

# Chapter 1: Digital Systems and Binary Numbers

- Digital age and information age
- Digital computers
  - general purposes
  - many scientific, industrial and commercial applications
- Digital systems
  - telephone switching exchanges
  - digital camera
  - electronic calculators, PDA's
  - digital TV
- Discrete information-processing systems
  - manipulate discrete elements of information

Reference : Digital Design: With an Introduction to the Verilog HDL, VHDL, and SystemVerilog, 6th Edition

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# Signal

- An information variable represented by physical quantity
- For digital systems, the variable takes on discrete values
  - Two level, or binary values are the most prevalent values
- Binary values are represented abstractly by:
  - digits 0 and 1
  - words (symbols) False (F) and True (T)
  - words (symbols) Low (L) and High (H)
  - and words On and Off.
- Binary values are represented by values or ranges of values of physical quantities

## Digital System

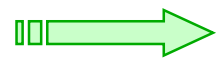
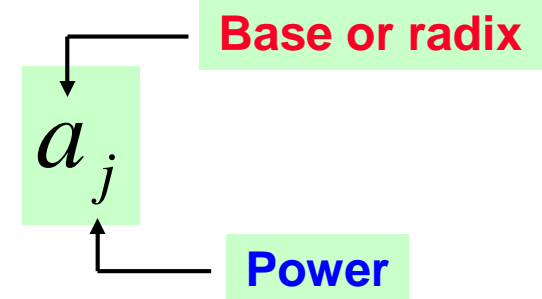
- A digital system is an interconnection of digital modules. **To understand the operation of each digital module, it is necessary to have a basic knowledge of digital circuits and their logical function.**
- A major trend in digital design methodology is the use of a HDL to describe and simulate the functionality of a digital circuit, **it is important that students become familiar with an HDL-based design methodology.**

# Binary Numbers

- Decimal number

$$\dots a_5 a_4 a_3 a_2 a_1 \cdot a_{-1} a_{-2} a_{-3} \dots$$

↑ **Decimal point**



$$\dots + 10^5 a_5 + 10^4 a_4 + 10^3 a_3 + 10^2 a_2 + 10^1 a_1 + 10^0 a_0 + 10^{-1} a_{-1} + 10^{-2} a_{-2} + 10^{-3} a_{-3} + \dots$$

**Example:**

$$7,329 = 7 \times 10^3 + 3 \times 10^2 + 2 \times 10^1 + 9 \times 10^0$$

- General form of base-r system

$$a_n \cdot r^n + a_{n-1} \cdot r^{n-1} + \dots + a_2 \cdot r^2 + a_1 \cdot r^1 + a_0 + a_{-1} \cdot r^{-1} + a_{-2} \cdot r^{-2} + \dots + a_{-m} \cdot r^{-m}$$

**Coefficient:**  $a_j = 0$  to  $r - 1$

# Binary Numbers

**Example: Base-2 number**

$$(11010.11)_2 = (26.75)_{10}$$

$$= 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 + 1 \times 2^{-1} + 1 \times 2^{-2}$$

**Example: Base-5 number**

$$(4021.2)_5$$

$$= 4 \times 5^3 + 0 \times 5^2 + 2 \times 5^1 + 1 \times 5^0 + 2 \times 5^{-1} = (511.5)_{10}$$

**Example: Base-8 number**

$$(127.4)_8$$

$$= 1 \times 8^3 + 2 \times 8^2 + 1 \times 8^1 + 7 \times 8^0 + 4 \times 8^{-1} = (87.5)_{10}$$

**Example: Base-16 number**

$$(B65F)_{16} = 11 \times 16^3 + 6 \times 16^2 + 5 \times 16^1 + 15 \times 16^0 = (46,687)_{10}$$

# Binary Numbers

**Example: Base-2 number**

$$(110101)_2 = 32 + 16 + 4 + 1 = (53)_{10}$$

## Special Powers of 2

- $2^{10}$  (1024) is Kilo, denoted "K"
- $2^{20}$  (1,048,576) is Mega, denoted "M"
- $2^{30}$  (1,073, 741,824) is Giga, denoted "G"

## Powers of two

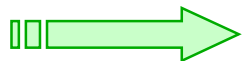


Table 1.1

**Table 1.1**  
*Powers of Two*

<b><math>n</math></b>	<b><math>2^n</math></b>	<b><math>n</math></b>	<b><math>2^n</math></b>	<b><math>n</math></b>	<b><math>2^n</math></b>
0	1	8	256	16	65,536
1	2	9	512	17	131,072
2	4	10	1,024 (1K)	18	262,144
3	8	11	2,048	19	524,288
4	16	12	4,096 (4K)	20	1,048,576 (1M)
5	32	13	8,192	21	2,097,152
6	64	14	16,384	22	4,194,304
7	128	15	32,768	23	8,388,608

## Arithmetic operation

Arithmetic operations with numbers in base  $r$  follow the same rules as decimal numbers.

# Binary Arithmetic

- Single Bit Addition with Carry
- Multiple Bit Addition
- Single Bit Subtraction with Borrow
- Multiple Bit Subtraction
- Multiplication
- BCD Addition

# Binary Arithmetic

- **Addition**

Augend: 101101

Addend: +100111

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Sum: 1010100

- **Subtraction**

Minuend: 101101

Subtrahend: -100111

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Difference: 000110

- **Multiplication**

<b>Multiplicand</b>	<b>1011</b>
<b>Multiplier</b>	<b>× 101</b>
<b>Partial Products</b>	<b>1011</b>
	<b>0000 -</b>
	<b>1011 - -</b>
<b>Product</b>	<b>110111</b>

## Number-Base Conversions

Name	Radix	Digits
Binary	2	0,1
Octal	8	0,1,2,3,4,5,6,7
Decimal	10	0,1,2,3,4,5,6,7,8,9
Hexadecimal	16	0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F

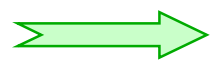
- The six letters (in addition to the 10 integers) in hexadecimal represent: 10, 11, 12, 13, 14, and 15, respectively.

# Number-Base Conversions

## Example 1.1

Convert decimal 41 to binary. The process is continued until the integer quotient becomes 0.

	<b>Integer Quotient</b>		<b>Remainder</b>		<b>Coefficient</b>
$41/2 =$	20	+	1/2		$a_0 = 1$
$20/2 =$	10	+	0		$a_1 = 0$
$10/2 =$	5	+	0		$a_2 = 0$
$5/2 =$	2	+	1/2		$a_3 = 1$
$2/2 =$	1	+	0		$a_4 = 0$
$1/2 =$	0	+	1/2		$a_5 = 1$



$$(41)_{10} = (a_5a_4a_3a_2a_1a_0)_2 = (101001)_2$$

# Number-Base Conversions

- ♣ The arithmetic process can be manipulated more conveniently as follows:

Integer	Remainder
41	
20	1
10	0
5	0
2	1
1	0
0	1

101001 = answer

# Number-Base Conversions

## Example 1.2

Convert decimal 153 to octal. The required base  $r$  is 8.

Integer	Remainder
153	
19	1
2	3
0	2

$= (231)_8$

## Example 1.3

Convert  $(0.6875)_{10}$  to binary.

The process is continued until the fraction becomes 0 or until the number of digits has sufficient accuracy.

# Number-Base Conversions

## Example 1.3

	Integer		Fraction		Coefficient
$0.6875 \times 2 =$	1	+	0.3750	$a_{-1} =$	1
$0.3750 \times 2 =$	0	+	0.7500	$a_{-2} =$	0
$0.7500 \times 2 =$	1	+	0.5000	$a_{-3} =$	1
$0.5000 \times 2 =$	1	+	0.0000	$a_{-4} =$	1



$$(0.6875)_{10} = (0.a_{-1}a_{-2}a_{-3}a_{-4})_2 = (0.1011)_2$$

- ♣ To convert a decimal fraction to a number expressed in base  $r$ , a similar procedure is used. However, multiplication is by  $r$  instead of 2, and the coefficients found from the integers may range in value from 0 to  $r - 1$  instead of 0 and 1.

# Number-Base Conversions

## Example 1.4

Convert  $(0.513)_{10}$  to octal.

$$0.513 \times 8 = 4.104$$

$$0.104 \times 8 = 0.832$$

$$0.832 \times 8 = 6.656$$

$$0.656 \times 8 = 5.248$$

$$0.248 \times 8 = 1.984$$

$$0.984 \times 8 = 7.872$$



$$(0.513)_{10} = (0.406517\dots)_8$$

♣ From Examples 1.1 and 1.3:  $(41.6875)_{10} = (101001.1011)_2$

♣ From Examples 1.2 and 1.4:  $(153.513)_{10} = (231.406517)_8$

# Octal and Hexadecimal Numbers

♣ Numbers with different bases: [Table 1.2](#).

**Table 1.2**  
*Numbers with Different Bases*

<b>Decimal (base 10)</b>	<b>Binary (base 2)</b>	<b>Octal (base 8)</b>	<b>Hexadecimal (base 16)</b>
00	0000	00	0
01	0001	01	1
02	0010	02	2
03	0011	03	3
04	0100	04	4
05	0101	05	5
06	0110	06	6
07	0111	07	7
08	1000	10	8
09	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

## Octal and Hexadecimal Numbers

- ♣ Conversion from binary to octal can be done by positioning the binary number into groups of three digits each, starting from the binary point and proceeding to the left and to the right.

$$\begin{array}{cccccccccccc} (10 & 110 & 001 & 101 & 011 & \cdot & 111 & 100 & 000 & 110) & _2 = & (26153.7406) & _8 \\ 2 & 6 & 1 & 5 & 3 & & 7 & 4 & 0 & 6 & & & \end{array}$$

- ♣ Conversion from binary to hexadecimal is similar, except that the binary number is divided into groups of four digits:

$$\begin{array}{cccccccc} (10 & 1100 & 0110 & 1011 & \cdot & 1111 & 0010) & _2 = & (2C6B.F2) & _{16} \\ 2 & C & 6 & B & & F & 2 & & & \end{array}$$

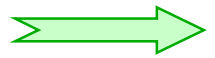
- ♣ Conversion from octal or hexadecimal to binary is done by reversing the preceding procedure.

$$\begin{array}{cccccccc} (673.124) & _8 = & (110 & 111 & 011 & \cdot & 001 & 010 & 100) & _2 \\ & & 6 & 7 & 3 & & 1 & 2 & 4 & \end{array}$$

$$\begin{array}{ccccccc} (306.D) & _{16} = & (0011 & 0000 & 0110 & \cdot & 1101) & _2 \\ & & 3 & 0 & 6 & & D & \end{array}$$

# Complements of Numbers

- ♣ There are two types of complements for each base- $r$  system: the radix complement and diminished radix complement.



the  $r$ 's complement and the second as the  $(r - 1)$ 's complement.

## ■ Diminished Radix Complement

Given a number  $N$  in base  $r$  having  $n$  digits, the  $(r - 1)$ 's complement of  $N$  is defined as  $(r^n - 1) - N$ . For decimal numbers,  $r = 10$  and  $r - 1 = 9$ , so the 9's complement of  $N$  is  $(10^n - 1) - N$ .

### Example:

The 9's complement of 546700 is  $999999 - 546700 = 453299$ .

The 9's complement of 012398 is  $999999 - 012398 = 987601$ .

- ♣ For binary numbers,  $r = 2$  and  $r - 1 = 1$ , so the 1's complement of  $N$  is  $(2^n - 1) - N$ .

### Example:

The 1's complement of 1011000 is 0100111

The 1's complement of 0101101 is 1010010

# Complements of Numbers

## ■ Radix Complement

The  $r$ 's complement of an  $n$ -digit number  $N$  in base  $r$  is defined as  $r^n - N$  for  $N \neq 0$  and as 0 for  $N = 0$ . Comparing with the  $(r - 1)$ 's complement, we note that the  $r$ 's complement is obtained by adding 1 to the  $(r - 1)$ 's complement, since  $r^n - N = [(r^n - 1) - N] + 1$ .

### Example: Base-10

The 10's complement of 012398 is 987602

The 10's complement of 246700 is 753300

### Example: Base-10

The 2's complement of 1101100 is 0010100

The 2's complement of 0110111 is 1001001

# Complements of Numbers

## ■ Subtraction with Complements

The subtraction of two  $n$ -digit unsigned numbers  $M - N$  in base  $r$  can be done as follows:

1. Add the minuend  $M$  to the  $r$ 's complement of the subtrahend  $N$ . Mathematically,  $M + (r^n - N) = M - N + r^n$ .
2. If  $M \geq N$ , the sum will produce an end carry  $r^n$ , which can be discarded; what is left is the result  $M - N$ .
3. If  $M < N$ , the sum does not produce an end carry and is equal to  $r^n - (N - M)$ , which is the  $r$ 's complement of  $(N - M)$ . To obtain the answer in a familiar form, take the  $r$ 's complement of the sum and place a negative sign in front.

## Complements of Numbers

### Example 1.5

Using 10's complement, subtract  $72532 - 3250$ .

$$\begin{array}{r} M = 72532 \\ 10\text{'s complement of } N = \underline{+96750} \\ \text{Sum} = 169282 \\ \text{Discard end carry } 10^5 = \underline{-100000} \\ \text{Answer} = 69282 \end{array}$$

### Example 1.6

Using 10's complement, subtract  $3250 - 72532$

$$\begin{array}{r} M = 03250 \\ 10\text{'s complement of } N = \underline{+27468} \\ \text{Sum} = 30718 \end{array} \quad \Rightarrow \quad \text{There is no end carry.}$$



Therefore, the answer is  $-(10\text{'s complement of } 30718) = -69282$ .

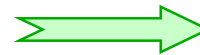
## Complements of Numbers

### Example 1.7

Given the two binary numbers  $X = 1010100$  and  $Y = 1000011$ , perform the subtraction (a)  $X - Y$  and (b)  $Y - X$  by using 2's complement.

$$\begin{array}{r} \text{(a)} \quad X = \quad 1010100 \\ \quad \quad 2\text{'s complement of } Y = \quad \underline{+0111101} \\ \quad \quad \text{Sum} = \quad 10010001 \\ \quad \quad \text{Discard end carry } 2^7 = \quad \underline{-10000000} \\ \quad \quad \text{Answer. } X - Y = \quad 0010001 \end{array}$$

$$\begin{array}{r} \text{(b)} \quad Y = \quad 1000011 \\ \quad \quad 2\text{'s complement of } X = \quad \underline{+0101100} \\ \quad \quad \text{Sum} = \quad 1101111 \end{array}$$



There is no end carry.  
Therefore, the answer is  
 $Y - X = -$  (2's complement  
of 1101111) =  $-0010001$ .

## Complements of Numbers

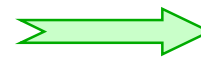
- Subtraction of unsigned numbers can also be done by means of the  $(r - 1)$ 's complement. Remember that the  $(r - 1)$ 's complement is one less than the  $r$ 's complement.

### Example 1.8

Repeat Example 1.7, but this time using 1's complement.

$$\begin{array}{r} \text{(a) } X - Y = 1010100 - 1000011 \\ X = 1010100 \\ \text{1's complement of } Y = \pm 0111100 \\ \text{Sum} = 10010000 \\ \text{End-around carry} = \quad \underline{+ \quad 1} \\ \text{Answer. } X - Y = 0010001 \end{array}$$

$$\begin{array}{r} \text{(b) } Y - X = 1000011 - 1010100 \\ Y = 1000011 \\ \text{1's complement of } X = \quad \underline{+ 0101011} \\ \text{Sum} = 1101110 \end{array}$$



There is no end carry,  
Therefore, the answer is  
 $Y - X = -$  (1's complement  
of 1101110) =  $- 0010001$ .

# Signed Binary Numbers

- ♣ To represent negative integers, we need a notation for negative values.
- ♣ It is customary to represent the sign with a bit placed in the leftmost position of the number.
- ♣ The convention is to make the sign bit 0 for positive and 1 for negative.

## Example:

Signed-magnitude representation:	10001001
Signed-1's-complement representation:	11110110
Signed-2's-complement representation:	11110111

- ♣ **Table 3** lists all possible four-bit signed binary numbers in the three representations.

# Signed Binary Numbers

**Table 1.3**  
*Signed Binary Numbers*

<b>Decimal</b>	<b>Signed-2's Complement</b>	<b>Signed-1's Complement</b>	<b>Signed Magnitude</b>
+7	0111	0111	0111
+6	0110	0110	0110
+5	0101	0101	0101
+4	0100	0100	0100
+3	0011	0011	0011
+2	0010	0010	0010
+1	0001	0001	0001
+0	0000	0000	0000
-0	—	1111	1000
-1	1111	1110	1001
-2	1110	1101	1010
-3	1101	1100	1011
-4	1100	1011	1100
-5	1011	1010	1101
-6	1010	1001	1110
-7	1001	1000	1111
-8	1000	—	—

# Signed Binary Numbers

## ■ Arithmetic Addition

The addition of two numbers in the signed-magnitude system follows the rules of ordinary arithmetic. If the signs are the same, we add the two magnitudes and give the sum the common sign. If the signs are different, we subtract the smaller magnitude from the larger and give the difference the sign of the larger magnitude.

- ♣ The addition of two signed binary numbers with negative numbers represented in signed-2's-complement form is obtained from the addition of the two numbers, including their sign bits.
- ♣ A carry out of the sign-bit position is discarded.

### Example:

+ 6	00000110	- 6	11111010
<u>+13</u>	<u>00001101</u>	<u>+13</u>	<u>00001101</u>
+ 19	00010011	+ 7	00000111
+ 6	00000110	- 6	11111010
<u>-13</u>	<u>11110011</u>	<u>-13</u>	<u>11110011</u>
- 7	11111001	- 19	11101101

# Binary Codes

## ■ BCD Code

**Table 1.4**  
*Binary-Coded Decimal (BCD)*

<b>Decimal Symbol</b>	<b>BCD Digit</b>
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001

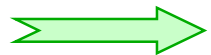
A number with  $k$  decimal digits will require  $4k$  bits in BCD. Decimal 396 is represented in BCD with 12bits as 0011 1001 0110, **with each group of 4 bits representing one decimal digit**. A decimal number in BCD is the same as its equivalent binary number only when the number is between 0 and 9. A BCD number greater than 10 looks different from its equivalent binary number, even though both contain 1's and 0's. Moreover, **the binary combinations 1010 through 1111 are not used and have no meaning in BCD.**

# Signed Binary Numbers

## ■ Arithmetic Subtraction

### ♣ In 2's-complement form:

1. Take the 2's complement of the subtrahend (including the sign bit) and add it to the minuend (including sign bit).
2. A carry out of sign-bit position is discarded.



$$(\pm A) - (+B) = (\pm A) + (-B)$$

$$(\pm A) - (-B) = (\pm A) + (+B)$$

### Example:

$$(-6) - (-13) \quad \longrightarrow \quad (11111010 - 11110011)$$

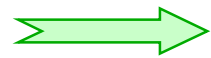
$$\quad \longrightarrow \quad (11111010 + 00001101)$$

$$\quad \longrightarrow \quad 00000111 (+7)$$

# Binary Codes

## Example:

Consider decimal 185 and its corresponding value in BCD and binary:



$$(185)_{10} = (0001\ 1000\ 0101)_{\text{BCD}} = (10111001)_2$$

## ■ BCD Addition

4	0100	4	0100	8	1000
<u>+5</u>	<u>+0101</u>	<u>+8</u>	<u>+1000</u>	<u>+9</u>	<u>+1001</u>
9	1001	12	1100	17	10001
			<u>+0110</u>		<u>+0110</u>
			10010		10111

# Binary Codes

## Example:

Consider the addition of  $184 + 576 = 760$  in BCD:

BCD	1	1		
	0001	1000	0100	184
	<u>+ 0101</u>	<u>0111</u>	<u>0110</u>	+576
Binary sum	0111	10000	1010	
Add 6	_____	<u>0110</u>	<u>0110</u>	_____
BCD sum	0111	0110	0000	760

## ■ Decimal Arithmetic

0	375
<u>+ 9</u>	<u>760</u>
0	135

# Binary Codes

## ■ Other Decimal Codes

**Table 1.5**  
*Four Different Binary Codes for the Decimal Digits*

<b>Decimal Digit</b>	<b>BCD 8421</b>	<b>2421</b>	<b>Excess-3</b>	<b>8, 4, -2, -1</b>
0	0000	0000	0011	0000
1	0001	0001	0100	0111
2	0010	0010	0101	0110
3	0011	0011	0110	0101
4	0100	0100	0111	0100
5	0101	1011	1000	1011
6	0110	1100	1001	1010
7	0111	1101	1010	1001
8	1000	1110	1011	1000
9	1001	1111	1100	1111
Unused bit combinations	1010	0101	0000	0001
	1011	0110	0001	0010
	1100	0111	0010	0011
	1101	1000	1101	1100
	1110	1001	1110	1101
	1111	1010	1111	1110

# Binary Codes

## ■ Gray Code

**Table 1.6**  
*Gray Code*

<b>Gray Code</b>	<b>Decimal Equivalent</b>
0000	0
0001	1
0011	2
0010	3
0110	4
0111	5
0101	6
0100	7
1100	8
1101	9
1111	10
1110	11
1010	12
1011	13
1001	14
1000	15

# Binary Codes

## ■ ASCII Character Code

**Table 1.7**

*American Standard Code for Information Interchange (ASCII)*

$b_4b_3b_2b_1$	$b_7b_6b_5$							
	000	001	010	011	100	101	110	111
0000	NUL	DLE	SP	0	@	P	`	p
0001	SOH	DC1	!	1	A	Q	a	q
0010	STX	DC2	“	2	B	R	b	r
0011	ETX	DC3	#	3	C	S	c	s
0100	EOT	DC4	\$	4	D	T	d	t
0101	ENQ	NAK	%	5	E	U	e	u
0110	ACK	SYN	&	6	F	V	f	v
0111	BEL	ETB	‘	7	G	W	g	w
1000	BS	CAN	(	8	H	X	h	x
1001	HT	EM	)	9	I	Y	i	y
1010	LF	SUB	*	:	J	Z	j	z
1011	VT	ESC	+	;	K	[	k	{
1100	FF	FS	,	<	L	\	l	
1101	CR	GS	-	=	M	]	m	}
1110	SO	RS	.	>	N	^	n	~
1111	SI	US	/	?	O	-	o	DEL

# Binary Codes

## ■ ASCII Character Code

## Control Characters

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NUL	Null	DLE	Data-link escape
SOH	Start of heading	DC1	Device control 1
STX	Start of text	DC2	Device control 2
ETX	End of text	DC3	Device control 3
EOT	End of transmission	DC4	Device control 4
ENQ	Enquiry	NAK	Negative acknowledge
ACK	Acknowledge	SYN	Synchronous idle
BEL	Bell	ETB	End-of-transmission block
BS	Backspace	CAN	Cancel
HT	Horizontal tab	EM	End of medium
LF	Line feed	SUB	Substitute
VT	Vertical tab	ESC	Escape
FF	Form feed	FS	File separator
CR	Carriage return	GS	Group separator
SO	Shift out	RS	Record separator
SI	Shift in	US	Unit separator
SP	Space	DEL	Delete

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# Binary Codes

## ■ ASCII Character Code

### Control characters

---

NUL	Null	DLE	Data-link escape
SOH	Start of heading	DC1	Device control 1
STX	Start of text	DC2	Device control 2
ETX	End of text	DC3	Device control 3
EOT	End of transmission	DC4	Device control 4
ENQ	Enquiry	NAK	Negative acknowledge
ACK	Acknowledge	SYN	Synchronous idle
BEL	Bell	ETB	End-of-transmission block
BS	Backspace	CAN	Cancel
HT	Horizontal tab	EM	End of medium
LF	Line feed	SUB	Substitute
VT	Vertical tab	ESC	Escape
FF	Form feed	FS	File separator
CR	Carriage return	GS	Group separator
SO	Shift out	RS	Record separator
SI	Shift in	US	Unit separator
SP	Space	DEL	Delete

---

## ASCII Character Codes

- American Standard Code for Information Interchange (**Refer to Table 1.7**)
- A popular code used to represent information sent as character-based data.
- It uses 7-bits to represent:
  - 94 Graphic printing characters.
  - 34 Non-printing characters
- Some non-printing characters are used for text format (e.g. BS = Backspace, CR = carriage return)
- Other non-printing characters are used for record marking and flow control (e.g. STX and ETX start and end text areas).

## ASCII Properties

**ASCII has some interesting properties:**

- **Digits 0 to 9 span Hexadecimal values  $30_{16}$  to  $39_{16}$  .**
- **Upper case A - Z span  $41_{16}$  to  $5A_{16}$  .**
- **Lower case a - z span  $61_{16}$  to  $7A_{16}$  .**
  - **Lower to upper case translation (and vice versa) occurs by flipping bit 6.**
- **Delete (DEL) is all bits set, a carryover from when punched paper tape was used to store messages.**
- **Punching all holes in a row erased a mistake!**

# Binary Codes

## ■ Error-Detecting Code

- ♣ To detect errors in data communication and processing, an eighth bit is sometimes added to the ASCII character to indicate its parity.
- ♣ A *parity* bit is an extra bit included with a message to make the total number of 1's either even or odd.

### Example:

Consider the following two characters and their even and odd parity:

	<b>With even parity</b>	<b>With odd parity</b>
ASCII A = 1000001	01000001	11000001
ASCII T = 1010100	11010100	01010100

# Binary Codes

## ■ Error-Detecting Code

- Redundancy (e.g. extra information), in the form of extra bits, can be incorporated into binary code words to detect and correct errors.
- A simple form of redundancy is parity, an extra bit appended onto the code word to make the number of 1's odd or even. Parity can detect all single-bit errors and some multiple-bit errors.
- A code word has even parity if the number of 1's in the code word is even.
- A code word has odd parity if the number of 1's in the code word is odd.

# Binary Storage and Registers

## ■ Registers

- ♣ A *binary cell* is a device that possesses two stable states and is capable of storing one of the two states.
- ♣ A *register* is a group of binary cells. A register with  $n$  cells can store any discrete quantity of information that contains  $n$  bits.

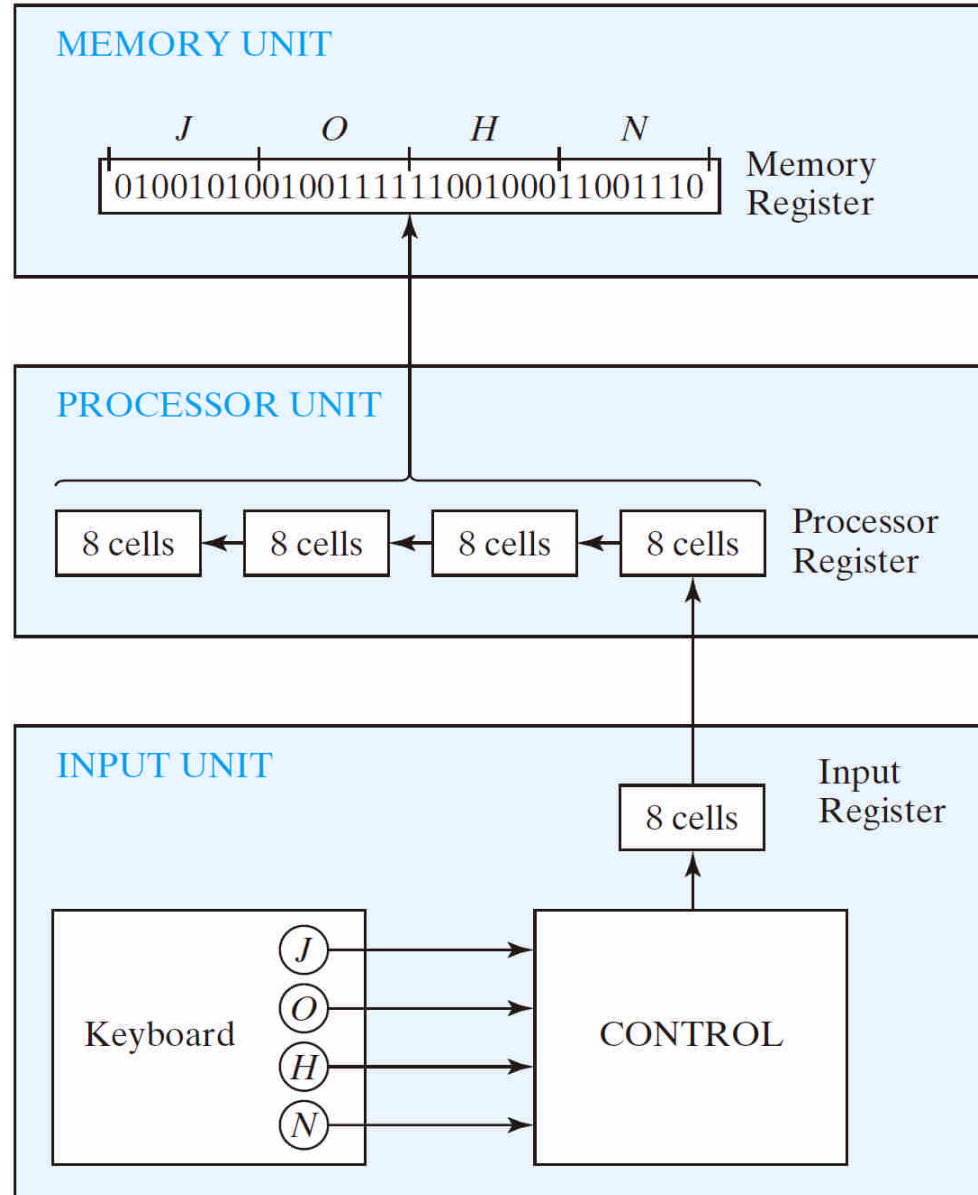
$n$  cells   $2^n$  possible states

- A binary cell
  - two stable state
  - store one bit of information
  - examples: flip-flop circuits, ferrite cores, capacitor
- A register
  - a group of binary cells
  - AX in x86 CPU

## ■ Register Transfer

- ♣ a transfer of the information stored in one register to another
- ♣ one of the major operations in digital system
- ♣ an example

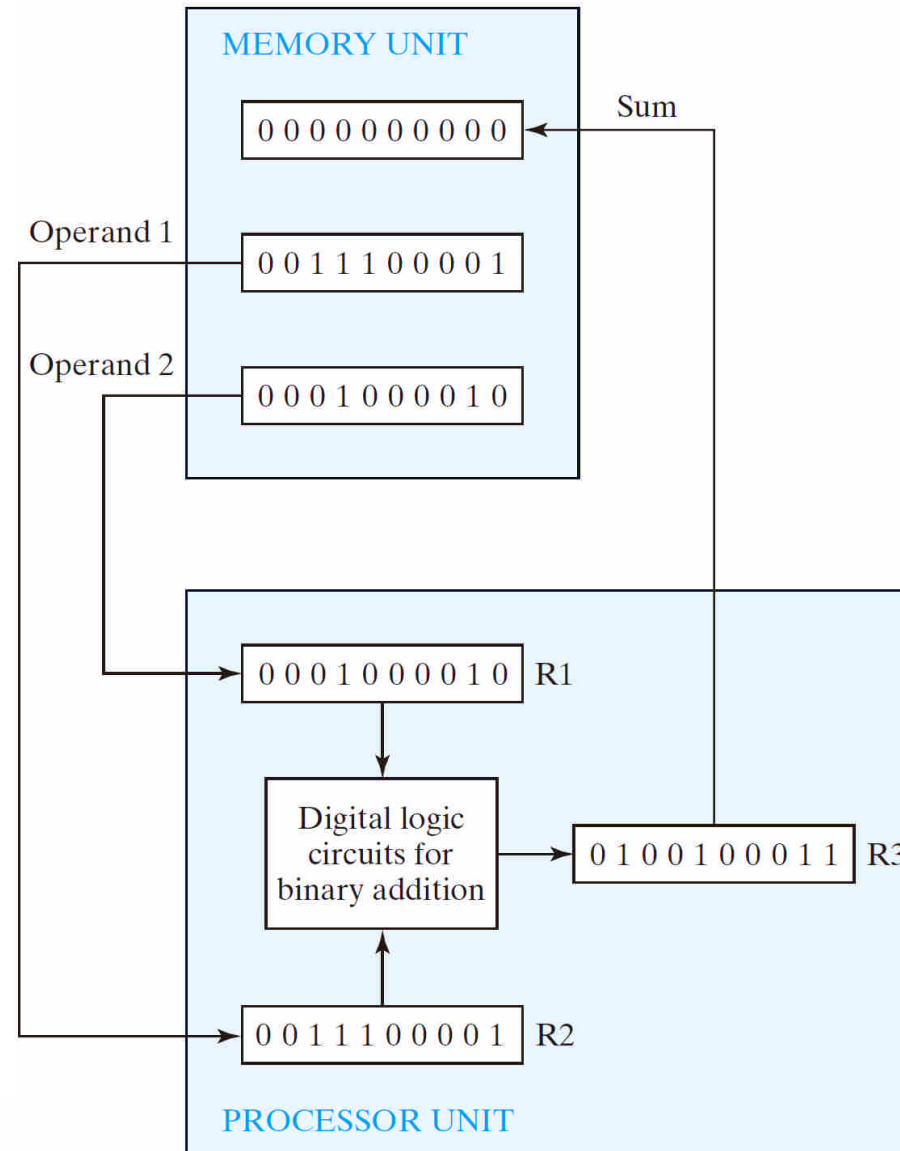
# Transfer of information



**FIGURE 1.1**

Transfer of information among registers

- **The other major component of a digital system**
  - **circuit elements to manipulate individual bits of information**



**FIGURE 1.2**  
Example of binary information processing

# Binary Logic

## ■ Definition of Binary Logic

- ♣ Binary logic consists of binary variables and a set of logical operations. The variables are designated by letters of the alphabet, such as A, B, C, x, y, z, etc, with each variable having two and only two distinct possible values: 1 and 0, There are three basic logical operations: AND, OR, and NOT.

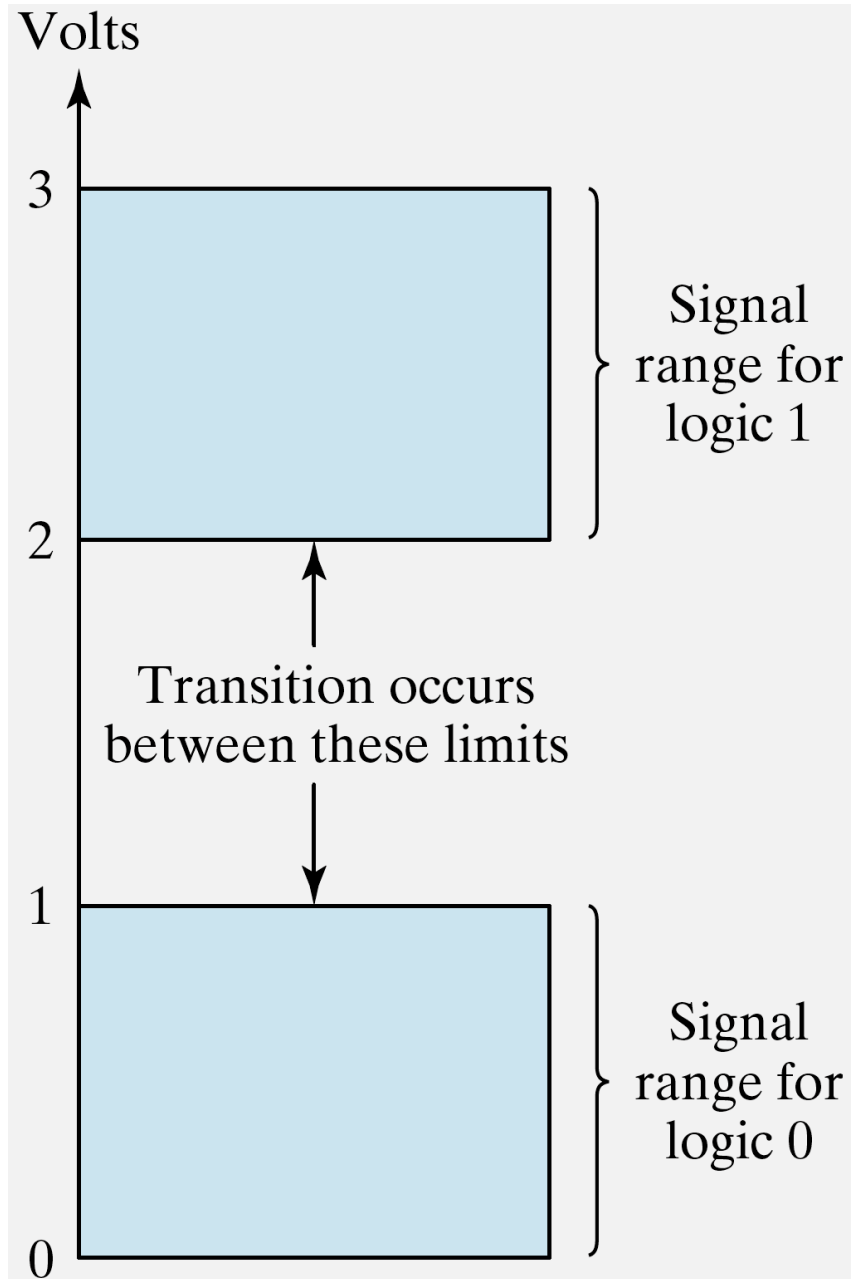
1. AND: This operation is represented by a dot or by the absence of an operator. For example,  $x \cdot y = z$  or  $xy = z$  is read “x AND y is equal to z,” The logical operation AND is interpreted to mean that  $z = 1$  if only  $x = 1$  and  $y = 1$ ; otherwise  $z = 0$ . (Remember that  $x$ ,  $y$ , and  $z$  are binary variables and can be equal either to 1 or 0, and nothing else.)
2. OR: This operation is represented by a plus sign. For example,  $x + y = z$  is read “x OR y is equal to z,” meaning that  $z = 1$  if  $x = 1$  or  $y = 1$  or if both  $x = 1$  and  $y = 1$ . If both  $x = 0$  and  $y = 0$ , then  $z = 0$ .
3. NOT: This operation is represented by a prime (sometimes by an overbar). For example,  $x' = z$  (or  $\bar{x} = z$ ) is read “not x is equal to z,” meaning that  $z$  is what  $x$  is not. In other words, if  $x = 1$ , then  $z = 0$ , but if  $x = 0$ , then  $z = 1$ , The NOT operation is also referred to as the complement operation, since it changes a 1 to 0 and a 0 to 1.



# Binary Logic

## ■ Logic gates

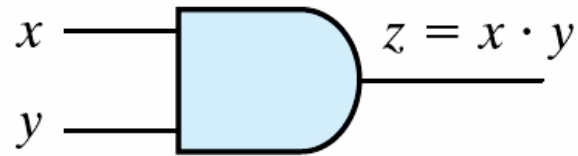
### ♣ Example of binary signals



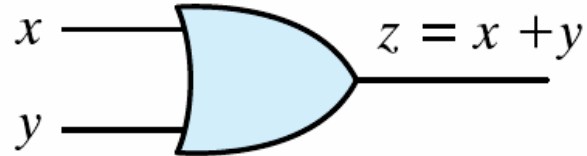
# Binary Logic

## Logic gates

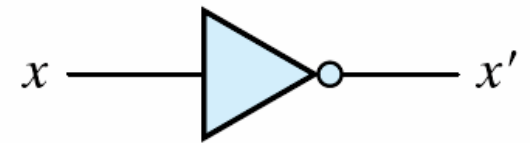
♣ Graphic Symbols and Input-Output Signals for Logic gates:



(a) Two-input AND gate



(b) Two-input OR gate

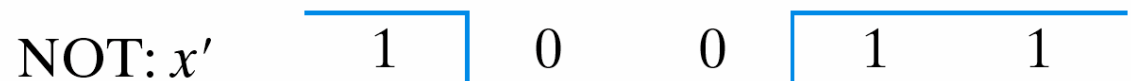
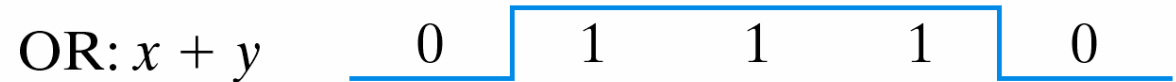
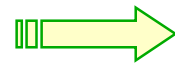


(c) NOT gate or inverter

Fig. 1.4 Symbols for digital logic circuits



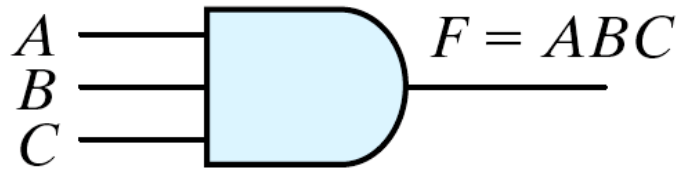
Fig. 1.5  
Input-Output signals  
for gates



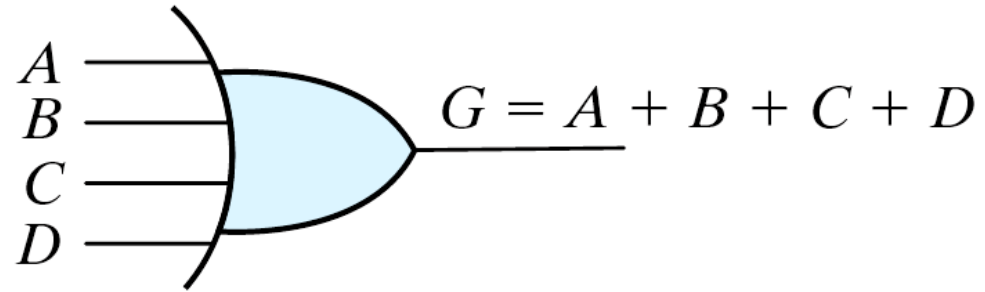
# Binary Logic

## ■ Logic gates

♣ Graphic Symbols and Input-Output Signals for Logic gates:



(a) Three-input AND gate



(b) Four-input OR gate

**Fig. 1.6 Gates with multiple inputs**

## Web Search Topic

- **BCD code**
- **ASCII**
- **Storage register**
- **Binary logic**
- **BCD addition**
- **Binary codes**
- **Binary numbers**
- **Excess-3 code**