passion is in feeling the quality of experience, not in trying to measure it.”