3 UNIT TRANSPORTATION & ASSIGNMENT PROBLEM

The TP is one inwhich the objective is to transport various quantities of a single homogeneous commodity, that are initially stored at various sources (origin), to different destinations in such a way that the total transportation cost is a minimum.

Suppose there are "m" sources of supply $S_1, S_2, \cdots, S_m$ having $a_1, a_2, \cdots, a_m$ units of supply to transported among "n" destinations $D_1, D_2, \cdots, D_n$ with $b_1, b_2, \cdots, b_n$ units of demand. Let c_{ij} be the cost of transporting one unit of commodity from the source S_i to the destination D_j and x_{ij} = quantity transported from the source S_i to the destination D_j.

Destination

origin (or) Source		D_1	D_2	$\cdots$	D_n	Supply
	S_1	x_{11} c_{11}	x_{12} c_{12}		x_{1n} c_{1n}	a_1
	S_2	x_{21} c_{21}	x_{22} c_{22}		x_{2n} c_{2n}	a_2
	$\vdots$					$\vdots$
	S_m	x_{m1} c_{m1}	x_{m2} c_{m2}		x_{mn} c_{mn}	a_m
Demand		b_1	b_2	$\cdots$	b_n	

Then the problem is to determine transportation schedule so as to minimize the total transportion cost satisfying supply and demand constraints.

ie) Minimize $Z = \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij} c_{ij}$ subject to the constraints:

$$\sum_{j=1}^{n} x_{ij} = a_i, \quad (i = 1, 2, \cdots, m)$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \quad (j = 1, 2, \cdots, n) \text{ and } x_{ij} \geq 0 \text{ for all } i \text{ and } j.$$

Note :- Since $x_{ij} \geq 0 \Rightarrow a_i \geq 0$ and $b_j \geq 0$.

If any $a_k = 0 \Rightarrow x_{kj} = 0$ for all $j = 1, 2, \cdots, n$ and thus can be eliminated from the problem. Therefore $a_i > 0$ and $b_j > 0$.

Example :- A company manufacturing air-coolers has two plants located at Hyderabad and Mumbai with a capacity of 300 units & 100 units per week respectively. The company supplies the air-coolers to its four showrooms situated at Bangalore, Chennai, Delhi and Ernakulam which have a maximum demand of 85, 150, 150 & 55 units respectively. Due to the differences in raw material cost and transportation cost, the profit per unit in rupees differs which is shown in the table below:

Destinations

Source (origin)	D_1 Bangalore	D_2 Chennai	D_3 Delhi	D_4 Ernakulam	Supply
Hyderabad (S_1)	x_{11} 110	x_{12} 90	x_{13} 75	x_{14} 55	300 (a_1)
Mumbai (S_2)	x_{21} 65	x_{22} 50	x_{23} 80	x_{24} 45	100 (a_2)
Demand:	85 (b_1)	150 (b_2)	150 (b_3)	55 (b_4)	

Let x_{ij} = the quantity of air-coolers to transported from S_1, S_2 to D_1, D_2, D_3, D_4 respectively.

The problem is to determine x_{ij} so as to maximize the total profit.

Maximize $Z = \sum_{i=1}^{2} \sum_{j=1}^{4} x_{ij} p_{ij}$ subject to the constraints:

$$\sum_{j=1}^{4} x_{ij} = a_i \quad (i = 1, 2)$$

$$\sum_{i=1}^{2} x_{ij} = b_j \quad (j = 1, 2, 3, 4) \text{ and all } x_{ij} \geq 0.$$

Existence of Feasible solution:- A necessary and sufficient condition for the existence of feasible solution to a general T.P is that Total supply = Total demand $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$

Existence of optimum solution:- when the total supply is equal to the total demand, there always exists an optimum solution to a T.P.

Basic feasible solutions:- out of $(m+n)$ equations (constraints), we have only $(m+n-1)$ linearly independent equations because of total demand = total supply. The basic feasible solution will consist of atmost $(m+n-1)$ positive variables, the rest being zero.

* A feasible solution involving exactly $(m+n-1)$ positive variables is known as non-degenerate basic feasible solution, otherwise it is said to be degenerate basic feasible solution.

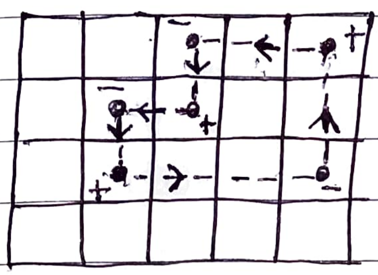
* when the total supply = total demand, the TP is called balanced T.P, otherwise it is said to be unbalanced.

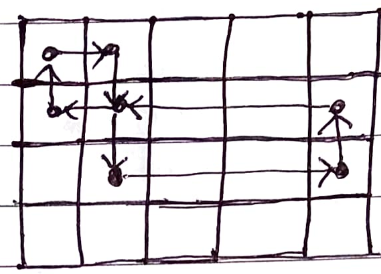
* The allocated cells in the transportation table will be called occupied cells and empty cells will be called non-occupied cells.

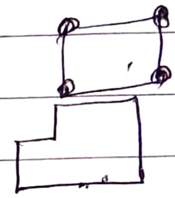
* In order to avoid difficulty to discriminate between degenerate and non-degenerate basic solution, we shall adopt the convention that while displaying a basic feasible solution in a transportation table, the value of any particular x_{ij} is actually entered if and only if x_{ij} is a basic variable.

* **Loops in T.P:-** In a TP, an ordered set of 4 or more cells is said to form a LOOP if (i) any two adjacent cells in the ordered set lie either in the same row or in the same Column and (ii) any 3 or more adjacent cells in the ordered set do not lie in the same row or the same column.

Examples:- The ordered set of cells L = {(3,2), (2,2), (2,3), (1,3), (1,5), (3,5), (3,2)} is a Loop whereas {(3,2), (3,5), (2,5), (2,2), (2,1), (1,1), (1,2), (2,2), (3,2)} is not a Loop.







Note: The first cell of the set is considered to follow the last in the set, i.e, each cell (except the first) must appear only once in the ordered set.

Remark:- 1) Every loop has an even number of cells.

(2) The allocations are said to be in independent positions if it is not possible to increase or decrease any independent individual allocation without changing the positions of these allocations. i.e) a closed Loop cannot be formed through these allocations.

(3) Each row & Column in the TP should have only one plus and minus sign. All cells that have a plus or minus sign, except the starting unoccupied (non-basic) cell are occupied (basic) cells.

(4) Closed loops may or may not be square in shape.

* Set containing a loop:- A set X of cells of a TP is said to contain a LOOP if the cells of X or a subset of X can be ordered so as to form a loop.

* A feasible solution to a TP is basic iff the corresponding cells in the TP do not contain a loop.

Solution of a TP:-

The solution to a TP involves the following major steps:

Step 1. Formulate the given problem as a LPP.

Step 2. Set up the given LPP in tabular form known as a TP.

Step 3. Examine whether total supply equals total demand. If not, Introduce a dummy row/column having all its cost elements as zero and supply / Demand as (+ve) difference of supply and demand.

Step 4: Find an initial basic feasible solution that must satisfy all the ~~empty~~ Supply and demand conditions.

Step 5: Examine the solution obtained in step 4 for optimality. ie) examine whether an improved transportation schedule with lower cost is possible.

Step 6: If the solution is not optimal, modify the shipping schedule by including that unoccupied cell whose inclusion may result in an improved solution.

Step 7: Repeat step 4 until no further improvement is possible.

Finding an Initial basic feasible solution:

1. NORTH - WEST corner method.

Step 1. Select the north-west (upper left hand) corner cell of the TP and allocate $x_{11} = \min(a_1, b_1)$

Step 2. If $b_1 > a_1$, we move down vertically to the 2^{nd} row and make the 2^{nd} allocation of magnitude $x_{21} = \min(a_2, b_1 - x_{11})$ in cell $(2,1)$

~~Step 3~~ If $b_1 < a_1$, move right horizontally to the 2^{nd} column and make the second allocation of magnitude $x_{12} = \min(a_1 - x_{11}, b_2)$ in the cell $(1,2)$

If $b_1 = a_1$, there is a tie for the 2^{nd} allocation and one can make 2^{nd} allocation of magnitude

$x_{12} = \min(a_1 - a_1, b_1) = 0$ in the cell $(1,2)$ (or)

$x_{21} = \min(a_2, b_1 - b_1) = 0$ in the cell $(2,1)$.

Step3: Repeat steps 1 and 2 moving down towards the lower/ right corner of the Transportion table until all the rim requirements are satisfied.

Note: 1) The cells which get allocation will be called basic cells.

2) The initial basic feasible solution obtained by means of north-west Corner rule may be far from optimum, because the costs completely ignored.

2. Least-cost method or Matrix minima method:- This method takes into account the minimum unit cost and can be summarized as follows:

Step1. Determine the smallest cost in the cost matrix of the transportation table. Let it be c_{ij}. Allocate $x_{ij} = \min(a_i, b_j)$ in the cell (i, j).

Step2: If $x_{ij} = a_i$, cross off the i^{th} row of the Transportation table and decrease b_j by a_i. Goto Step 3.

If $x_{ij} = b_j$, cross off the j^{th} column of the transportion table and decrease a_i by b_j. Goto Step 3.

If $x_{ij} = a_i = b_j$, cross off either the i^{th} row or j^{th} Column but not both.

Step3: Repeat steps 1 and 2 for the resulting reduced transportation table until all the rim requirements are satisfied. whenever the minimum cost is not unique, make an arbitrary choice among the minimum.

3. Vogel's approximation method (VAM).

The VAM takes into account not only the least cost c_{ij} but also the cost that just exceed c_{ij}. The steps of the method are given below:

Step1:- For each row of the transportation table, identity the smallest and the next-to-smallest costs. Determine the difference between them for each row. Display them along side the transportation table by enclosing them in parenthesis against the respective rows. Similary, compute the differences for each column.

Step 2: Identify the row or column with the largest difference among all the rows & columns. If a tie occurs, use any arbitrary tie-breaking choice. Let the greatest difference correspond to i^{th} row and let c_{ij} be the smallest cost in the i^{th} row. Allocate the maximum feasible amount $x_{ij} = \min(a_i, b_j)$ in the (i,j) cell and cross off the i^{th} row or the j^{th} column in the usual manner.

Step 3 Recompute the column and row difference for the reduced transportation table and goto step 2. Repeat the procedure until all the rim requirements are satisfied.

Problem:-

1) obtain an initial basic feasible solution to the following transportation problem using the North-west Corner rule:-

	D	E	F	G	Available (a_i)
A	11	13	17	14	250
B	16	18	14	10	300
C	21	24	13	10	400
Requirements (b_j)	200	225	275	250	

Solution:- Since $\sum_{i=1}^{3} a_i = \sum_{j=1}^{4} b_j = 950$, there exists a feasi[ble] solution to the TP.

	D	E	F	G	Supply (a_i)
A	200 11	13	17	14	250
B	16	18	14	10	300
C	21	24	13	10	400
Demands (b_j)	200	225	275	250	

	E	F	G	
A	50 13	17	14	50
B	18	14	10	30
C	24	13	10	40
	~~225~~ 175	275	250	

	E	F	G	
B	175 18	14	10	~~300~~ 125
C	24	13	10	400
	~~175~~	275	250	

	F	G	
B	125 14	10	~~125~~ 0
C	13	10	400
	~~275~~ 150	250	

	F	G	
C	150 13	10	~~400~~ 250
	~~150~~	250	

	G	
C	250 10	250
	250	

(OR)

11	13	17	14	250
16	18	14	10	300
21	24	13	10	400
200	225	275	250	

$m = 3$ $\quad m+n-1 = 6.$

$n = 4$

$x_{11} = 200, \quad x_{12} = 50$

$x_{22} = 175, \quad x_{23} = 125$

$x_{33} = 150, \quad x_{34} = 250$

Hence the initial basic feasible solution to the TP has been obtained and is displayed in in table given below

	D	E	F	G
A	200 11	50 13	17	14
B	16	175 18	125 14	10
C	21	24	150 13	250 10

The transportation cost according the above route is

$Z = 200\times 11 + 50\times 13 + 175\times 18$
$\quad + 125\times 14 + 150\times 13 + 250\times 10$
$\quad = Rs.\ 12,200$

2) obtain an initial basic feasible solution to the following TP using the matrix minima method:-

	D_1	D_2	D_3	D_4	Capacity
O_1	1	2	3	4	6
O_2	4	3	2	0	8
O_3	0	2	2	1	10
Demand	4	6	8	6	

Solution:- Since $\sum_{i=1}^{3} a_i = \sum_{j=1}^{4} b_j = 24$, The TP has basic feasible solution

	D_1	D_2	D_3	D_4	
O_1	1	2	3	4	6
O_2	4	3	2	0	8
O_3	$\boxed{4}$ 0	2	2	1	~~10~~ 6
	~~4~~ 0	6	8	6	

	D_2	D_3	D_4	
O_1	2	3	4	6
O_2	3	2	$\boxed{6}$ 0	~~8~~ 2
O_3	2	2	1	6
	6	8	~~6~~ 0	

	D_2	D_3	
O_1	$\boxed{6}$ 2	3	~~6~~ 0
O_2	3	2	2
O_3	2	2	6
	~~6~~ 0	8	

	D_2	D_3	
O_2	3	2	2
O_3	$\boxed{0}$ 2	2	6 6
	0 0	8	

	D_3	
O_2	$\boxed{2}$ 2	~~2~~ 0
O_3	2	6
	~~8~~ 6	

	D_3	
O_3	$\boxed{6}$ 2	~~6~~ 0
	~~6~~ 0	

Hence the initial basic basic feasible solution is given in the following table:

	D_1	D_2	D_3	D_4
O_1	1	$\boxed{6}$ 2	3	4
O_2	4	3	$\boxed{2}$ 2	$\boxed{6}$ 0
O_3	$\boxed{4}$ 0	2	$\boxed{6}$ 2	1

$x_{12} = 6$, $x_{23} = 2$, $x_{24} = 6$

$x_{31} = 4$, $x_{33} = 6$

$m + n - 1 = 3 + 4 - 1 = 6$

The number of basic variables are $5 < m+n-1$

$\therefore$ This basic feasible solution is degenerate

The transportation cost according to this schedule is

$Z = 6\times 2 + 2\times 2 + 6\times 0 + 4\times 0 + 6\times 2$
$= 28.$

3) Use vogel's approximation method to obtain an initial basic feasible solution for the following TP:

	D	E	F	G	Available
A	11	13	17	14	250
B	16	18	14	10	300
C	21	24	13	10	400
Demand	200	225	275	250	

Solution: Since total demand = 950 = Total Availability

∴ This TP has an ibfs.

	D	E	F	G		Row difference
A	200 / 11	13	17	14	250 ~~50~~	(2)
B	16	18	14	10	300	(4)
C	21	24	13	10	400	(3)
	200	225	275	250		
Column difference	* (5)	(5)	(1)	(0)		

	E	F	G		row Difference
A	50 / 13	17	14	50	(1)
B	18	14	10	300	(4)
C	24	13	10	400	(3)
	225 175	275	250		
Column diff.	* (5)	(1)	(0)		

	E	F	G		Row diff
B	[175] 18	14	10	~~300~~ 125	(4)
C	24	13	10	400	(1)
	175 / 0	275	250		
Column diff	(6)*	(1)	(0)		

	F	G		Row diff
B	14	[125] 10	~~125~~ 0	(4)*
C	13	10	400	(3)
	275	~~250~~ 125		
Column diff	(1)	(0)		

	F	G	
C	[275] 13	[125] 10	~~400~~ ~~275~~ 0
	~~275~~ 0	~~125~~ 0	

An initial basic feasible solution is shown below:

	D	E	F	G
A	[200] 11	[50] 13	17	14
B	16	[175] 18	14	[125] 10
C	~~275~~ 21	24	[275] 13	[125] 10

The initial basic solution is $x_{11} = 200$, $x_{12} = 50$

$x_{22} = 175$, $x_{24} = 125$

$x_{33} = 275$, $x_{34} = 125$

$m = 3$

$n = 4$

$m+n-1 = 6$

Also the initial transportation cost is

$$Z = 200 \times 11 + 50 \times 13 + 175 \times 18 + 125 \times 10 + 275 \times 13 + 125 \times 10$$

$$= 12075.$$

4) Find an initial basic feasible solution to the following TP by using VAM:

	D_1	D_2	D_3	D_4	Supply
S_1	20	25	28	31	200
S_2	32	28	32	41	180
S_3	18	35	24	32	110
Demand	150	40	180	170	

Solution: Since total demand = 540 ≠ 490 = total Supply.
So The TP is Unbalanced.
Supply is less & the difference between Supply & demand is 50.
So introduce a dummy row with transportation cost of all the cells equals 0 & The corresponding supply is 50.

	D1	D2	D3	D4	Supply	Row diff
S_1	20	25	28	31	200	(5)
S_2	32	28	32	41	180	(4)
S_3	18	35	24	32	110	(6)
dummy	0	0	0	50 0	~~50~~ 0	(0)
Demand	150	40	180	170 120		
Column diff	(18)	(25)	(24)	(31)*		

	D1	D2	D3	D4		Row diff
S_1	20	25	28	31	200	(5)
S_2	32	28	32	41	180	(4)
S_3	110 18	35	24	32	~~110~~ 0	(6)*
	150 40	40	180	120		
Column diff:	(2)	(3)	(4)	(1)		

	D1	D2	D3	D4		Row diff
S_1	40 20	25	28	31	200 160	(5)
S_2	32	28	32	41	180	(4)
	~~40~~ 0	40	180	120		
Column diff	(12)*	(3)	(4)	(10)		

	D_2	D_3	D_4		Row diff
S_1	25	28	[120] 31	~~160~~ 40	(3)
S_2	28	32	41	180	(4)
	40	180	~~120~~ 0		
Column diff	(3)	(4)	(10)*		

	D_2	D_3		Row diff.
S_1	25	28	40	(3)
S_2	[40] 28	32	~~180~~ 140	(4)*
	~~40~~ 0	180		
Column diff	(3)	(4)		

	D_3	
S_1	[40] 28	~~40~~ 0
S_2	[140] 32	140
	~~180~~ 140	

The initial basic feasible allocation is shown below:

40 20	25	40 28	120 31
32	40 28	140 32	41
110 18	35	24 50	32
0	0	0	0

The number of allocated cells is $7 = m+n-1$

The solution is non-degenerate

The transportation cost associated with the above allocation is $Z = 40\times20 + 40\times28 + 120\times31 + 40\times28 + 140\times32 + 110\times18$
$+ 0\times50 = 13,220$.

Test for optimality:- After finding the initial basic feasible solution to a T.P, the next question is how to arrive at the optimal solution. For the optimum solution, we follow the steps given below:

(i) Determining the net evaluations for non-basic variables (unoccuppied cells).

(ii) Choosing the net evaluation which may improve the current basic feasible solution.

(iii) Determining the current the current occuppied cells which leaves the basis (i.e, becoming unoccuppied cell) and repeat steps (i) Through (iii) until an optimum solution is attained.

Determining Net evaluations (uv-method):-

For each supply centre S_i, introduce a variable u_i, which is called location rent. and similarly, for each demand centre D_j, introduce a variable v_j, which is termed as market price.

The values of these $(m+n)$ variables u_i & v_j are determined by fact That

"At the occuppied cells (i,j), $u_i + v_j = c_{ij}$"

By using the above fact, we can get the values of u_i and v_j by taking the value of any one of u_i and v_j to be zero.

Now compute the net evaluations $z_{ij} - c_{ij} = u_i + v_j - c_{ij}$ at all the unoccuppied cells (i,j).

Selecting the entering variable:-

If all $z_{ij} - c_{ij} \leq 0$, then the current solution is optimum, otherwise choose the nonbasic variable x_{rs} whose $z_{rs} - c_{rs}$ is maximum positive.

This x_{rs} becomes the new basic variables.

$$z_{rs} - c_{rs} = \max_{i,j} \{ z_{ij} - c_{ij} > 0 \}.$$

Selecting the leaving Variable:-

Form a closed loop which starts and ends at the cell (r,s) connecting only the basic cells. Allocate a quantity "θ" to the cell (r,s). Start at the cell (r,s), add and subtract the quantity θ in the closed loops successively. Now θ = minimum of allocations made in the cells having a "−" sign. Add this value of θ to all cells having "+" sign and subtract θ from the cells having a "−" sign. One of the cells in the loop becomes unoccuppied cells and the corresponding variable x_{pq} leaves the basis.

(5) **Removing degeneracy in TP:-** If the number of occuppied cells is less than $m+n-1$, then the solution is degenerate. To resolve degeneracy, a very small quantity $\epsilon > 0$ is allocated in an unoccuppied cells so as to get $m+n-1$ number of occuppied cells. Once the purpose is over, ϵ must be removed from the scene.

Transportation Algorithm (MODI method):-

Step 1:- Find the initial basic feasible solution by using North-west Corner rule, Matrix minima method or VAM method.

2. Check the number of occuppied cells. If there are less than $m+n-1$, there exists degeneracy and we introduce a very small quantity $\epsilon > 0$ in suitable independent positions so that the number of occuppied cells is exactly equal to $m+n-1$.
3. For each occuppied cells (i,j), solve the system of equations $u_i + v_j = c_{ij}$ starting initially with some $u_i = 0$ or $v_j = 0$ and entering successively the values of u_i and v_j in the transportation table margins.

4. Compute the net evaluations $Z_{ij} - C_{ij} = U_i + V_j - C_{ij}$ for all unoccuppied cells and enter them in lower left Corner of the Corresponding cells.
5. Examine the sign of each $Z_{ij} - C_{ij}$. If all $Z_{ij} - C_{ij} \leq 0$, then the current solution is optimum. If atleast one $Z_{ij} - C_{ij} > 0$, select the unoccuppied cell (r,s) having the largest +ve net evaluation. to the basis.
6. Let (r,s) be the new occuppied cell and allocate an unknown quantity θ to the cell (r,s). Identify the loop that start and ends at the cell (r,s) and connects some of the basic cells. Add and subtract interchangeably θ to and from the transition cells of the loop in such a way that the rim requirements remains satisfied.
7. Assign a maximum value to θ in such a way that the value of one basic variable becomes zero and the other basic variables in the loop remains non-negative. The basic cell (p,q) whose allocation has been reduced to zero, leave the basis.
8. Return to step3 and repeat the process until an optimum basic feasible solution has been obtained.

Problems:- 1) Find the starting solution the following TP by using vogels Approximation method and also Obtain the optimum solution:-

	D_1	D_2	D_3	D_4	Supply
S_1	3	7	6	4	5
S_2	2	4	3	2	2
S_3	4	3	8	5	3
Demand	3	3	2	2	

Solution: **Step1. (IBFS).**

	D_1	D_2	D_3	D_4	Supply	Row difference			
S_1	$\boxed{3}$ 3	7	6	$\boxed{2}$ 4	~~5~~ ~~2~~ 0	(1)	(1)	(1)	(1)*
S_2	2	4	$\boxed{2}$ 3	$\boxed{\epsilon_1}$ 2	~~2~~ ~~ϵ_1~~ 0	(1)	(2)	–	–
S_3	4	$\boxed{3}$ 3	8	$\boxed{\epsilon_2}$ 5	~~3~~ ϵ_2	(1)	(1)	(1)	(1)
Demand	~~3~~ 0	~~3~~ 0	~~2~~ 0	~~2~~					

Column difference	(1)	(1)	(3)*	(2)
	(1)	(1)	–	(2)*
	(1)	(4)*	–	(1)
	(1)	–	–	(1)

The initial basic feasible solutions are $x_{11}=3$, $x_{14}=2$

$x_{23}=2$, $x_{24}=\epsilon_1$, $x_{32}=3$, $x_{34}=\epsilon_2$.

The initial transportation cost is $Z = 3\times 3 + 2\times 4 + 2\times 3 + 2\epsilon_1 + 3\times 3 + 5\epsilon_2 = 32 + 2\epsilon_1 + 5\epsilon_2$

$Z = 32$ as $\epsilon_1 \to 0$ & $\epsilon_2 \to 0$.

Step2 The IBFS is degenerate & to overcome this, we introduce small quantities ϵ_1 & ϵ_2 at the unoccupied cells (2,4) and (3,4) respectively.

Step3

$u_1+v_1=3$ $v_4+u_2=2$

	$\boxed{3}$ 3	-5 7	-1 6	$\boxed{2}$ 4	$u_1=4$
	-1 2	-4 4	$\boxed{2}$ 3	$\boxed{\epsilon_1}$ 2	$u_2=2$
	0 4	$\boxed{3}$ 3	-2 8	$\boxed{5}$ $\boxed{\epsilon_2}$	$u_3=5$
	$v_1=-1$	$v_2=$ ~~3~~ -2	$v_3=1$	$v_4=0$	

or $u_3=5$

At the occupied cells
$u_i + v_j = c_{ij}$
Take first $v_4 = 0$.

Step4: At the unoccupied cells,
$z_{ij} - c_{ij} = u_i + v_j - c_{ij}$

Step5: Since all $Z_{ij} - C_{ij} \leq 0$ at the unoccuppied cells, the Current solution is optimum.

$\therefore$ The optimum allocations are $x_{11} = 3$, $x_{14} = 2$, $x_{32} = 3$, $x_{23} = 2$ and minimum transportation Cost is $Z = 32$.

2. Given $x_{13} = 50$ units, $x_{14} = 20$ units, $x_{21} = 55$ units, $x_{31} = 30$ units, $x_{32} = 35$ units and $x_{34} = 25$ units. Is it an optimal solution to the T.P:

$$\begin{array}{c} \\ \begin{bmatrix} 6 & 1 & 9 & 3 \\ 11 & 5 & 2 & 8 \\ 10 & 12 & 4 & 7 \end{bmatrix} \\ \text{Required Units: } \; 85 \;\; 35 \;\; 50 \;\; 45 \end{array} \begin{array}{c} \text{Available units} \\ 70 \\ 55 \\ 90 \\ \\ \end{array}$$

If not, modify it to obtain a better feasible solution.

Solution:-

0 6	7 1	$\boxed{50}$ $-\theta$ 9	$\boxed{20}$ $+\theta$ 3	$u_1 = -4$
$\boxed{55}$ $-\theta$ 11	8 5	θ 12 2	0 8	$u_2 = 1$
$\boxed{30}$ $+\theta$ 10	$\boxed{35}$ 12	9 4	$\boxed{25}$ $-\theta$ 7	$u_3 = 0$
$v_1 = 10$	$v_2 = 12$	$v_3 = 13$	$v_4 = 7$	

$m+n-1 = 3+4-1 = 6 =$ no. of allocations

$\therefore$ given solution is non-degenerate.

At the occuppied cells

$u_i + v_j = c_{ij}$

Take $u_3 = 0$ initially.

At the unoccuppied cells,

net evaluation $z_{ij} - c_{ij} = u_i + v_j - c_{ij}$

All $z_{ij} - c_{ij}$ is not less than or equal to zero.

$\therefore$ Given solution is not an optimal solution.

maximum of +ve $(z_{ij} - c_{ij})$ is 12 which is in the cell (2,3)

$\therefore$ x_{23} is the entering variable & allocate a +ve unknown quantity "θ" at the cell (2,3). Form a loop which starts and ends at the cell (2,3).

Select $\theta = \min\{$allocations in "$-\theta$" cells$\} = \min\{55, 25, 50\} = 25$

Add $\theta = 25$ to the allocations at the "$+\theta$" cells & Subtract $\theta = 25$ from allocations at the "$-\theta$" cells.

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(12) 6	+θ (19) 1	[25] −θ 9	[45] 3	$u_1 = 0$
[30] −θ 11	(8) 5	[25] +θ 2	(−12) 8	$u_2 = -7$
[55] +θ 10	[35] −θ 12	(−3) 4	(−12) 7	$u_3 = -8$
$v_1 = 18$	$v_2 = 20$	$v_3 = 9$	$v_4 = 3$	

Again no. of allocations $= 6 = m+n-1$.
Solution is non degenerate.
At the occuppied cells, $u_i + v_j = c_{ij}$
Taking initially $u_1 = 0$
At the unoccuppied cells,
$$z_{ij} - c_{ij} = u_i + v_j - c_{ij}.$$
All $z_{ij} - c_{ij}$ is not less than or equal to zero.
So current allocations is not optimal.

maximum of +ve $z_{ij} - c_{ij}$ is 19 which is in the cell (1,2).
∴ x_{12} is entering the basis. Form a closed loop which starts and ends at the cell (1,2). Starting at cell (1,2), allocate +θ & −θ ~~successively~~ alternatively in the closed loop.
choose θ = min{allocations in the "−θ" cells} = min{25, 30, 35} = 25
Add θ = 25 in the "+θ" cells & subtract θ = 25 from "−θ cells".

(−7) 6	[25] 1	(−19) 9	[45] 3	$u_1 = 0$
[5] −θ 11	+θ (8) 5	[50] 2	(7) 8	$u_2 = 12$
[80] +θ 10	[10] −θ 12	(−3) 4	(7) 7	$u_3 = 11$
$v_1 = -1$	$v_2 = 1$	$v_3 = -10$	$v_4 = 3$	

No. of allocations $= 6 = m+n-1$.
Solution is non-degenerate.
All $z_{ij} - c_{ij}$ is not less than or equal to zero
current allocation is not optimal
max{+ve $(z_{ij} - c_{ij})$} = 8 at the cell (2,2).
∴ x_{22} is the entering variable
θ = min{allocations in "−θ" cells} = 5
So Add θ = 5 in the "+θ" cell & Subtract θ = 5 from "−θ" cell.

(−7) 6	[25] +θ 1	(−11) 9	[45] −θ 3	$u_1 = 1$
(−8) 11	[5] 5	[50] 2	(−1) 8	$u_2 = 5$
[85] 10	[5] −θ 12	(5) 4	+θ (7) 7	$u_3 = 12$
$v_1 = -2$	$v_2 = 0$	$v_3 = -3$	$v_4 = 2$	

No. of allocations $= 6 = m+n-1$.
Solution is non-degenerate.
So Test current solution for optimality.
All $z_{ij} - c_{ij}$ is not less than or equal to zero.
current allocation is not optimal
max{+ve $(z_{ij} - c_{ij})$} = 7 at cell (3,4) & ∴ x_{34} enters the basis.

θ = min{allocations in "−θ" cells} = 5

	[0] 6	[30] 1	[-11] 9	[40] 3	$u_1=0$
	[-1] 11	[5] 5	[50] 2	[-1] 8	$u_2=4$
	[85] 10	[-7] 12	[-2] 4	[5] 7	$u_3=4$
	$v_1=6$	$v_2=1$	$v_3=-2$	$v_4=3$	

No. of allocations $= 6 = m+n-1$
Solution is non-degenerate.
Test the current allocations for optimality.

All $Z_{ij} - C_{ij} \le 0$.
The current allocation is optimal.

$\therefore$ The optimal allocation is $x_{12}=30$, $x_{14}=40$, $x_{22}=5$, $x_{23}=50$, $x_{31}=85$, $x_{34}=5$ and the minimum transportation cost is $Z = 30\times1 + 40\times3 + 5\times5 + 50\times2 + 85\times10 + 5\times7 = \text{Rs.}1160$.

3) ABC Limited has three production shops supplying a product to Five warehouses. The cost of production varies from shop to shop and cost of transportation from one shop to a warehouse also varies. Each shop has a specific production capacity and each warehouses has certain amount of requirements. The cost of transportation are as given below:

	Warehouses					
Shop	I	II	III	IV	V	Supply
A	6	4	4	7	5	100
B	5	6	7	4	8	125
C	3	4	6	3	4	175
Demand	60	80	85	105	70	

The cost of manufacturing of the product at different production Shop is

Shop	Variable cost	Fixed cost
A	14	7000
B	16	4000
C	15	5000

Find the optimum quantity to be supplied from each shop to different warehouses at a minimum total cost.

Solution:- In the transportion cost matrix, the data pertaining fixed cost is of no use. we shall include only the transportation cost plus the variable cost as shown below:

Shop	Warehouses I	II	III	IV	V	Supply
A	6+14=20	4+14=18	4+14=18	7+14=21	5+4=19	100
B	5+16=21	6+16=22	7+16=23	4+16=20	8+16=24	125
C	3+15=18	4+15=19	6+15=21	3+15=18	4+15=19	175
Demand	60	80	85	105	70	

Total supply = 400 = total demand ⇒ TP is balanced.

Using VAM, the following initial basic feasible solution is obtained.

						Supply	Row difference
	20	$\boxed{15}$ 18	$\boxed{85}$ 18	21	19	~~100~~ ~~15~~ 0	(1) (2) (2) (3)* — —
	21	$\boxed{20}$ 22	23	$\boxed{105}$ 20	24	~~125~~ ~~20~~ 0	(1) (1) (1) (2) (2)
	$\boxed{60}$ 18	$\boxed{45}$ 19	21	18	$\boxed{70}$ 19	~~175~~ ~~105~~ ~~45~~ 0	(1) (1) (1) (1) (1) —
Demand	~~60~~ 0	~~80~~ ~~65~~ ~~20~~ 0	~~85~~ 0	~~105~~ 0	~~70~~ 0		
Column difference	(2)	(1)	(3)	(2)	(5)*		
	(2)	(1)	(3)*	(2)	—		
	(2)*	(1)	—	(2)	—		
	—	(1)	—	(2)	—		
	—	(3)*	—	(2)	—		

No. of allocations = 7 = m+n−1 & ∴ the solution is non-degenerate.

Test the solutions for optimality.

	−3 20	$\boxed{15}$ 18	$\boxed{85}$ 18	−5 21	−1 19	$u_1 = 1$
	0 21	$\boxed{20}$ 22	−1 23	$\boxed{105}$ 20	−2 24	$u_2 = 3$
	$\boxed{60}$ 18	$\boxed{45}$ 19	−2 21	−1 18	$\boxed{70}$ 19	$u_3 = 0$
	$v_1 = 18$	$v_2 = 19$	$v_3 = 19$	$v_4 = 17$	$v_5 = 19$	

All $z_{ij} - c_{ij} \le 0$.

The current allocation is optimal allocation.

The optimal solution is $x_{12} = 15$, $x_{13} = 85$, $x_{22} = 20$, $x_{24} = 105$, $x_{31} = 60$, $x_{32} = 45$, $x_{35} = 70$

The minimum total cost is = Rs 7605.

optimum transport is

Shop A → 15 units for warehouse II, 85 units for warehouse III

Shop B → 20 Units for warehouse II, 105 units for warehouse IV

Shop C → 60 units for warehouse I, 45 units for warehouse II, 70 units for warehouse V.

—x—

Assignment Problem

The assignment problem is a special case of the TP in which the objective is to assign a number of resources to the equal number of activities at a minimum cost (or maximum profit).

Assignment problem is completely degenerate form of a TP. The units available at each origin (resource) and units demanded at each destination (activity) are equal to one. That means exactly one occuppied cell in each row and each column of TP.

Mathematical formulation of Assignment Problem: — consider a problem of assignment of "n" resources (workers) to "n" activities (jobs) so as to minimize the overall cost or time in such a way that each resources can associate with one and only one job. The cost (or effectiveness) matrix (c_{ij}) is given as under:

Activity (JOB)

		A_1	A_2	...	A_n	Availability
Resources (WORKERS)	R_1	c_{11}	c_{12}	...	c_{1n}	1
	R_2	c_{21}	c_{22}	...	c_{2n}	1
	⋮	⋮	⋮		⋮	⋮
	R_n	c_{n1}	c_{n2}	...	c_{nn}	1
Required		1	1	...	1	

4×6

This cost matrix is same as that of a TP except that availability at each of resources and the requirement at each of the destination is unity (due to the fact that assignments are made on a one-to-one basis.

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Let x_{ij} = assignment of i^{th} resources (workers) to the j^{th} activity (JOB), such that.

$$x_{ij} = \begin{cases} 1 & \text{if resources } i \text{ is assign to } j^{th} \text{ activity} \\ 0 & \text{otherwise.} \end{cases}$$

∴ The mathematical formulation of AP is minimize $Z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij}$ subject to the constraints

$$\sum_{j=1}^{n} x_{ij} = 1 \text{ for } i = 1 \text{ to } n, \quad \sum_{i=1}^{n} x_{ij} = 1 \text{ for } j = 1 \text{ to } n.$$

(Here c_{ij} - cost associated with assigning i^{th} resources to j^{th} activity).

<u>Theorem (Reduction theorem)</u>:- In an AP, if we add or subtract a constant to every element of any row (or column) of the cost matrix (c_{ij}), then an assignment that minimizes the total cost of one matrix also minimizes the total cost on the other matrix.

<u>Theorem</u> If $c_{ij} \geq 0$ such that minimum $\sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij} = 0$, the feasible solution x_{ij} provides an optimum solution.

<u>Solution of an AP by Hungarian assignment method</u>:-

<u>Step1</u>: Determine the cost table from given problem.
(i) If the number of sources = number of destinations, goto step3.
(ii) If the number of sources ≠ number of destination, go to step2.

<u>Step2</u>:- Add a dummy sources or destinations so that the cost table becomes square matrix. The cost entries of dummy source or destination are always zero.

<u>Step3</u>: Locate the smallest element in each row of the given cost matrix and then subtract it from each element of that row.

<u>Step4</u>:- In the reduced matrix obtained from steps, locate smallest element of each column and subtract it from every element of that column. Now each row & column has atleast zero.

Step5:- In the modified matrix obtained from step4, Search for an optional assignments as follows:

(a) Examine the rows successively until a row with a single zero is found. Enrectangle this zero (□) and cross off (x) all other zeros in its column. Continue in this manner until all the rows have been taken care of.

(b) Repeat the procedure for each column of reduced matrix.

(c) If a row and/or column has two or more zeros and one cannot be chosen by inspection, then assign arbitrary any one of these zeros and cross off all other zeros of that row / column.

(d) Repeat (a) through (c) above successively until the chain of assigning (□) or cross (x) ends.

Step6: If the number of assignments (□) is equal to "n" (the ~~number~~ order of the matrix), an optimal assign is reached. If the number of assignment (□) is less than "n", goto Step 7.

Step7:- Draw the minimum number of horizontal and or vertical lines to cover all the zeros of the reduced matrix.

Step8:- Develop the new revised cost matrix as follows:

(a) Find the smallest element of the reduced matrix not covered by any of the lines.

(b) Subtract this element from all the uncovered elements and add the same to all the elements lying at the intersection of any two lines.

Step9:- Goto ~~8~~ Step6 and repeat the procedure until an optimum solution is attained.

Problems

1). A departmental head has 4 subordinates and 4 task to be performed. The subordinates differ in efficiency and the task differ in their intrinsic difficulty. His estimate of time each man would take to perform each task, is given in the matrix below:

~~Men~~

Tasks	Men			
	E	F	G	H
A	18	26	17	11
B	13	28	14	26
C	38	19	18	15
D	19	26	24	10

How should the tasks be allocated, one to a man, so as to minimize the total man hours?

Solution Here number of tasks = 4 = number of subordinates, the AP is balanced (we move to step 3).

Subtract smallest element of each row from all elements in that row

7	15	6	0
0	15	1	13
23	4	3	0
9	16	14	0

Subtract smallest element of each column in the reduced matrix from all elements in that column

7	11	5	0
0	11	0	13
23	0	2	0
9	12	13	0

Test for optimality

7	11	5	$\boxed{0}$
$\boxed{0}$	11	$\cancel{0}$	13
23	$\boxed{0}$	2	$\cancel{0}$
9	12	13	$\cancel{0}$

Number of assignments (□) is $3 \neq 4$.

∴ Now cover all zeros by drawing minimum number of horizontal and or vertical lines.

7	11	5	0
0	11	0	13
23	0	2	0
9	12	13	0

Develop a number of new cost matrix. Smallest element among uncovered element is 5. Subtract 5 from all the uncovered elements and add 5 to the elements at the intersection of two lines.

2	6	0	0
0	11	0	18
23	0	2	5
4	7	8	0

Do the assignment ($\square$ or X) now

	E	F	G	H
A	2	6	$\boxed{0}$	~~0~~
B	$\boxed{0}$	11	~~0~~	18
C	23	$\boxed{0}$	2	5
D	4	7	8	$\boxed{0}$

Number of assignment is 4 = order of the matrix.
$\therefore$ optimum is reached and the optimal assignments is

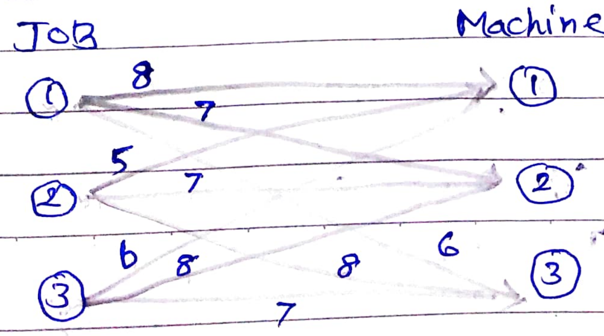
A $\rightarrow$ G
B $\rightarrow$ E
C $\rightarrow$ F
D $\rightarrow$ H

The mimum total time for optimal assignment is 17+13+19+10 = 59 Man-Hour.

2) A company wishes to assign 3 jobs to 3 machines in such a way that each job is assigned to some machine and no m/c works on more than one job. The cost of assigning job i to m/c j is given by the matrix below: Draw the associated network. Formulate the network LPP and find the minimum cost of making assignment.

$$\begin{array}{c c} & \text{M/C} \\ \text{JOB} & \begin{bmatrix} 8 & 7 & 6 \\ 5 & 7 & 8 \\ 6 & 8 & 7 \end{bmatrix} \end{array}$$

Solution:- (a) Network formulation:-



(b) LPP formulation:-

$$x_{ij} = \begin{cases} 1 & \text{if assigning } i^{th} \text{ job to } j^{th} \text{ machine} \\ 0 & \text{otherwise} \end{cases}$$

Minimize $Z = (8x_{11} + 7x_{12} + 6x_{13}) + (5x_{21} + 7x_{22} + 8x_{23}) + (6x_{31} + 8x_{32} + 7x_{33})$ subject to the

Constraints
$$x_{i1} + x_{i2} + x_{i3} = 1, \quad (i = 1, 2, 3)$$
$$x_{1j} + x_{2j} + x_{3j} = 1 \quad (j = 1, 2, 3)$$

(c) Optimum assignment:-

$$\begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 3 \\ 0 & 2 & 1 \end{bmatrix} \quad \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} \quad \begin{bmatrix} 2 & \boxed{0} & \cancel{0} \\ \boxed{0} & 1 & 3 \\ \cancel{0} & 1 & 1 \end{bmatrix}$$

Number of assignments ($\square$) is $= 2 <$ order of the matrix $= 3$.
This assignment is not optimum.
Develop a new cost matrix. cover all zeros by drawing minimum number of Horizontal & or vertical lines.

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

Smallest uncovered element = 1.
Subtract 1 from all uncovered elements and add 1 to the elements at point of intersection of two lines.

$$\begin{bmatrix} 3 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

Assign ($\square$ or x) to zeros.

$$\begin{bmatrix} 3 & \boxed{0} & \cancel{0} \\ \boxed{0} & \cancel{0} & 2 \\ \cancel{0} & \cancel{0} & \boxed{0} \end{bmatrix}$$

Number of assignments ($\square$) = 3 = order of the matrix.
∴ optimum is reached. optimum assignment is

JOB 1 → M/C 2
JOB 2 → M/C 1
JOB 3 → M/C 3

Total minimum Cost = 7 + 5 + 7 = 19

3. A department head has 4 tasks to be performed and 3 subordinates, the subordinates differ in efficiency. The estimates of time, each subordinate would take to perform is given in the following matrix. How should he allocate the task one to each man, so as to minimize the total man-hours.

		Men 1	Men 2	Men 3
Task (Job)	I	9	26	15
	II	13	27	6
	III	35	20	15
	IV	18	30	20

Solution:- no. of task ≠ no. of men.

Introduce a dummy man 4 with all ~~cost equ~~ times = 0.

	1	2	3	4
I	9	26	15	0
II	13	27	6	0
III	35	20	15	0
IV	18	30	20	0

$\boxed{0}$	6	9	$\cancel{0}$
4	7	$\boxed{0}$	$\cancel{0}$
26	$\boxed{0}$	9	$\cancel{0}$
9	10	14	$\boxed{0}$

Number of assignments ($\square$) is 4 = order of the matrix

The optimum is reached.

The optimum assignment is Task I → Man 1, Task II → Man 3, Task III → Man 2, Task IV → dummy man, which means Task IV to be done. The minimum total time is $9 + 6 + 20 = 35$ man-hours

Home work: 1103.

4) A pharmaceutical Company is producing a single product and is selling it through 5 agencies located in different cities. All of sudden, there is a demand for the product in another 5 cities not having any agency of the Company. The company is faced with the problem of deciding on how to assign the existing agencies to despatch the product to needy cities in such a way that the travelling distance is minimised. The distance between the surplace and deficit cities (in km) is given in the following table:

Deficit cities

Surplus cities	a	b	c	d	e
A	85	75	65	125	75
B	90	78	66	132	78
C	75	66	57	114	69
D	80	72	60	120	72
E	76	64	56	112	68

Determine the optimum assignment schedule.

5 Four prefessors are each capable of teaching any one of the four different courses. class preparation time in hours for different topics varies from professor to professor and is given in the table below. Each professor is assigned only one course. Determine an assignment schedule so as to minimize the total course preparation time for all courses:

professors	Linear programmes	Queueing Theory	Dynamic programme	Regression Analysis
A	2	10	9	7
B	15	4	14	8
C	13	14	16	11
D	4	15	13	9

Special cases in assignment problems

Maximization case in assignment problem:- In some cases, the payoff elements of the assignment problem may represent revenues or profits instead of costs so that the objective will be to maximize the total revenue or profit. The Haungarian method explained earlier can also be used for maximization case. The problem of maximization can be converted into a minimization one by selecting the largest element of the profit matrix and then subtracting ~~it~~ from it all the elements of the matrix. This gives us an opportunity loss matrix. we then proceed as usual and obtain the optimum assignment schedule. Maximum profit is then obtained by restoring the original values of those cells inwhich the assignments have been made.

prohibited assignments:- Sometimes due to certain reasons, a particular resources (say a man or a machine) cannot be assigned to perform a particular activity (say a territory or ajob). In such cases, the cost of performing that particular activity by a particular resources is considered to be very large (written as M or ∞) so as to prohipit the entry of this pair of resource-activity into the final solution.

1) A manufacturing company has four zones A, B, C, D and four sales engineers P, Q, R, S respectively for assigment. Since the zones are not equally rich in sales potential, it is estimated that a particular engineer operating in particular zone will bring the following sales: Zone A: 4,20,000; ZoneB: 3,36,000; Zone C: 2,94,000; Zone D: 4,62,000.

The engineers are having different sales ability. Working under same conditions, their yearly sales are proportional to 14, 9, 11 and 8 respectively. The criteria of maximum expected total sales is to be met by assigning the best engineers to the richest zone, the next best to the next richest Zone and so on. Find the optimum assignment and the maximum sales.

Solution:-

Take Rs.1000 as one unit.

Zone A : 420; Zone B : 336; Zone C : 294; Zone D : 462.

Sales by engineer P : $\frac{14}{14+9+11+8}$ x sales in particular zone.

" Q : $\frac{9}{42}$ x sales in particular zone

" R : $\frac{11}{42}$ x sales in particular zone

" S : $\frac{8}{42}$ x sales in particular zone.

∴ The effectiveness table is

Sales engineer	ZONE A	B	C	D
P	140	112	98	154
Q	90	72	63	99
R	110	88	77	121
S	80	64	56	88

The objective is to assign sales engineers to zones so as to maximize the sales.

So convert this maximization problem into minimization problem by subtracting from the highest element (ie) 154), all the elements of the given table. The resulting opportunity loss matrix is:

	A	B	C	D
P	14	42	56	0
Q	64	82	91	55
R	44	66	77	33.
S	74	90	98	66

Apply Hungarian method for optimum assignments. Subtract minimum element of each row from all the element of that row. Then subtract the minimum element of each column from all the element of that column.

14	42	56	0
9	27	36	0
11	33	44	0
8	24	32	0

6	18	24	0
1	3	4	0
3	9	12	0
0	0	0	0

Make assigments now in the reduced matrix.

6	18	24	$\boxed{0}$
1	3	4	$\cancel{0}$
3	9	12	$\cancel{0}$
$\boxed{0}$	$\cancel{0}$	$\cancel{0}$	$\cancel{0}$

Now number of assignments($\square$) = 2 < order of the matrix optimum is not reached. So Modify the matrix. Cover all zeros by drawing minimum number of horizantal (or) vertical lines.

6	18	24	0
1	3	4	0
3	9	12	0
0	0	0	0

The smallest uncovered element is 1 and ∴ Subtract 1 from all the uncovered elements and add 1 to the element at the intersection of two lines.

5	17	23	0
0	2	3	0
2	8	11	0
0	0	0	1

Make an assignments ($\square$) now.

5	17	23	$\boxed{0}$
$\boxed{0}$	2	3	$\cancel{0}$
2	8	11	0
$\cancel{0}$	$\boxed{0}$	$\cancel{0}$	1

Number of assignments is 3 < order of the matrix optimum is not reached. Modify the table.

5	17	23	0
0	2	3	0
2	8	11	0
0	0	0	1

The smallest uncovered element = 2.

3	15	21	$\boxed{0}$
$\cancel{0}$	$\boxed{0}$	$\cancel{0}$	2
$\boxed{0}$	6	9	$\cancel{0}$
$\cancel{0}$	$\cancel{0}$	$\boxed{0}$	3

Number of assignments = 4, equal to the order of matrix. Optimum is reached optimum assignment is
P → D ; Q → B ; R → A
S → C.

Maximum Sales = 154 + 72 + 110 + 56
= Rs. 392 thousands.

Teacher's Signature

2) The following is the cost matrix of assigning 4 clerks to 4 key punching jobs. Find the optional assignment if clerk I cannot be assigned to job 1. What is the minimum total cost.

clerk	JOB 1	2	3	4
1	–	5	2	0
2	4	7	5	6
3	5	8	4	3
4	3	6	6	2

Solution:

$$\begin{bmatrix} - & 5 & 2 & 0 \\ 0 & 3 & 1 & 2 \\ 2 & 5 & 1 & 0 \\ 1 & 4 & 4 & 0 \end{bmatrix} \quad \begin{bmatrix} - & 2 & 1 & 0 \\ 0 & 0 & 0 & 2 \\ 2 & 2 & 0 & 0 \\ 1 & 1 & 3 & 0 \end{bmatrix} \quad \begin{bmatrix} - & 2 & 1 & \boxed{0} \\ \boxed{0} & \cancel{0} & \cancel{0} & 2 \\ 2 & 2 & \boxed{0} & \cancel{0} \\ 1 & 1 & 3 & \cancel{0} \end{bmatrix}$$

Number of assignments = 3 < 4 = order of the matrix.
Modify the matrix.

$$\begin{bmatrix} - & 2 & 1 & 0 \\ 0 & 0 & 0 & 2 \\ 2 & 2 & 0 & 0 \\ 1 & 1 & 3 & 0 \end{bmatrix} \quad \begin{bmatrix} - & 1 & \cancel{0} & \boxed{0} \\ \boxed{0} & \cancel{0} & \cancel{0} & 3 \\ 2 & 2 & \boxed{0} & 1 \\ \cancel{0} & \boxed{0} & 2 & \cancel{0} \end{bmatrix}$$

Number of assignment is 4 = order of the matrix optimum is reached.

Optimum assignment is clerk 1 → Job 4
" 2 → " 1
" 3 → " 3
" 4 → " 2.

Minimum total cost = 0 + 4 + 4 + 6 = 14.

Home work: 1127, 1128, 1129.