19

Inventory Control—I

"None but the productive can be strong and none but the strong can survive"

19:1. INTRODUCTION

Inventory consists of usable but idle resources. The resource may be of any type — men, materials, machines, etc. For example, if a company purchases a machine or appoints an expert in anticipation of the requirement of their services in future, these resources work as inventory. Generally, inventory of men or machines is not carried and managements hire machines or consult experts whenever required, as an economical alternative. When the resource involved is material or goods, it is referred as stock or simply as 'Inventory'. The term is generally used to indicate raw materials in process, finished product, packaging, spares and others — stocked in order to meet an expected demand or distribution in the future. Though inventory of materials is an idle resource — it is not meant for immediate use — it is almost essential to maintain some inventories for the smooth functioning of an enterprise. Traditionally, inventory is viewed as a necessary evil — too little of it causes costly interruptions and too much of it results in idle capital. Inventory control aims at maintaining the balance between these two extremes.

19:2. TYPES OF INVENTORIES

Inventories may be held for a variety of purposes, but, in general, there are following five types of inventories that an enterprise can use for serving these purposes :

Movement inventories. These, also called *transit or pipeline inventories*, arise due to shipment of inventory items to distribution centres and from various production centres. The amount of movement inventories depend on the time consumed in transportation and the nature of demand. For example, when coal is transported from coal fields to an industrial unit by trains, then the coal, while in transit, cannot provide any service to the customers for power generation or for burning.

Buffer inventories. These are maintained to meet uncertainties of demand and supply. Such 'buffer' inventories which are in excess of those necessary to just meet the average demand during the *lead time* (the time elapsing between placing an order and having the goods in stock ready for use), held for protecting against the fluctuations in demand are also termed as *safety stocks.*

Anticipation inventories. These, also known as *seasonal inventories*, are held because a future demand for the product is anticipated. Production of specialized items like crackers well before Diwali, electric fans and air-conditioners when summers are approaching, are some examples of *anticipation inventories*. The basic idea underlying is to smoothen the production process producing for a longer duration on a continuous scale rather than operating with excessive overtime in one period and then let the system to be idle or close down for the reason of inadequate or no demand for another period.

Decoupling inventories. If various production stages operate successively then in the event of breakdown of one or any disturbance at some stage can affect the entire system. This kind of inter-dependence is not only costly but also disruptive for the entire system. The inventories used to reduce the interdependence of various stages of production system are known as *decoupling inventories*.

Decoupling inventories are not only in the form of in-process inventories to 'decouple' successive production stages but also in the form of raw material which are used to decouple the producers from the suppliers, and finished goods to decouple the consumer from producer. Inventories may also be carried to take advantage of quantity discounts or raw material price in the event of an expected rise in price.

Lot-size inventories. These are held for the reason that purchases are usually made in lots rather than for the exact amounts which may be needed at a point of time. Lot-size inventories are also called *cycle inventories*. The amount of such inventories depend upon the production lot size, economical shipment quantities, storage space limitation, price-quantity discount schedules, and inventory carrying cost, and so on.

19:3. REASONS FOR CARRYING INVENTORIES

Inventories, in general, are built up to provide a cushion between supply and demand. They are meant to take care of requirements of demand till next arrival, probable delay in delivery and sudden increase in demand. A manufacturing firm carries inventories because of the following major reasons :

1. To ensure of an adequate supply of items to customers and avoids the shortages as far as possible at the minimum cost.
2. To reduce the risk of loss due to the changes in prices of items stocked at the time of making the stock.
3. To take advantages of quantity discounts on bulk purchases.
4. To carry buffer stocks (reserve stocks) in case of delayed deliveries by the suppliers.
5. To help in minimizing the loss due to deterioration, obsolescence, damages or pilferage of goods, etc.
6. To make use of available capital and/or storage space in a most effective way and avoids an unnecessary expenditure on high inventories.
7. To utilize the benefits of price fluctuations. Thus, in conclusion, with a good inventory, a firm is able to make purchases in economic lots, maintain continuity of operations, avoid small-time consuming orders and guarantee for the prompt delivery of finished goods.
8. To satisfy other business constraints, such as :
   (*i*) *Supplier's condition of minimum quantity*. Many times, a supplier insists on certain minimum quantity. This can happen under one of the following conditions :
   — The manufacturer has the monopoly and hence the order quantities are dictated to by the seller.
   — The set-up cost is very high.
   (*ii*) *Government regulations*. Too much capital gets locked in inventories of items that are subjected to government regulations (*e.g.*, imported items).
   (*iii*) *Seasonal availability*. There are certain items which are available in plenty and hence cheaply only during certain months (*e.g.*, ground nuts, coco-nuts, soyabeans, etc.) or are not available in certain months. The firm is thus forced to buy large quantity and hence create inventories.

19:4. THE INVENTORY DECISIONS

Inventory Control is the process of deciding what and how much of various items are to be kept in stock. It also determines the time and quantity of various items to be procured. The basic objective of inventory control is to reduce investment in inventories and ensuring that production process does not suffer at the same time. To attain various objectives in an inventory control situation, there are two basic questions to be answered. They are :

(*i*) How much to order? That is what is the optimum quantity of an item that should be ordered.
(*ii*) When should the order be placed?

The answer to the first question determines the *economic order quantity* (*EOQ*) by minimizing the total inventory cost, which is given by

Purchase Cost + Set-up Cost (Ordering Cost) + Carrying Cost + Shortage Cost

When price discounts are not offered, the purchase cost remains constant and is independent of the quantity purchased. The total variable inventory cost is, then, given by

Ordering Cost + Carrying Cost + Shortage Cost

The answer to the second question (when to order?) depends on the type of inventory system with which we are dealing. If the system requires *periodic review* (*e.g.*, every month or year), the time for receiving a new order coincides with the start of each period. Alternatively, if the system is based on *continuous review*, new orders are placed when the inventory level drops to a pre-specified level, called the *reorder point*.

19:5. OBJECTIVES OF SCIENTIFIC INVENTORY CONTROL

A fundamental objective of a good inventory control system is to determine '*what to order*', '*how much to order*', '*when to order*', and '*how much to carry in stock*' so as to gain economy in purchasing, storing, manufacturing and selling. These fundamental objectives may be amplified into the following objectives :

Service to the customers. Sufficient stock of finished product is to be maintained to meet the requirements of customers.

Effective use of capital. The system should enable the management to make an effective use of its capital, *i.e.*, lock-up of capital should be barest minimum.

Continuity of productive operations. The system should be made to ensure the continuity of productive operations by ensuring uniform flow of materials and eliminating the possibility of stock outs.

Reduction of administrative work load. The administrative work load on the purchasing, receiving, inspection, stores, accounts and other related departments should be barest minimum.

Economy in purchasing. The system should enable the management to gain economy in bulk purchasing and take advantage of price discount.

Minimization of risk obsolescence and deterioration. The possibility of the risk of loss on account of obsolescence and deterioration should be minimized. In-built checks in the system should enable the management to weed out obsolete and non-moving items periodically and automatically.

Up-to-date accurate records. In order to enable the company to prepare periodical financial statements, minimising of discrepancies between physical stock and book balance an up-to-date stock records must be maintained.

19:6. COSTS ASSOCIATED WITH INVENTORIES

Various costs associated with inventory control are often classified as follows :

Set-up cost. This is the cost associated with the setting up of machinery before starting production. Set-up cost is generally assumed to be independent of the quantity ordered for or produced.

Ordering cost. This is a cost associated with ordering of raw material for production purposes. Advertisements, consumption of stationery and postage, telephone charges, telegrams, rent for space used by the purchasing department, travelling expenditures incurred, etc., constitute the *ordering cost.*

Purchase (or production cost). The cost of purchasing (or producing) a unit of an item is known as *purchase (or production) cost*. The purchase price will become important when quantity discounts are allowed for purchases above a certain quantity or when economies of scale suggest that the per unit production cost can be reduced by a larger production run.

Carrying (or holding) cost. The carrying cost is associated with carrying (or holding) inventory. This cost generally includes the costs such as rent for space used for storage, interest on the money locked-up, insurance of stored equipment, production, taxes, depreciation of equipment and furniture used, etc.

Shortage (or stock out) cost. The penalty cost for running out of stock (*i.e.*, when an item cannot be supplied on the customer's demand) is known as *shortage cost*. This cost includes the loss of potential profit through sales of items and loss of goodwill, in terms of permanent loss of customers and its associated lost profit in future sales.

Salvage cost (or selling price). When the demand for certain commodity is affected by the quantity stocked, decision problem is based on a profit maximization criterion that includes the revenue from selling. Salvage value may be combined with the cost of storage and hence is generally neglected.

Revenue cost. When it is assumed that both the price and the demand of the product are not under control of the organisation, the revenue from the sales is independent of the company's inventory policy and may be neglected except for the situation when the organisation cannot meet the demand and the sale is lost. Therefore, the revenue cost may or may not be included in the study of inventory policy.

19:7. FACTORS AFFECTING INVENTORY CONTROL

Besides the costs that determine the profitability, other factors which play an important role in the study of inventory control are the following :

Demand. The number of units required per period is called *demand*. The demand pattern of a commodity may be either deterministic or probabilistic.

Lead time. The time gap between placing of an order and its actual arrival in the inventory is known as *lead time.*

Lead time has two components, namely the *administrative lead time* — from initiation of procurement action until the placing of an order, and the *delivery lead time* — from placing of an order until the delivery of the ordered material.

Order cycle. The time period between placement of two successive orders is referred to as an *order cycle*. The order may be placed on the basis of following two types of inventory review systems :

(*a*) *Continuous review.* The record of the inventory level is checked continuously until a specified point (called reorder point) is reached where a new order is placed. This is often referred to as the *two-bin system*. This divides the inventory into two parts and places it physically, or on paper, in two bins. Items are drawn from only one bin and when it is empty, a new order is placed. Demand is then satisfied from the second bin until the order is received. Upon receipt of the order, enough items are placed in the second bin to make up the earlier total. The remaining items are placed in the first bin. This procedure is then repeated.

(b) *Periodic review.* In this system the inventory levels are viewed at equal time intervals and orders are placed at such intervals. The quantity ordered each time depends on the available inventory level at the time of review.

Time horizon. The time period over which the inventory level will be controlled is called the *time horizon.* This horizon may be finite or infinite depending upon the nature of the demand for the commodity.

Re-order level. The level between maximum and minimum stock, at which purchasing (or manufacturing) activities must start for replenishment, is known as *re-order level.*

Stock replenishment. Although an inventory problem may operate with lead time, the actual replacement of stock may occur instantaneously or uniformly. Instantaneous replenishment occurs in case the stock is purchased from outside sources whereas the uniform replenishment may occur when the product is manufactured by the company.

Inventory Turnover. Inventory turnover is defined as the ratio of the value of materials consumed to the average investment in inventories for the same period.

$$\text{Inventory turnover} = \frac{\text{Value of the material consumed}}{\text{Value of average inventory}}$$

Inventory turnover is a very useful index of inventory control. The higher the index, the lower the inventory levels and lower the cost of maintaining the inventories. The top management can keep itself informed of the broad situation through the use of such an index.

Standardisation. In inventory control, standardisation is the determination of fixed sizes, shapes, quantity and dimensions of material. As it is desirable to encourage the use of standards prescribed for the type of industry in which the company operates, by keeping the material to standards, a great deal of reductions are possible in inventory space, obsolescence, handling costs, and inventory as well as improved quality.

19:8. AN INVENTORY CONTROL PROBLEM

The major steps to build up a suitable inventory problem and then to derive decision rules are as follows :

Step 1. Collect the data regarding the pattern of demand, the replenishment policy, time horizon, relevant inventory costs, etc.

Step 2. Build up an appropriate relationship using above information which can be representative of the behaviour (or objective) of the inventory system. Relationships so developed may have an objective function of minimizing the total inventory costs subject to changes in inventory reorder policy and constraints of limited resources (such as floor space for storage, capital investment, etc.). Thus, the problem would either be an unconstrained optimization problem or a mathematical programming problem depending upon whether constraints are imposed or not.

Step 3. Derive an optimal inventory policy (*i.e.*, economic order quantity) by using an appropriate solution procedure so as to balance among inventory costs.

Remark. When the future demand is known exactly, the inventory problem is said to be *under certainty*. If the probability distribution of future demand (from past records) is known, the inventory problem is said to be *under risk*. In case the levels of future demand are totally unknown, the inventory problem is termed to be *under uncertainty*.

19:9. THE CONCEPT OF *EOQ*

The inventory problems in which demand is assumed to be fixed and completely pre-determined are usually referred to as the *economic order quantity (EOQ)* or *lot size problems.*

By the 'order quantity' we mean the quantity produced or procured during one production cycle. When the size of order increases, the ordering costs (cost of purchasing, inspection, etc.) will decrease whereas the inventory carrying costs (cost of storage, insurance, etc.) will increase. Thus, in the production process there are two opposite costs, one encourages the increase in the order size and the other discourages. *Economic Order Quantity* (EOQ) is that size of order which minimizes total annual costs of carrying inventory and cost of ordering.

The two opposite costs can be shown graphically by plotting them against the order size as shown in Fig. 19.1 below :

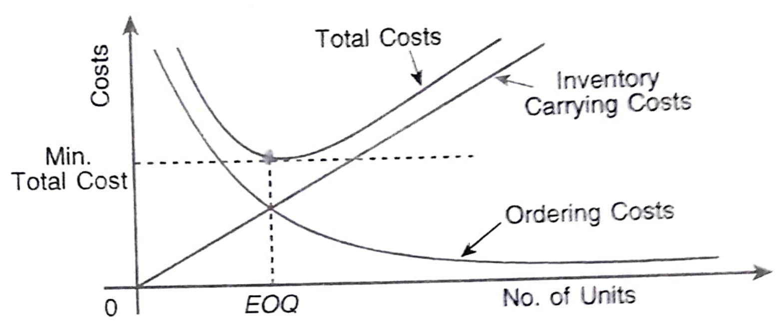

Fig. 19.1. Graph of *EOQ*

It is evident from above that the minimum total cost occurs at the point where the ordering costs and inventory carrying costs are equal.

19:10. DETERMINISTIC INVENTORY PROBLEMS WITH NO SHORTAGES

This section presents five variations of the *EOQ* problem when demand is assumed to be fixed and completely pre-determined.

Case 1. The Fundamental Problem of *EOQ*

The objective of the study of this problem is to determine an optimum order quantity (*EOQ*) such that the total inventory cost is minimized. We illustrate the problem, under consideration, using the following assumptions :

(*i*) Demand is known and uniform.

(*ii*) Let D denote the total number of units purchased/produced or supplied per time period and Q denote the lot size in each production run.

(*iii*) Shortages are not permitted, *i.e.*, as soon as the level of the inventory reaches zero, the inventory is replenished.

(*iv*) Production or supply of commodity is instantaneous (Abundant Availability).

(*v*) Lead time is zero.

(*vi*) Set-up cost (or ordering cost) per production run on procurement cost is C_o (or A).

(*vii*) Holding cost is C_1 per unit in inventory for a unit, *i.e.*, $C_1 = IC$, where C is the unit cost, I is called inventory carrying charge expressed as a % of the value of the average inventory.

This fundamental situation can be shown on an inventory-time diagram, with Q on the vertical axis and time on the horizontal axis. The total time period (one year) is divided into n parts :

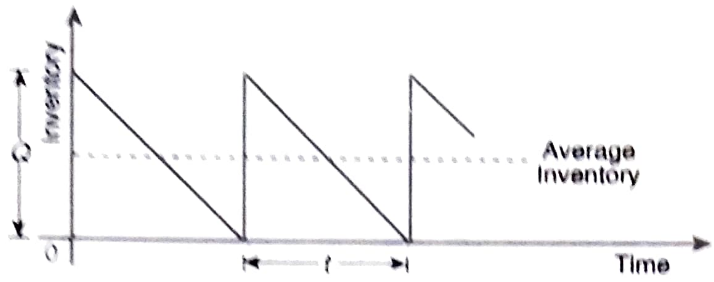

Fig. 19.2. Problem of *EOQ* with uniform demand

Here, it is assumed that after each time t, the quantity Q is produced/purchased or supplied throughout the entire time period, say one year. Now, if n denotes the total number of runs of the quantity produced or purchased during the year, then clearly we have

$$1 = nt \text{ and } D = nQ.$$

It may be clear that the *average* amount of inventory at hand on any day is then $\frac{1}{2}Q$, as shown by dotted line in Fig. 19.2. Total inventory over the time period t days is clearly the area of the first triangle $(= \frac{1}{2}Qt)$. Thus, the average inventory at any time on any given day in the t period is $\frac{1}{2}Qt/t = \frac{1}{2}Q$.

Now, since each of the triangles in Fig. 19.2 over a year period looks the same, $\frac{1}{2}Q$ remains the average amount of inventory in each interval of length t during the entire period. Annual inventory holding cost is therefore given by

$$f(Q) = \frac{1}{2}QC_1$$

Annual costs associated with runs of size Q are given by

$$g(Q) = nC_o = \frac{D}{Q}C_o, \text{ since } n = \frac{D}{Q}.$$

Since, the minimum total cost occurs at the point where ordering cost and the total inventory carrying cost are equal, we must have

$$f(Q) = g(Q).$$

This implies, that

$$\frac{1}{2}QC_1 = \frac{D}{Q}C_o$$

Hence, the optimum value of Q is

$$Q^o = \sqrt{2DC_o/C_1}$$

This is known as the *economic (optimum) lot size formula* due to R.H. Wilson.

The above *EOQ* formula can also be expressed in terms of the *economic order value* as follows :

$$Q^o = \sqrt{\frac{2AD}{IC}}$$

Characteristics of Case 1.

1. Optimum number of orders placed per year

$$n^o = D/Q^o = \sqrt{DC_1/2C_o}$$

2. Optimum length of time between orders

$$t^o = T/n^o = T\sqrt{2C_o/DC_1} \quad \text{or} \quad \sqrt{2C_o/DC_1},$$

when T (total time horizon) is one year.

The optimum length of time between orders is also called the 'economic review period' (*ERP*).

3. Total annual expected cost (TEC) is given by

$$(TEC)_0 = \text{Annual inventory carrying cost} + \text{Annual ordering cost}$$

$$= \frac{1}{2} Q^\circ C_1 + \frac{D}{Q^\circ} C_0$$

$$= \frac{1}{2} \times \sqrt{\frac{2DC_0}{C_1}} \times C_1 + D \times \sqrt{\frac{C_1}{2DC_0}} \times C_0$$

$$= \sqrt{2DC_1C_0}.$$

This represents the total annual cost associated with holding inventory and setting the machine when the optimum lot size, Q°, is produced in each run. The optimum (minimum) total cost is demonstrated graphically in Fig. 19.3.

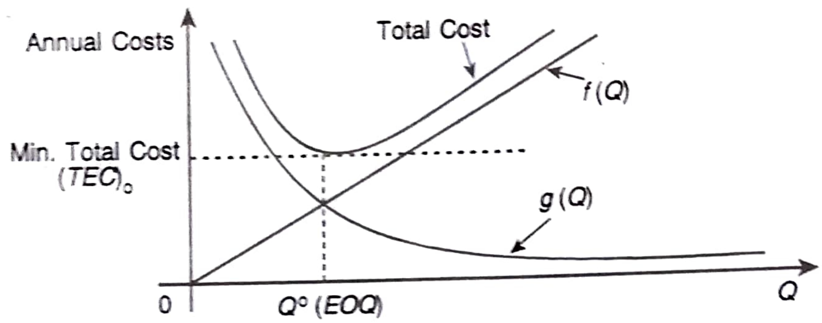

Fig. 19.3

Remark. *1.* Since the holding cost $C_1 > 0$, annual inventory costs are linear functions of Q with a positive slope; the smaller the average inventory, the lower are these costs. In contrast to these costs, ordering cost increases as Q increases.

2. If the carrying cost is given as a percentage of average value of inventory held, then total annual carrying cost may be expressed as $C_1 = C \times I$ or $P \times I$. The total annual inventory cost then becomes

$$TC = \frac{1}{2} Q \times C \times I + DC_0/Q$$

The optimum order quantity Q° will, then, be

$$Q^\circ = \sqrt{2DC_0/CI}.$$

3. Generally, the annual demand for inventory item(s) is expressed in rupee value rather than in units. In such cases, the demand may be expressed in units as follows (provided unit cost of the item is known) :

$$\text{Demand in units} = \frac{\text{Rupee value of demand}}{\text{Unit cost of the item}}$$

When the unit cost of the item is not known, then determine EOQ in rupee terms as follows :

$$Q^\circ = \sqrt{\frac{2 \times \text{Annual demand in rupees} \times \text{Ordering cost}}{\text{Inventory carrying cost}}}$$

Corollary 1. *In the EOQ problem discussed above, if the set-up (ordering) cost is $C_0 + bQ$ instead of being fixed (where b is set-up cost per unit item produced) then there is no change in the optimum order quantity produced due to change in the set-up cost.*

Proof. In this case, the annual cost is given by

$$TC = \frac{1}{2} QC_1 + \frac{D}{Q}(C_0 + bQ).$$

For the optimum value of Q, we see that

$$\frac{d}{dQ}(TC) = 0 \Rightarrow Q = \sqrt{\frac{2DC_0}{C_1}} \quad \text{and} \quad \frac{d^2}{dQ^2}(TC) > 0 \quad \text{for} \quad Q > 0.$$

Hence
$$Q^\circ = \sqrt{\frac{2DC_0}{C_1}}.$$

This shows that there is no change in Q° in spite of change in the set-up cost.

Corollary 2. *In the above EOQ problem lead time has been assumed to be zero. But in most of the business situations, there exists a positive lead time, say L, from the time the order is placed until it is actually supplied.*

Proof. If the inventory consumption rate is 'K' units per day and L is the lead time in days, the total inventory requirements during the lead time will be 'LK'. Therefore, as soon as the inventory level becomes 'LK' an order Q is placed. This is called the re-order point $p = LK$. This is equivalent to continuously observing the level of inventory, until the re-order point is obtained. Because of this reason, *EOQ* problem is sometimes called the *continuous review problem*. Fig. 19.4 shows the re-order points :

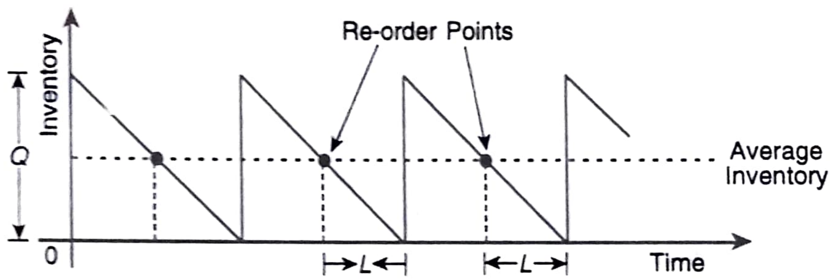

Fig. 19.4

Figure given above assumes that the lead time is less than the cycle length, which is not necessarily the case in general. To account for this situation, we define the effective lead time as

$$L_e = L - mt^\circ,$$

where m is the largest integer not exceeding L/t°. This result is justified because after m cycles of t° each, the inventory situation acts as if the interval between placing an order and receiving another is L_e.

Case 2. Problem of *EOQ* with Several Production Runs of Unequal Length

In this problem all the assumptions are same as in *Case 1* except that the demand is uniform and the production runs differ in units.

Let $t_1, t_2, t_3, \ldots, t_n$ denote the times of successive production runs, such that

$$t_1 + t_2 + \ldots + t_n = 1 \text{ year}.$$

Thus, the fundamental situation can be represented graphically as shown in Fig. 19.5 :

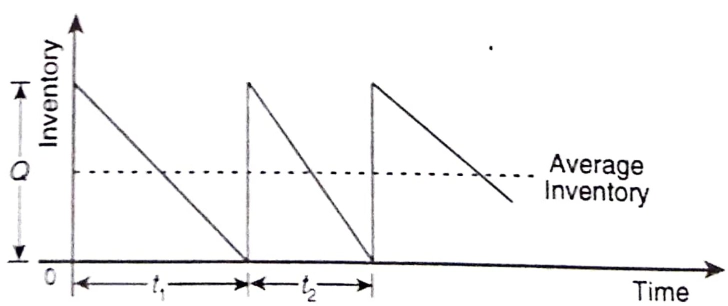

Fig. 19.5

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Obviously, the annual inventory holding cost is given by

$$f(Q) = \left(\tfrac{1}{2}Q t_1\right)C_1 + \left(\tfrac{1}{2}Q t_2\right)C_1 + \dots + \left(\tfrac{1}{2}Q t_n\right)C_1$$
$$= \tfrac{1}{2}Q(t_1 + t_2 + \dots + t_n)C_1 = \tfrac{1}{2}QC_1$$

and the set-up costs associated with runs of size Q are given by

$$g(Q) = \frac{D}{Q}C_o, \quad \text{since} \quad nQ = D.$$

$\therefore$ Total annual cost is

$$TC = f(Q) + g(Q) = \tfrac{1}{2}QC_1 + \frac{D}{Q}C_o.$$

This cost is the same as was obtained in *Case 1* and hence the optimum quantities are

$$Q^o = \sqrt{2DC_o/C_1} \quad \text{and} \quad TC^o = \sqrt{2DC_1C_o}$$

Remark. If the total time period is T instead of one year, then the optimum order quantity becomes $Q^o = \sqrt{2C_o D/C_1 T}$ and the minimum cost becomes $TC^o = \sqrt{2C_1 C_o D/T}$. Thus, the uniform rate of demand is replaced by average rate of demand, *i.e.*, D is replaced by D/T.

SAMPLE PROBLEMS

1901. *An oil engine manufacturer purchases lubricants at the rate of Rs. 42 per piece from a vendor. The requirement of these lubricants is 1,800 per year. What should be the order quantity per order, if the cost per placement of an order is Rs. 16 and inventory carrying charge per rupee per year is only 20 paise.* ***[Delhi B.Sc. (Stat.) 2000]***

Solution. We are given

D = 1,800 lubricants per year, C_o = Rs. 16 per order,

C_1 = Rs. $42 \times$ Re. 0.20 = Rs. 8.40

$\therefore \quad Q^o = \sqrt{2 \times 1800 \times 16/8.40} = 82.8$ or 83 lubricants.

1902. *A manufacturing company purchases 9,000 parts of a machine for its annual requirements, ordering one month usage at a time. Each part costs Rs. 20. The ordering cost per order is Rs. 15 and the carrying charges are 15% of the average inventory per year. You have been assigned to suggest a more economical purchasing policy for the company. What advice would you offer and how much would it save the company per year?*

[Delhi B.Sc. (Stat.) 2004; Kerala M.Com. 1993; Madras M.C.A. 1999]

Solution. We are given

D = 9,000 parts per year, C_o = Rs. 15 per order

C_1 = 15% of the average inventory per year

= Rs. $20 \times 15/100$ = Rs. 3 each part per year

$$\therefore \quad Q^o = \sqrt{\frac{2 \times 15 \times 9{,}000}{3}} = 300 \text{ units}$$

$$\text{and} \quad t^o = \frac{300}{9{,}000} = \frac{1}{30} \text{ year}$$

Minimum average yearly cost = $\sqrt{2 \times 3 \times 15 \times 9{,}000}$ = Rs. 900. If the company follows the policy of ordering every month, then the annual ordering cost is Rs. 12×15 or Rs. 180, and lot size of inventory each month = 9,000/12 = 750 (= Q).

Average inventory at any time = $\tfrac{1}{2}Q$ = 750/2 = 375.

$\therefore$ Storage cost at any time = $375\,C_1 = 375 \times 3$ = Rs. 1,125.

Total annual cost = Rs. 1,125 + Rs. 180 = Rs. 1,305.

Hence, the company should purchase 300 parts at time intervals of 1/30 year instead of ordering 750 parts each month. The net saving of the company will be

$$\text{Rs. } 1{,}305 - \text{Rs. } 900 = \text{Rs. } 405 \text{ per year.}$$

1903. *A Company operating 50 weeks in a year is concerned about its stocks of copper cable. This costs Rs. 240 a metre and there is a demand for 8,000 metres a week. Each replenishment costs Rs. 1,050 for administration and Rs. 1,650 for delivery, while holding costs are estimated at 25 per cent of value held a year. Assuming no shortages are allowed, what is the optimal inventory policy for the company?*

How would this analysis differ if the company wanted to maximize profit rather than minimize cost? What is the gross profit if the company sell cable for Rs. 360 a metre.

Solution. From the data of the problem, we have

Demand rate $(D) = 8{,}000 \times 50 = 4{,}00{,}000$ metres a year

Purchase cost (C) = Rs. 240 a metre; Ordering cost $(C_o) = 1{,}050 + 1{,}650 =$ Rs. 2,700

Holding cost $= 0.25 \times 240 =$ Rs. 60 a metre a year

$$\therefore \quad \text{Optimal order quantity, } Q^o = \sqrt{2DC_o/C_1} = \sqrt{\frac{2 \times 4{,}00{,}000 \times 2{,}700}{60}} = 6{,}000 \text{ metres,}$$

$$\text{Total inventory cost } (TC) = \text{Purchase cost} + \text{Total expected cost}$$

$$= D \times C + \left(\frac{1}{2} Q^o C_1 + \frac{D}{Q^o} C_o\right)$$

$$= 4{,}00{,}000 \times \text{Rs. } 240 + \left(\frac{1}{2} \times 6{,}000 \times \text{Rs. } 60 + \frac{4{,}00{,}000}{6{,}000} \times \text{Rs. } 2{,}700\right)$$

$$= \text{Rs. } 9{,}60{,}00{,}000 + \text{Rs. } 3{,}60{,}000 = \text{Rs. } 9{,}63{,}60{,}000.$$

With a turnover in excess of 96 million rupees a year inventory costs are only Rs. 3,60,000 or 0.36 per cent. This figure is usually low but any well run organisation should try to make all the waivings it can.

If the company wanted to maximize profit rather than minimize cost, the analysis used would remain exactly the same. This can be demonstrated by defining selling price (SP) per unit so that gross profit per unit time becomes :

$$\text{Profit} = \text{Revenue} - \text{Cost} = D \times SP - \left[D \times C + \frac{D}{Q} C_o + \frac{1}{2} QC_1\right]$$

$$= D \times SP - \text{Rs. } 9{,}63{,}60{,}000.$$

If the company sells the cable for Rs. 360 a metre, its revenue is

$$\text{Rs. } 360 \times 4{,}00{,}000 = \text{Rs. } 14{,}40{,}00{,}000 \text{ per year.}$$

$$\therefore \quad \text{Gross profit} = \text{Rs. } 14{,}40{,}00{,}000 - \text{Rs. } 9{,}63{,}60{,}000 = \text{Rs. } 4{,}76{,}40{,}000 \text{ a year.}$$

1904. *A company plans to consume 760 pieces of a particular component. Past records indicate that purchasing department haa used Rs. 12,000 for placing 15,000 orders. The average inventory was valued at Rs. 45,000 and the total storage cost was Rs. 7,650 which included wages, rent, taxes, insurance, etc. related to store department. The company borrows capital at the rate of 10% a year.*

If the price of a component is Rs. 12 and the order size is of 10 components, determine : purchase cost, purchase expenses, storage expenses, capital cost and total cost per year.

Solution. Using the given information, we get

$$\text{Ordering cost } (C_o) = \frac{\text{Rs. } 12{,}555}{15{,}000} = \text{Re. } 0.80$$

$$\text{Inventory carrying cost } (I) = \frac{7{,}650}{45{,}000} \times 100 = 17\%$$

$$\therefore \quad \text{Purchase cost} = \text{Rs. } 12 \times 760 = \text{Rs. } 9{,}120$$

5

$$\text{Purchase expenses} = \frac{\text{Re. } 0.80 \times 760}{10} = \text{Rs. } 60.80$$

$$\text{Storage expenses} = \frac{1}{2} Q \times I \times C = \frac{1}{2} \times 10 \times 0.17 \times 12 = \text{Rs. } 10.20$$

$$\text{Capital cost} = \frac{1}{2} Q \times 0.10 \times 12 = \frac{1}{2} \times 10 \times 0.10 \times 12 = \text{Rs. } 6.$$

$$\text{Total cost} = \text{Rs. } (9{,}120 + 60.80 + 10.20 + 6.00) = \text{Rs. } 9{,}197.$$

1905. *Neon lights in an industrial park are replaced at the rate of 100 units per day. The physical plant orders the neon lights periodically. It costs Rs. 100 to initiate a purchase order. A neon light kept in storage is estimated to cost about Re. 0.02 per day. The lead time between placing and receiving an order is 12 days. Determine the optimum inventory policy for ordering the neon lights.*

Solution. We are given *[Meerut M.Sc. (Math.) 2008]*

D = 100 units per day, C_o = Rs. 100 per order, C_1 = Re. 0.02 per unit per day, L = 12 days.

$$\therefore \quad Q^o = \sqrt{\frac{2 \times 100 \times 100}{0.02}} = 1{,}000 \text{ neon lights}$$

$$t^o = \frac{Q^o}{D} = 10 \text{ days}$$

Now, since the lead time, L (= 12 days) exceeds the cycle length (= 10 days), we compute L_e.

Thus, $$L_e = L - mt^o = 12 - 1 \times 10 = 2 \text{ days},$$

where m = (largest integer $\leq L/t^o$) = (largest integer $\leq 12/10$) = 1.

The re-order point, therefore, occurs when the inventory level drops to

$$L_e \times D = 2 \times 100 = 200 \text{ neon lights.}$$

Hence, the inventory policy for ordering neon lights is : Order 100 units whenever the inventory level drops to 200 units.

1906. *A chemical company holds its inventory of raw materials in special containers, with each container occupying 10 square feet of floor space. There are only 5,000 square feet of storage space available. Each year, this company uses 9,000 special containers of raw materials, paying Rs. 8 per container of raw material. If ordering cost is Rs. 40 per order and annual holding costs are 20 per cent of the average inventory value, how much is it worth for this company to increase its container of raw material storage area? How many days (maximum) supply of inventory can be stored with the 5,000 square feet storage limitation, assuming that this company works a 300-day year?*

[Kurukshetra M.B.A. 2008]

Solution. We are given

D = 9,000 containers, C_o = Rs. 40 per order, C_1 = Rs. 8.00 × Re. 0.20 = Rs. 1.60

$$\therefore \quad Q^o = \sqrt{\frac{2 \times 9000 \times 40}{1.60}} = 670.8 \text{ or } 671 \text{ approx.}$$

Hence, the optimum order size is 671 containers. Since each container occupies 10 square feet of floor space, the company will require 6,710 square feet of floor space to store the optimum order quantity. The total storage space available with the company currently is only 5,000 square feet, therefore, it can order only 500 containers at a time. Hence, in order to calculate how much is worth for this company to increase the storage area, we will calculate the total variable cost that the company will have to incur for two alternatives, *viz.*, 671 containers per order and 500 containers per order.

Alternatives	*Annual ordering cost (Rs.)*	*Annual carrying cost (Rs.)*	*Total cost (Rs.)*
671 per order	$\frac{9{,}000}{671} \times 40 = 537$	$\frac{671}{2} \times 8 \times 0.2 = 537$	1,074
500 per order	$\frac{9{,}000}{500} \times 40 = 720$	$\frac{500}{2} \times 8 \times 0.2 = 400$	1,120

Thus, the company will save Rs. (1,120 − 1,074) = Rs. 46 if it follows economic order policy and it is Rs. 46 worth for the company to increase the storage space.

With 5,000 sq. ft. limitation only, 500 units can be ordered and each day usage is 30 (= 9,000/300) units and so nearly 500/30 ≈ 17 days supply of inventory will be there in each order.

PROBLEMS

1907. Given the annual consumption of material is 3,600 units, ordering costs are Rs. 400 per order. Cost per unit of material is Rs. 64 and storage cost or annual carrying cost is 50% of inventory value. Find out Economic Order Quantity, *E.O.Q.* in (*i*) units, (*ii*) rupees. *[Jammu M.B.A. 2008]*

1908. A manufacturer has to supply his customer with 600 units of his product per year. Shortages are not allowed and the storage cost amounts to Re. 0.60 per unit per year. The set-up cost per run is Rs. 80.00. Find the optimum run-size and the minimum average yearly cost. *[Annamalai M.B.A. (Nov.) 2002]*

1909. A shopkeeper has a uniform demand of an item at the rate of 600 items per year. He buys from a supplier at a cost of Rs. 8 per item and the cost of ordering is Rs. 12 each time. If the stock holding cost is 20% per year of stock value, how frequently should he replenish his stock and what is the optimal order quantity? *[Madras M.B.A. 1996]*

1910. A certain item costs Rs. 235 per tonne. The monthly requirement is 5 tonnes and each time the stock is replenished, there is a set-up cost of Rs. 1,000. The cost of carrying inventory has been estimated at 10% of the value of the stock per year. What is the Optimal Order Quantity? *[Meerut M.Sc. (Math.) 1997]*

1911. A company manufacturers a product having a monthly demand of 2,000 units. For one unit of product 2 kg of a particular raw material item is needed. The purchase price of the material is Rs. 20 per kg. The ordering cost is Rs. 120 per order and the holding cost is 10 per cent per annum.

Calculate :

(*i*) Economic Ordering Quantity (EOQ), and

(*ii*) Annual cost of purchasing and storage of the raw material at the quantity. *[Marathwada M.B.A. 2009]*

1912. An aircraft uses rivets at an approximately constant rate of 5,000 kg. per year. The rivets cost Rs. 20 per kg. and the company personnel estimate that it costs Rs. 200 to place an order, and the carrying cost of inventory is 10% per year.

(*i*) How frequently should orders for rivets be placed, and what quantities should be ordered for?

(*ii*) If the actual costs are Rs. 500 to place an order and 15% for carrying cost, the optimum policy would change. How much is the company losing per year because of imperfect cost information? *[A.I.M.A. (Dip. in Management) Dec. 1995; Delhi B.F.I.A. 2007]*

1913. A factory follows an economic order quantity system for maintaining stocks of one of its component requirements. The annual demand is for 24,000 units, the cost of placing an order is Rs. 300, and the component cost is Rs. 60 per unit. The factory has imputed 24 per cent as the inventory carrying rate.

(*i*) Find the optimal interval for placing orders, assuming a year is equivalent to 360 days.

(*ii*) If it is decided to place only one order per month, how much extra cost does the factory incur per year as a consequence of this decision? *[Delhi M.Com. (O.C.) 2010]*

1914. A manufacturer of food processors has annual sales of 20,000 units. The manufacturer operates 250 days a year and produces 200 units per day. The set up cost is Rs. 600 for a production run. The food processors cost Rs. 400 to produce. The inventory holding cost fraction is 0.025 per year.

(*a*) What is the most economical batch size?

(*b*) What is the minimum annual total of inventory holding costs and set up costs?

(*c*) What is the number of production runs per year?

(*d*) How many days are required for the production of each batch?

(*e*) What is the maximum inventory on hand for the EOQ? *[Delhi M.Com. 2008]*

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1915. (*a*) Find the *EOQ* for the following data :

Annual usage = 1,000 pieces,	Cost per piece = Rs. 250
Ordering cost = Rs. 6 per order,	Expediting cost = Rs. 4 per order
Inventory holding cost = 20% of average inventory	
Material holding cost = Re. 1 per piece.	

[Shivaji M.B.A. 2009]

(*b*) The Deena Paints Ltd. would like to improve its inventory management policies for its supply of paint used for automobiles. Annual demand for such paint is 50,000 litres, and the paint, which costs Rs. 20 per litre, is used at a constant rate. Annual carrying costs are estimated at 15 per cent of the value of paint held. Each order costs Rs. 80. Determine :

(*i*) How much paint should be ordered each time?

(*ii*) How often should paint be ordered?

(*iii*) Time between two consecutive orders.

(*iv*) What is the total annual cost associated with this policy? *[B.H.U. M.B.A. (Nov.) 2009]*

1916. Two items have the following characteristics :

Item	*Annual demand*	*Unit price*	*Order cost*	*Holding cost*
X	9 units	Re. 0.25	Rs. 8.00	25%
Y	27 units	Rs. 3.00	Rs. 18.00	25%

(*a*) For each item, compute the annual ordering cost, the annual holding cost, and the annual total cost for order quantities of 24, 36, 48 and 60 units.

(*b*) Compute *EOQ* for each item and verify the computation by computing total costs for cone unit more than *EOQ* and one unit less than *EOQ*.

[***Hint*** : Total cost = Ordering cost + Holding cost = $\frac{D}{Q} \times C_o + \frac{1}{2} Q \times C \times I$]

1917. A purchase manager places order each time for a lot of 500 units of a particular item. From the available data the following results are obtained :

Inventory carrying cost = 40 %	Ordering cost per order = Rs. 600
Cost per unit = Rs. 40	Annual demand = 6 orders each of 200 units.

Find out the loss to the organization due to his ordering policy. *[Osmania M.B.A. 1998]*

1918. Find the *EOQ* for the following data :

Annual usage = 1,000 pieces	Cost per piece = Rs. 250
Ordering cost = Rs. 6 per order	Expediting cost = Rs. 4 per order
Inventory holding cost = 20% of average inventory	Material holding cost = Re. 1 per piece.

[Shivaji M.B.A. 1996]

Case 3. Problem of *EOQ* with Finite Replenishment (Production)

In this problem all the assumptions are same as in *case 1*, except that of instantaneous replenishment. Assume that each production run of length t consists of two parts, say t_1 and t_2, such that

(*i*) the inventory is building up at a constant rate of $(k-r)$ units, per unit of time during t_1, $k > r$;

(*ii*) there is no replenishment (or production) during time t_2 and the inventory is decreasing at the rate of r per unit of time.

The graphical representation of the situation is shown in Fig. 19.6.

Here, the total (order) quantity Q is produced over a period, t_1, which is defined by the production rate k. Since the inventory does not pile up in one shot but rather continuously over a time period and is also consumed simultaneously, the average inventory level would be determined not only by the lot size Q, but also be affected by the production rate k and depletion (demand) rate r.

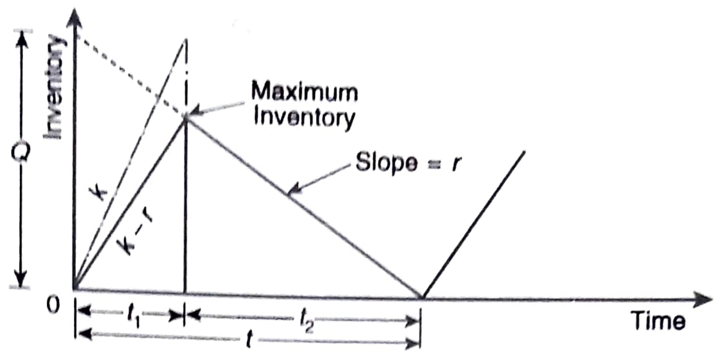

Fig. 19.6

To determine the average inventory, we proceed as follows :

Since t_1 is the time required to produce Q at a rate k, we shall have

$$Q = kt_1 \quad \text{or} \quad t_1 = Q/k.$$

During production period t_1, inventory is increasing at the rate of k and simultaneously decreasing at the rate of r. Thus, inventory accumulates at the rate of $(k - r)$ units. Therefore, the maximum inventory level shall be equal to $t_1\,(k - r)$.

$\therefore$ $$\text{Average inventory} = \tfrac{1}{2} t_1 (k - r) = \tfrac{1}{2} Q (1 - r/k), \qquad \text{since } t_1 = Q/k$$

Now, with C_1 as the holding cost per unit per year, the total annual holding cost is given by

$$f(Q) = \text{Average inventory} \times C_1 = \tfrac{1}{2} QC_1 (1 - r/k)$$

Annual ordering cost is given by

$$g(Q) = C_o \times D/Q,$$

since D is the total demand in a year.

Since the minimum total cost occurs at the point where annual ordering cost and annual holding cost are equal, we must have

$$f(Q) = g(Q)$$

This implies that

$$\frac{1}{2} QC_1 \left(1 - \frac{r}{k}\right) = C_o \frac{D}{Q}$$

Hence, the optimum value of Q is

$$Q^o = \sqrt{\frac{2\,DC_o}{C_1\,(1 - r/k)}} = \sqrt{\frac{2DC_o}{C_1}\left(\frac{k}{k-r}\right)}$$

Characteristics of Case 3

1. Optimum number of production runs per year

$$n^o = D/Q^o = \sqrt{\frac{DC_1}{2C_o}(1 - r/k)}$$

2. Optimum length of each lot size production run

$$t_1{}^o = Q^o/k = \sqrt{\frac{2DC_o}{C_1\,k\,(k-r)}}$$

3. Total minimum production inventory cost

$$TC^o = \frac{C_o D}{Q^o} + \frac{1}{2} Q^o \left(1 - \frac{r}{k}\right) C_1 = \sqrt{2DC_oC_1\,(1 - r/k)}$$

Note. If $k \to r$, then $C \to 0$. This shows that there will be no holding cost and no set-up cost.

If $k \to \infty$, *i.e.*, when the production rate becomes infinite, the above problem reduces to the one considered in *case 1*. The inventory holding cost per unit of time is reduced from the cost discussed in *case 1* in the ratio $(1 - r/k) : 1$ for minimum cost, although the set-up cost remains the same.

SAMPLE PROBLEMS

1919. *A contractor has to supply 10,000 bearings per day to an automobile manufacturer. He finds that, when he starts a production run, he can produce 25,000 bearings per day. The cost of holding a bearing in stock for one year is Rs. 2 and the set-up cost of a production run is Rs. 1,800. How frequently should production run be made?*

[Delhi B.Sc. (Stat.) 2006; Madras M.B.A. (April) 2006]

Solution. We are given

$D = 10{,}000 \times 300 = 30{,}00{,}000$ bearings (Assuming 300 working days in a year)

C_1 = Rs. 2 per bearing per year, C_o = Rs. 1,800 per production run

$r = 10{,}000$ bearings per day, $k = 25{,}000$ bearings per day

$$\therefore \quad Q^o = \sqrt{\frac{2 \times 30{,}00{,}000 \times 1{,}800}{2}} \sqrt{\frac{25{,}000}{25{,}000 - 10{,}000}} = 94{,}868 \text{ bearings}$$

$$t^o = \frac{94{,}868}{10{,}000} = 9.49 \text{ days}$$

$$\text{Length of production cycle} = \frac{94{,}868}{25{,}000} = 4 \text{ days (approx.)}$$

Thus, the production cycle starts at an interval of 9.49 days and production continues for 4 days so that in each cycle a batch of 94,868 bearings is produced.

1920. *A manufacturing company needs 2,500 units of a particular component every year. The company buys it at the rate of Rs. 30 per unit. The order processing cost for this part is estimated at Rs. 15 and the cost of carrying a part in stock comes to about Rs. 4 per year.*

The company can manufacture this part internally. In that case, it saves 20% of the price of the product. However, it estimates a set-up cost of Rs. 250 per production run. The annual production rate would be 4,800 units. However, the inventory holding costs remain unchanged.

(*i*) *Determine the EOQ and the optimal number of orders placed in a year.*

(*ii*) *Determine the optimum production lot size and the average duration of the production run.*

(*iii*) *Should the company manufacture the component internally or continue to purchase it from the supplier?* **[Delhi M.Com. 1993]**

Solution. (*i*) Here, we have

D = 2,500 units, C_o = Rs. 15 per order, and C_1 = Rs. 4 per unit per year.

$$\therefore \quad Q^o = \sqrt{\frac{2DC_o}{C_1}} = \sqrt{\frac{2 \times 2{,}500 \times 15}{4}} = 137 \text{ units.}$$

$$n^o = \text{Optimal number of orders} = \frac{D}{Q^o} = \frac{2{,}500}{137} = 18.$$

(*ii*) We are given :

D = 2,500 units, C_o = Rs. 250 per production run, C_1 = Rs. 4 per unit per year,

r = 2,500 units, and k = 4,800 units.

$$\therefore \quad \text{Economic lot size, } Q_1^o = \sqrt{\frac{2DC_o}{C_1}} \times \sqrt{\frac{k}{k-r}}$$

$$= \sqrt{\frac{2 \times 2{,}500 \times 250}{4}} \times \sqrt{\frac{4{,}800}{4{,}800 - 2{,}500}} = 808 \text{ units.}$$

$$t^o = \text{Average duration of the production run} = \frac{808}{2{,}500} = 0.32 \text{ year}$$

(*iii*) When item is purchased from outside :

$$\text{Total cost} = D \times C + \left(\frac{D}{Q^o} \times C_o + \tfrac{1}{2} Q^o C_1\right)$$

$$= 2{,}500 \times 30 + \left(\frac{2{,}500}{137} \times 15 + \tfrac{1}{2} \times 137 \times 4\right)$$

$$= 75{,}000 + 547.72$$

$$= \text{Rs. } 75{,}547.72 \text{ or Rs. } 75{,}548 \text{ approx.}$$

When item is produced internally :

Cost per unit, C = 80% of Rs. 30 = Rs. 24, and C_1 = Rs. 250 per production run.

$$\therefore \quad \text{Total cost} = D \times C + \frac{D}{Q_1^o} \times C_o + \tfrac{1}{2} Q_1^o \times \frac{k-r}{k} \times C_1$$

$$= 2{,}500 \times 24 + \frac{2{,}500}{808} \times 250 + \tfrac{1}{2} \times 808 \times \frac{4{,}800 - 2{,}500}{4{,}800} \times 4$$

$$= 60{,}000 + 1{,}547.84 = \text{Rs. } 61{,}547.84$$

From the above, we observe that the company should manufacture the product internally.

1921. (*a*) *At present a company is purchasing an item 'X' from outside suppliers. The assumption is 10,000 units/year. The cost of the item is Rs. 5 per unit and the ordering cost is estimated to be Rs. 100 per order. The cost of carrying inventory is 25%. If the consumption rate is uniform, determine the economic purchasing quantity.*

(*b*) *In the above problem, assume that company is going to manufacture the item with the equipment that is estimated to produce 100 units per day. The cost of the unit thus produced is Rs. 3.50 per unit. The set-up cost is Rs. 150 per set-up and the inventory carrying charge is 25 per cent. How has your answer changed?* ***[Jammu M.B.A. (Oct.) 1990]***

Solution. We are given

(*a*) D = 10,000 units per year, C_o = Rs. 100 per order, and C_1 = Rs. 5×0.25 = Rs. 1.25.

$$\therefore \quad EOQ = Q^o = \sqrt{\frac{2DC_o}{C_1}} = \sqrt{\frac{2 \times 10{,}000 \times 100}{1.25}} = 1{,}265 \text{ units.}$$

(*b*) Here, C_o = Rs. 150 per set-up, C_1 = Rs. 3.50×0.25 = Re. 0.875, k = 100 units per day, and

$$r = \frac{10{,}000}{250} \,(= 40) \text{ units per day (assuming 250 working days in a year).}$$

$$\therefore \quad EOQ = Q^o = \sqrt{\frac{2 \times 10{,}000 \times 150}{0.875}} \sqrt{\frac{100}{100 - 40}} = 2{,}391 \text{ units.}$$

The increase in EOQ in case of (*b*) may be due to following reasons :

(*i*) Increased ordering cost from Rs. 100 to Rs. 150 per set-up.

(*ii*) Consumption of inventory simultaneously with production, reduces average inventory from $\frac{Q}{2}$ to $\left(1 - \frac{r}{k}\right)\frac{Q}{2}$.

PROBLEMS

1922. An item is produced at the rate of 50 items per day. The demand occurs at the rate of 25 items per day. If the set-up cost is Rs. 100.00 and holding cost is Re. 0.01 per unit of item per day, find the economic lot size for one run, assuming that the shortages are not permitted. Also find the time of cycle and minimum total cost for one run. *[Madras B.Sc. (Math.) 1995]*

1923. An item is produced at the rate of 128 units per day. The annual demand is 6,400 units. The set-up cost for each production run is Rs. 24 and inventory carrying cost is Rs. 3 pr unit per year. There are 250 working days for production each year. Develop an inventory policy for this item. *[Amravathi B.E. (Rul.) 1994]*

1924. A contractor has to supply 20,000 units per day. He can produce 30,000 units per day. The cost of holding a unit in stock is Rs. 3 per year and the set-up cost per run is Rs. 50. How frequently, and of what size, the production runs be made?

1925. Amit manufactures 50,000 bottles of tomato ketchup in an year. The factory cost per bottle is Rs. 5, the set-up cost per production run is estimated to be Rs. 90, and the carrying costs on finished goods inventory amount to 20% of the cost per annum. The production rate is 600 bottles per day, and sales amount to 150 bottles per day. What is the optimum production lot size and the number of production runs?

If the factory costs increase to Rs. 7.50 per bottle, what will be the optimum production lot size? *[Punjabi M.B.A. 1996]*

1926. The details of a part to be machined are as follows :

Annual requirement = 2,400 pieces, Machine rate = 10 pieces/shift
No. of working days in the year = 320 shifts
Cost of machining a component = Rs. 100 per piece
Inventory carrying cost per annum = 12% of value
Set-up cost per production run = Rs. 400

Find the optimum run size for machining. *[Madras M.B.A. (Nov.) 2008]*

1927. A manufacturing company uses an *EOQ* (Economic order quantity) approach in planning its production of gears. The following information is available. Each gear costs Rs. 250 per unit, annual demand is 60,000 gears, set-up cost are Rs. 4,000 per set-up and the inventory carrying cost per month is established at 2 per cent of the average inventory value. When in production, these gears can be produced at the rate of 400 units per day and this company works only for 300 days in a year. Determine the economic lot size, the number of production runs per year and the total inventory costs. *[Gujarat M.B.A. 2009]*

1928. A product is to be manufactured on a machine. The cost, production and demand etc. are as follows :

Fixed costs per lot	= Rs. 30
Variable costs per unit	= Re. 0.10
Percentage charges for interest, taxes, insurance and storage	= 50%
Production rate	= 1,00,000 units per year
Demand rate	= 10,000 units per year

Determine the economic manufacturing quantity.

1929. Company *A* wants to know what production cost its major competitor, company *B*, has assigned to product item p_7. After a bit of investigation, company *A* has collected the following data about company *B*'s production of item p_7 :

Production lot size	= 2,600 units
Set-up cost	= Rs. 135
Annual demand	= 30,000 units
Daily demand	= 100 units
Production rate	= 200 units per day
Inventory holding costs	= 28% of the average value per year.

Company *A* has further learnt that company *B* produces according to 'economic lot size' model.

What is the company *B*'s cost of producing product item p_7? *[Delhi PG Dip. in Glob. Bus. Oper. 2011]*

19:11. DETERMINISTIC INVENTORY PROBLEM WITH SHORTAGES

In a business concern, if shortages occur then these can be classified into the following two categories :

(*a*) As soon as the desired units of a certain commodity arrive in inventory, the back orders are satisfied,

(*b*) Shortages are lost sales.

In the first category demand of the customer is met in the beginning of new production run, whereas in the second category the customer moves to some other firm to fulfil his requirements. This case deals with those problems of shortages where back orders are entertained.

Case 1. Problem of *EOQ* with Instantaneous Production and Variable Order Cycle Time

The problem that we now discuss is same as was discussed in Section *19:10 (Case 1)* with the difference that the shortages are now permitted. Let C_2 be the shortage cost per unit of time per unit quantity. This inventory situation can also be illustrated graphically as shown in Fig. 19.7 :

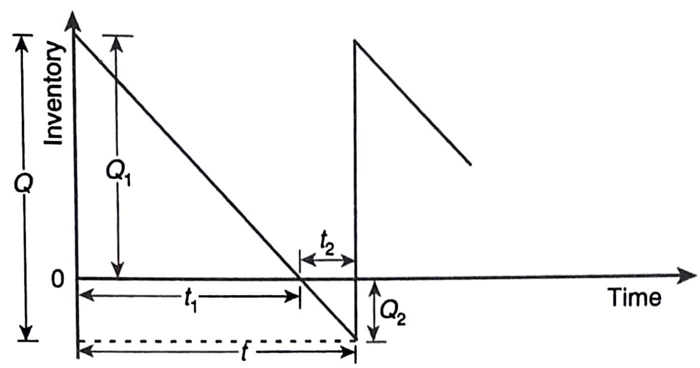

Fig. 19.7

Here the total time period is one year and is divided into equal parts, say of interval t. Further, this time interval t is divided into two parts t_1 and t_2, such that $t = t_1 + t_2$.

During the interval t_1, the items are drawn from the inventory as needed and during t_2, orders for the item are being accumulated but not filled. Then at the end of the interval t an amount Q is produced (or delivered). The amount Q has been divided into Q_1 and Q_2 such that $Q = Q_1 + Q_2$, where Q_1 denotes the amount which goes into inventory, and Q_2 denotes the amount which is immediately taken to satisfy past orders or unfilled demand.

The problem now is concerned with the areas of triangles above the time axis (representing items in inventory) and below the same axis (representing items in shortage).

Now, Total inventory over the time period t $= \frac{1}{2} Q_1 t_1$

Average inventory at any time $= \left(\frac{1}{2} Q_1 t_1\right)/t$

Annual inventory holding cost $= C_1\left(\frac{1}{2} Q_1 t_1\right)/t$

Similarly,

Total amount of shortage over time period t $= \frac{1}{2} Q_2 t_2$

Annual shortage costs $= C_2\left(\frac{1}{2} Q_2 t_2\right)/t$

Annual costs associated with runs of size $Q = nC_o = \frac{D}{Q} C_o$,

since D/Q runs are produced in each year.

$\therefore$ Total annual cost is given by

$$TC = \left[C_1\left(\frac{1}{2} Q_1 t_1\right) + C_2\left(\frac{1}{2} Q_2 t_2\right)\right]/t + \frac{D}{Q} C_o.$$

Now, using the relationship for similar triangles, we have

$$\frac{t_1}{t} = \frac{Q_1}{Q} \quad \text{and} \quad \frac{t_2}{t} = \frac{Q_2}{Q}$$

i.e.,

$$t_1 = \frac{Q_1}{Q}t \quad \text{and} \quad t_2 = \frac{Q_2}{Q}t.$$

Making use of these values, we get

$$TC = \frac{1}{2}C_1\left(\frac{Q_1^2}{Q}\right) + \frac{1}{2}C_2\left[\frac{(Q - Q_1)^2}{Q}\right] + C_o\left(\frac{D}{Q}\right),$$

since $Q_2 = Q - Q_1$.

For determining the optimum values of Q_1 and Q so as to optimize TC, we have

$$\frac{\partial (TC)}{\partial Q_1} = 0 \quad \Rightarrow \quad Q_1 = C_2 Q/(C_1 + C_2),$$

$$\frac{\partial (TC)}{\partial Q} = 0 \quad \Rightarrow \quad Q = \sqrt{\frac{2C_o D + C_1 Q_1^2}{C_2} + Q_1^2}$$

and $$\frac{\partial^2 (TC)}{\partial Q_1^2} > 0, \quad \frac{\partial^2 (TC)}{\partial Q^2} > 0 \text{ for these values of } Q_1 \text{ and } Q.$$

Thus, the optimum quantities are given by (on simplification)

$$Q^o = \sqrt{\frac{2C_o D}{C_1}}\sqrt{\frac{(C_1 + C_2)}{C_2}}$$

$$Q_1^o = \left(\frac{C_2}{C_1 + C_2}\right)Q^o = \sqrt{\frac{C_2}{C_1 + C_2}}\sqrt{\frac{2C_o D}{C_1}} \qquad \textit{(Optimum Stock Level)}$$

Characteristics of Case 1

1. Time between receipt of orders (when to order)

$$t^o = Q^o/D = \sqrt{\frac{2C_o}{DC_1}}\sqrt{\frac{C_1 + C_2}{C_2}}$$

2. Total optimum inventory cost

$$TC^o = \sqrt{2DC_oC_1}\sqrt{\frac{C_2}{C_1 + C_2}}$$

3. Maximum inventory level

$$Q^o - Q_1^o = Q^o\left(1 - \frac{C_1}{C_1 + C_2}\right).$$

$$= \sqrt{\frac{2C_o D}{C_1}}\sqrt{\frac{C_2}{C_1 + C_2}}$$

Remarks 1. If $C_1 > 0$ and $C_2 \to \infty$, *shortages are prohibited.* In this case $Q_1^o = Q^o = \sqrt{2C_o D/C_1}$ and each batch of Q^o is used entirely for inventory.

2. If $C_1 \to \infty$ and $C_2 > 0$, *inventories are prohibited.* In this case $Q_1^o = 0$, $Q^o = \sqrt{2C_o D/C_2}$ and each batch is used only to fill back orders.

3. If *shortage costs are negligible*, then $C_1 > 0$ and $C_2 \to 0$. In this case $Q_1^o \to 0$ and $Q^o \to \infty$.

4. If *inventory costs are negligible*, then $C_1 \to 0$ and $C_2 > 0$. In this case $Q^o \to \infty$ and $Q_1^o \to \infty$, *i.e.*, $Q_1^o \to Q^o$. Thus as inventory costs become very small, increasingly large batches should be produced and used entirely as inventory for future demands.

5. When *inventories and shortages are equally costly, i.e.*, when

$$C_1 = C_2, \frac{C_2}{C_1 + C_2} = \frac{1}{2}.$$

Thus, in this case

$$Q^o = \sqrt{2}\sqrt{\frac{2C_o D}{C_1}} = (1.414)\sqrt{\frac{2C_o D}{C_1}}$$

This shows that the lot size is 1.414 times as large as earlier when no shortages were allowed.

Case 2. Problem of *EOQ* with Instantaneous Production and Fixed Order Cycle

Let t be fixed, *i.e.*, inventory is to be replenished after every time period t. Also, let us assume that items are being supplied or produced at the rate of r units per unit of time during this fixed time period.

Here, total inventory over the time period $t_1 = \frac{1}{2} Q_1 t_1$

and total amount of shortages over time period $t_2 = \frac{1}{2} Q_2 t_2$

$\therefore$ Total production cost is given by

$$TC = \frac{1}{2} Q_1 t_1 C_1 + \frac{1}{2} Q_2 t_2 C_2$$

Using now the relationship of similar triangles, *viz.*,

$$\frac{t_1}{t} = \frac{Q_1}{Q} \quad \text{and} \quad \frac{t_2}{t} = \frac{Q_2}{Q}$$

the cost equation reduces to

$$TC = \frac{1}{2Q} \cdot C_1 Q_1^2 t + \frac{1}{2Q} \cdot C_2 Q_2^2 t + \frac{1}{2r} C_1 Q_1^2 + \frac{1}{2r} C_2 (rt - Q_1)^2,$$

since $Q_2 = Q - Q_1$ and $Q = rt$.

The optimum value Q_1^o is obtained as follows :

$$\frac{\partial (TC)}{\partial Q_1} > 0 \Rightarrow C_1 Q_1 + C_2 (Q_1 - rt) = 0 \quad \text{or} \quad Q_1 = rt C_2/(C_1 + C_2).$$

Now $\frac{\partial^2 (TC)}{\partial Q_1^2} > 0$ for all values of Q_1, therefore,

$$Q_1^o = rt\left(\frac{C_2}{C_1 + C_2}\right). \qquad \textit{(Optimum Inventory Level)}$$

Note : In this problem, set-up cost is not considered, because of it being fixed as the time of one production run is fixed. Substituting the value of Q_1^o in the total production cost equation, the optimum inventory cost is

$$TC^o = \left(\frac{C_1 C_2}{C_1 + C_2}\right) \cdot rt$$

Case 3. Problem of *EOQ* with Finite Replenishment (Production)

In this problem all the assumptions are same as in *case 1* except that the rate of replenishment of inventory is finite, say k units per unit of time.

Assume that each production run of length t consists of two parts t_1 and t_2 which are further sub-divided into two parts say t_{11} and t_{12}, t_{21} and t_{22}, where :

(*i*) inventory is building up at a constant rate of $(k - r)$ units per unit of time during time t_{11},

(*ii*) no replenishment during time t_{12} and inventory is decreasing at the rate r per unit of time,

(*iii*) shortage is building up at a constant rate of r per unit of time during time t_{21},

(*iv*) shortages are being filled immediately at the rate of $(k - r)$ units per unit of time during time t_{22}.

The graphical representation of the situation is as follows :

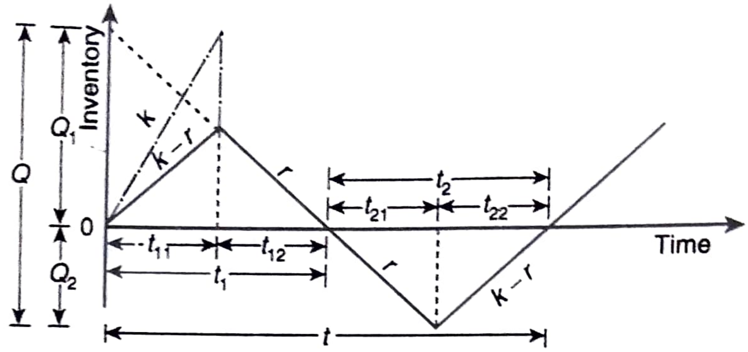

Fig. 19.8

From Fig. 19.8, we see that at the end of t_{11}, the level of inventory is Q_1 and at the end of period t_{12} inventory becomes nil. Now shortages start and suppose that the shortages build up of quantity Q_2 up to time t_{21} and let then these shortages be filled up during time t_{22}.

Then obviously,

$$Q_1 = t_{11}(k-r), \quad Q_1 = t_{12}r,$$
$$Q_2 = t_{21}r \quad \text{and} \quad Q_2 = t_{22}(k-r).$$

Now, if Q is the lot size, then

$$Q_1 = Q - Q_2 - rt_{11} - rt_{22}.$$

Eliminating t_{11} and t_{22} from this, we have

$$Q_1 = Q - Q_2 - r\left(\frac{Q_1}{k-r} + \frac{Q_2}{k-r}\right) \quad \text{or} \quad Q_1 + Q_2 = (k-r)\,Q/k$$

Production cycle is

$$t = t_{11} + t_{12} + t_{21} + t_{22} = \frac{Q_1}{k-r} + \frac{Q_1}{r} + \frac{Q_2}{r} + \frac{Q_2}{k-r}$$
$$= k(Q_1 + Q_2)/r(k-r)$$

Substituting the value of $Q_1 + Q_2$, we get $t = Q/r$.

The average inventory and amount of shortage during production cycle time t are :

Average inventory $= \frac{1}{2}Q_1(t_{11} + t_{12})/t$, and *Average shortage* $= \frac{1}{2}Q_2(t_{21} + t_{22})/t$.

$\therefore$ The total inventory cost is

$$TC = \frac{1}{2}Q_1C_1\frac{(t_{11} + t_{12})}{t} + \frac{1}{2}Q_2C_2\frac{(t_{21} + t_{22})}{t} + rC_o/Q$$
$$= \frac{1}{2Q} \times \frac{k}{k-r}\left[C_1\left(\frac{k-r}{k}Q - Q_2\right)^2 + C_2Q_2^2\right] + \frac{r}{Q}C_o$$

since $$Q_1 + Q_2 = \frac{r(k-r)}{k}t = \frac{r(k-r)}{k} \times \frac{Q}{r} = \frac{k-r}{k}Q.$$

Now, $$\frac{\partial\,TC}{\partial\,Q_2} = 0 \Rightarrow Q_2 = \frac{C_1Q}{C_1 + C_2}(1 - r/k)$$

and $$\frac{\partial\,TC}{\partial\,Q} = 0 \Rightarrow Q = \sqrt{\frac{2C_o(C_1 + C_2)}{C_1C_2}}\sqrt{\frac{kr}{k-r}}$$

Since $\dfrac{\partial^2 TC}{\partial Q^2} > 0$, and $\dfrac{\partial^2 (TC)}{\partial Q_2^2} > 0$ for all values of Q and Q_2, the optimum values of Q and Q_2 so as to minimize TC are given by

$$Q^o = \sqrt{\frac{2\,C_o\,(C_1 + C_2)}{C_1 C_2}}\sqrt{\frac{kr}{k - r}} \qquad \textit{(Optimum production lot size)}$$

and

$$Q_2{}^o = Q^o\left(1 - \frac{r}{k}\right)\left(\frac{C_1}{C_1 + C_2}\right)$$

$$= \sqrt{\frac{2rC_oC_1}{(C_1 + C_2)\,C_2}}\sqrt{\frac{k - r}{k}}\,. \qquad \textit{(Optimum level of shortage)}$$

Characteristics of Case 3

1. Production cycle time is

$$t^o = \frac{Q^o}{r} = \sqrt{\frac{2\,C_o\,(C_1 + C_2)}{C_1 C_2}}\sqrt{\frac{k}{r\,(k - r)}}\,.$$

2. Maximum inventory level is

$$Q_1{}^o = \left(\frac{k - r}{k}\right)Q^o - Q_2{}^o = \sqrt{\frac{2\,C_2\,C_o}{C_1\,(C_1 + C_2)}}\sqrt{r\left(1 - \frac{r}{k}\right)}\,.$$

3. Total minimum production inventory cost is

$$TC^o = \frac{1}{2Q^o}\times\left(\frac{k}{k - r}\right)(C_1\,Q_1^{o\,2} + C_2\,Q_2^{o\,2}) + \frac{r}{Q^o}\,C_o$$

$$= \sqrt{\frac{2\,C_1\,C_2\,C_o}{C_1 + C_2}}\sqrt{r\left(1 - \frac{r}{k}\right)}\,.$$

Remarks. *1.* If $k \to \infty$, the problem is in complete agreement with the problem discussed in *case 1*.

2. If $C_3 \to \infty$, the problem reduces to that of *case 3* of previous section.

3 If $k \to \infty$ and $C_2 \to \infty$, the problem reduces to that discussed in *case 1* of previous section.

SAMPLE PROBLEMS

1930. *The demand for a certain item is 16 units per period. Unsatisfied demand causes a shortage cost of Re. 0.75 per unit per short period. The cost of initiating purchasing action is Rs. 15.00 per purchase and the holding cost is 15% of average inventory valuation per period. Item cost is Rs. 8.00 per unit. (Assume that shortages are being back ordered at the above mentioned cost). Find the minimum cost purchase quantity.*

Solution. We are given D = 16 units, C_2 = Re. 0.75 per unit, C_1 = Rs. 8 × 0.15 = Rs. 1.20, C_o = Rs. 15.

$$\therefore \qquad Q^o = \sqrt{\frac{2C_oD}{C_1}}\times\sqrt{\frac{C_1 + C_2}{C_2}} = \sqrt{\frac{2\times15\times16}{1.20}}\sqrt{\frac{1.25 + 0.75}{0.75}} = 32 \text{ units approx.}$$

Minimum inventory cost is

$$TC = \sqrt{2DC_oC_1}\sqrt{\frac{C_2}{C_1 + C_2}}$$

$$= \sqrt{2\times16\times15\times1.20}\sqrt{\frac{0.75}{1.25 + 0.75}}$$

$$= \sqrt{576\times0.375} = \sqrt{216} = \text{Rs. } 15 \text{ approx.}$$

1931. *A dealer supplies you the following information with regard to a product dealt in by him :*

Annual demand : 10,000 units; ordering cost : Rs. 10 per order; price : Rs. 20 per unit.

Inventory carrying cost : 20% of the value of inventory per year.

The dealer is considering the possibility of allowing some back-order (stock-out) to occur. He has estimated that the annual cost of back-ordering will be 25% of the value of inventory.

(i) *What should be the optimum number of units of the product he should buy in one lot?*

(ii) *What quantity of the product should be allowed to be back-ordered, if any?*

(iii) *What would be the maximum quantity of inventory at any time of the year?*

(iv) *Would you recommend to allow back-ordering? If so, what would be the annual cost saving by adopting the policy of back-ordering.* **[Delhi M.Com. 1994]**

Solution. In the usual notations, we are given :

D = 10,000 units, C_o = Rs. 10 per order, C_1 = 20% of Rs. 20 = Rs. 4 per unit per year, and

C_2 = 25% of Rs. 20 = Rs. 5 per unit per year.

(i) (a) When stock-outs are not permitted :

$$Q^o = \sqrt{\frac{2DC_o}{C_1}} = \sqrt{\frac{2 \times 10{,}000 \times 10}{4}} = 223.6 \text{ units}$$

(b) When back-ordering is permitted :

$$Q^o = \sqrt{\frac{2DC_o}{C_1} \times \frac{C_1 + C_2}{C_2}} = \sqrt{\frac{2 \times 10{,}000 \times 10}{4} \times \frac{4+5}{5}} = 300 \text{ units}$$

(ii) Optimum quantity of the product to be back-ordered is given by :

$$Q_2^o = 300 \times \frac{4}{4+5} = 133 \text{ units (approx.)}$$

(iii) Maximum inventory level = 300 − 133 = 167 units.

(iv) Minimum total variable inventory cost in the cases (a) and (b) are :

$$TC\,(223.6) = \sqrt{2 \times 10{,}000 \times 10 \times 4} = \text{Rs. } 894.43$$

$$TC\,(300) = \sqrt{2 \times 10{,}000 \times 10 \times 4 \times \frac{5}{4+5}} = \text{Rs. } 666.67$$

Since, $TC\,(223.6) > TC\,(666.67)$, the dealer should accept the proposal for back-ordering as this will result in a saving of (894.43 − 666.67) = Rs. 227.76 per year.

1932. *A contractor undertakes to supply Diesel engines to a truck manufacturer at the rate of 25 per day. There is a clause in the contract penalizing him Rs. 10 per engine per day late for missing the scheduled delivery date. He finds that the cost of holding a complete engine in stock is Rs. 16 per month. His production process is such that each month he starts a batch of engines through the shops, and all these engines are available for delivery any time after the end of the month. What should his inventory level be at the beginning of each month?* **[Meerut M.Sc. (Math.) 2000]**

Solution. We are given

C_1 = Rs. 16 per engine per month, C_2 = Rs. 10 per engine per day,

r = 25 engines per day, and t = one month (= 30 days)

The optimal value of Q_1 is,

$$Q_1^o = rt\,\frac{C_2}{C_1 + C_2} = 25 \times 30\,\frac{10}{10 + 16/30} = \frac{10 \times 25 \times 30 \times 30}{316} = 712 \text{ engines.}$$

1933. *The demand for an item in a company is 18,000 units per year, and the company can produce the items at a rate of 3,000 per month. The cost of one set-up is Rs. 500.00 and the holding cost of 1 unit per month is 15 paise. The shortage cost of one unit is Rs. 20.00 per month. Determine (i) Optimum production batch quantity and the number of strategies, (ii) Optimum cycle time and production time, (iii) Maximum inventory level in the cycle, and (iv) Total associated cost per year if the cost of the item is Rs. 20 per unit.* **[Bharathidasan B.Com. 1999; Karnataka B.E. (Ind.) 1994]**

Solution. Here,

C_1 = Re. 0.15 per month, C_2 = Rs. 20, C_o = Rs. 500, k = 3,000 units per month,

r = 18,000 units per year or 1,500 units per month.

∴ (*i*) Optimum production batch quantity is given by

$$\therefore \quad Q^o = \sqrt{\frac{2C_o(C_1 + C_2)}{C_1 C_2}} \cdot \sqrt{\frac{kr}{k - r}}$$

$$= \sqrt{\frac{2 \times 500\,(0.15 + 20)}{0.15 \times 20}} \cdot \sqrt{\frac{1{,}500 \times 3{,}000}{3{,}000 - 1{,}500}}$$

$$= 4{,}489 \text{ units approx.}$$

Number of shortages is given by

$$Q_2^o = \frac{C_1}{C_1 + C_2} Q^o \left(1 - \frac{r}{k}\right) = \frac{0.15}{0.15 + 20} \times 4{,}889 \left[1 - \frac{1{,}500}{3{,}000}\right]$$

$$= 18 \text{ units approx.}$$

(*ii*) Optimum production time $= \dfrac{Q^o}{k} = \dfrac{4{,}489}{3{,}000} = 1.5$ months.

Optimum cycle time between set-ups $= \dfrac{Q^o}{r} = \dfrac{4{,}489}{1{,}500} = 3$ months.

(*iii*) Maximum inventory level is

$$Q_1^o = \frac{k - r}{k} Q^o - Q_2^o = \frac{1{,}500}{3{,}000} \times 4{,}489 - 18 = 2{,}227 \text{ units approx.}$$

(*iv*) Total associated cost is Rs. 20 × 2,227 = Rs. 44,540.

PROBLEMS

1934. The demand for a purchased item is 1,000 units/month, and shortages are allowed. If the unit cost is Rs. 1.50 per unit, the cost of making one purchase is Rs. 600, the holding cost for one unit is Rs. 2 per year, and the cost of one shortage is Rs. 10 per year, determine :

(*i*) The optimum purchase quantity. (*ii*) The number of orders per year.

(*iii*) The optimum total yearly cost.

1935. A commodity is to be supplied at a constant rate of 200 units per day. Supplies of any amounts can be had at any required time, but each ordering costs Rs. 50.00, cost of holding the commodity in inventory is Rs. 2.00 per unit per day while the delay in the supply of the item induces a penalty of Rs. 10.00 per unit per delay of 1 day. (*i*) Find the optimal policy (q, t), where t is the re-order cycle period and q is the inventory level after re-order. (*ii*) What would be the best policy, if the penalty cost becomes ∞?

1936. The demand for product is 25 units per month and the items are withdrawn uniformly. The set-up cost each time a production is run is Rs. 15. The inventory holding cost is Re. 0.30 per item per month :

(*i*) Determine how often to make production run, if shortages are not allowed.

(*ii*) Determine how often to make production run, if shortages cost Rs. 1.50 per item per month.

[Meerut M.Sc. (Math.) 1999]

1937. A manufacturer has to supply his customer with 24,000 units of his product every year. This demand is fixed and known. Since the unit is used by the customer in an assembly operation and the customer has no storage space for units the manufacturer must supply a day's requirement each day. If the manufacturer fails to supply the required units, the shortage cost is Rs. 2 per unit per month. The inventory carrying cost is Re. 1 per unit per month, and the set-up cost per run is Rs. 3,500. Determine the optimum run size (Q), the optimum level of inventory (S) at the beginning of any period, the optimum scheduling period, and the minimum total expected relevant yearly cost (TC). ***[Pune M.B.A. 2008]***

1938. A TV manufacturing company produces 2,000 TV sets in a year for which it needs an equal number of tubes of a certain type. Each tube costs Rs. 10 and the cost to hold a tube in stock for a year is Rs. 2.40. Besides, the cost of placing an order is Rs. 150, which is not related to its size. The shortage cost is known to be Rs. 1.60 per unit per annum. Determine EOQ, Optimum level of shortage, maximum inventory level, and total inventory cost.

1939. A dealer supplies you the following information with regard to an item of inventory :

Annual demand	:	5,000 units
Ordering costs	:	Rs. 250 per order
Inventory-holding costs	:	30% of the value of inventory per year
Inventory stock-out costs	:	Rs. 10 per unit per year
Price	:	Rs. 100 per unit

Find out : (*i*) Economic ordering quality. (*ii*) What quantity should be allowed to be back-ordered? (*iii*) What will be the maximum inventory at any particular time of the year? and (*iv*) Cost savings, if any, through back ordering. *[Delhi M.Com. 2000]*

1940. Suppose that Philips India has a product for which the assumptions of the inventory model with planned shortages are valid. The following data is given for the company :

Annual demand (D) = 2,000 units per year, Cost of unit (C) = Rs. 50 per unit

Ordering cost (C_0) = Rs. 25 per order, Holding cost (C_h) = Rs. 10 per unit per year,

Back order cost (C_b) = Rs. 30 per unit per year.

Calculate the following : (*i*) Minimum cost order quantity, (*ii*) maximum number of back order units, (*iii*) maximum inventory level, (*iv*) time between orders, (*v*) total annual costs. *[Jammu M.B.A. 2009]*

1941. Higley Radio Components Company has a product for which the assumptions of the inventory model with backorders are valid. Information obtained by the company is as follows :

Annual demand = 20,000 units, Inventory carrying charge = 20 per cent per annum

Unit cost = Rs. 50, Ordering cost = Rs. 250 per order

Backorder cost = Rs. 30 per unit per year.

Calculate : (*i*) Economic order quantity, (*ii*) Maximum stock level, (*iii*) Maximum number of units backordered, and (*iv*) Total relevant cost per annum.

If the company decides to prohibit backorders and adopts the regular EOQ model, what effect will it have on order quantity, maximum stock level and total relevant cost. *[Delhi M.Com. 2009]*

1942. The demand of an item is uniform at a rate of 20 units per month. The fixed cost is Rs. 10 each time a production run is made. The production cost is Re. 1 per item, and the inventory carrying cost is Re. 0.25 per item per month. If the shortage cost is Rs. 1.25 per item per month, determine how often to make a production run and of what size it should be. *[Madras M.B.A. 1999; Delhi B.Sc. (Stat.) 1995]*

1943. A company has a demand of 12,000 units/year for an item and it can produce 2,000 such items per month. The cost of one set-up is Rs. 400 and the holding cost per unit month is Re. 0.15. The shortage cost of one unit is Rs. 20 per year. Find the optimum lot size and the total cost per year, assuming the cost of 1 unit as Rs. 4. Also find the maximum inventory, manufacturing time and total time. *[Madras B.Sc. (Appl. Sc.) 1999]*

19:12. PROBLEMS OF *EOQ* WITH PRICE BREAKS

In the real world, it is not always true that the unit cost of an item is independent of the quantity procured. Often, *discounts* are offered for the purchase of large quantities. These discounts take the form of *price breaks.*

Let us now consider a manufacturer who is encountered with a problem of determining an optimum production quantity for each production run and an optimal interval between successive runs. The following conditions are assumed to hold : (*i*) Demand is known and uniform, (*ii*) Shortages are not allowed, and (*iii*) Production for supply commodities is instantaneous.

Let Q be the lot size in each production run, D the total number of units produced or supplied over the time period, C_0 the cost per production run, K_i the cost of manufacturing (or purchasing) per unit and I the monthly holding cost per unit.

This model when represented graphically for any one value of the unit purchase cost K_1, has the same shape as in Fig. 19.2. Evidently, we have

$$1 = nt \quad \text{and} \quad D = nQ$$

where n is the total number runs during the entire time period.

The total annual cost for a discount level is given by

$$TC = [n\,Q\,k_1 + n\,C_o] + \tfrac{1}{2}Q\,C_1, \qquad \text{where } C_1 = k_1 \times P$$

$$= \frac{D}{Q}(Q\,k_1 + C_o) + \tfrac{1}{2}Q\,(k_1 \times I)$$

$$= D\,k_1 + \frac{D}{Q}C_o + \tfrac{1}{2}Q\,k_1\,I$$

For the optimum order quantity Q^o that gives the minimum total annual cost $TC(Q)$, we have

$$\frac{d}{dQ}[TC(Q)] = 0 \quad \text{and} \quad \frac{d^2}{dQ^2}[TC(Q)] > 0$$

$$\therefore \qquad Q^o = \sqrt{\frac{2DC_o}{k_1\,I}}$$

Using this value of Q^o in TC, we get

$$TC(Q^o) = D k_1 + \sqrt{2\,DC_o\,k_1\,I}$$

Case 1. Problem of EOQ with One Price Break

When there is only one price break (one quantity discount), the situation may be illustrated as follows :

Range of quantity	*Purchase cost per unit*
$0 \le Q_1 < b_1$	K_{11}
$b_1 \le Q_2 \le b_2$	K_{12}

where b is that quantity at and beyond which the quantity discount applies and $K_{12} < K_{11}$.

The procedure for obtaining EOQ may be summarized in the following steps :

Step 1. Compute Q_2^o, *i.e.*, optimum order quantity for the lowest price (highest discount), *i.e.*, K_{12} and compare it with the quantity b_1.

Step 2. If $Q_2^o > b_1$, then optimum order quantity will be Q_2^o, *i.e.*, $Q^o = Q_2^o$.

Step 3. If $Q_2^o < b_1$, we cannot place order at the reduced price K_{12}. Therefore, in order to obtain the optimum order quantity, we need only to compare the total inventory cost for $Q = Q_1^o$ (for price K_{11}) with $Q = b_1$

The values of $TC(Q_1^o)$ and $TC(b_1)$ may be determined as follows :

$$TC(Q_1^o) = DK_{11} + \frac{D}{Q_1^o}C_o + \tfrac{1}{2}Q_1^o \times K_{11} \times I$$

$$TC(b_1) = DK_{12} + \frac{D}{b}C_o + \tfrac{1}{2}b_1 \times K_{12} \times I$$

If $TC(Q_1^o) > TC(b_1)$, then $Q^o = b_1$, otherwise $Q^o = Q_1^o$.

SAMPLE PROBLEMS

1944. *Find the optimum order quantity for a product for which the price breaks are as follows :*

Quantity	*Unit cost (Rs.)*
$0 \le Q_1 < 500$	*10.00*
$500 \le Q_2$	*9.25*

The monthly demand for the product is 200 units, the cost of storage is 2% of the unit cost and the cost of ordering is Rs. 350. ***[Bharathidasan M.Com. 2007]***

Solution. We are given

C_o = Rs. 350, D = 200 units per month, I = Re. 0.2, K_{11} = Rs. 10 and K_{12} = Rs. 9.25.

Step 1. The highest discount available is Rs. 9.25. So, we compute Q_2^o by taking into consideration K_{12} as Rs. 9.25.

$$\therefore \qquad Q_2^o = \sqrt{\frac{2 \times 350 \times 200}{9.25 \times 0.02}} = 870 \text{ units.}$$

Now, Q_2^o = 870 and b = 500 indicate that $Q_2^o > b$, *i.e.*, Q_2^o is within the range of $Q_2^o \geq 500$. Therefore, the optimum purchase quantity is given by

$$Q^o \equiv Q_2^o = 870 \text{ units.}$$

1945. *Find the optimum order quantity for a product for which the price breaks are as follows :*

Quantity	*Unit cost (Rs.)*
$0 \leq Q_1 < 800$	Re. 1.00
$800 \leq Q_2$	Re. 0.98

The yearly demand for the product is 1,600 units per year, cost of placing an order is Rs. 5, the cost of storage is 10% per year.

Solution. We are given

D = 1,600 units per year, C_o = Rs. 5, I = Re. 0.10, K_{11} = Re. 1.00 and K_{12} = Re. 0.98.

Step 1. The highest discount available is K_{12} (= Re. 0.98). So we compute Q_2^o at K_{12}.

$$\therefore \qquad Q_2^o = \sqrt{\frac{2 \times 5 \times 1{,}600}{(0.10) \times (0.98)}} = 404 \text{ units.}$$

Now, $\qquad Q_2^o$ = 404 and b = 800 indicate that $Q_2^o < b$.

Step 2. Considering now K_{11} = Re. 1.00; the optimum order quantity Q_1^o is obtained as follows :

$$Q_1^o = \sqrt{\frac{2 \times 5 \times 1{,}600}{(0.10) \times (1.00)}} = 400 \text{ units.}$$

Since Q_1^o = 400 and b = 800 indicate that $Q_1^o < b$, we compare the optimum cost of procuring the least quantity which will entitle us a price break (in this case, b = 800). Now,

$$TC\,(Q_1^o) = 1{,}600 \times 1 + \frac{1{,}600}{400} \times 5 + \tfrac{1}{2} \times 400 \times 1 \times 0.10 = \text{Rs. } 1{,}640.$$

$$TC\,(b) = 1{,}600 \times 0.98 + \frac{1{,}600}{800} \times 5 + \tfrac{1}{2} \times 800 \times 0.98 \times 0.10 = \text{Rs. } 1{,}617.20.$$

Since, $TC\,(Q_1^o) > TC\,(b)$; the optimum purchase quantity is given by

$$Q^o = b = 800 \text{ units.}$$

1946. *A company uses 8,000 units of a product as raw material, costing Rs. 10 per unit. The administrative cost per purchase is Rs. 40. The holding costs are 28% of the average inventory. The company is following an optimal purchase policy and places orders according to EOQ. It has been offered a quantity discount of one per cent if it purchases its entire requirement only four times a year.*

Should the company accept the offer of quantity discount of one per cent? If not, what minimum discount should the company demand?

[Delhi M.Com. 1993]

Solution. We are given

D = 8,000 units, C_o = Rs. 40 per order, C_1 = 28% of Rs. 10 = Rs. 2.80 per unit per year

$$\therefore \qquad Q^o = \sqrt{\frac{2 \times 40 \times 8{,}000}{2.80}} = 478 \text{ units (approx.)}$$

Total cost associated with Q^0 is

$$TC = D \times C + \frac{DC_0}{Q^0} + \frac{1}{2} Q^0 \times C_1$$

$$= 8{,}000 \times 10 + \frac{8{,}000}{478} \times 40 + \frac{1}{2} \times 478 \times 0.28 \times 10$$

$$= 80{,}000 + 1{,}338.66 = \text{Rs. } 81{,}338.66.$$

When 1% discount is offered, the unit price will be 99% of Rs. 10, *i.e.*, Rs. 9.90. In this case,

$$TC = 8{,}000 \times 9.90 + \frac{8{,}000}{2{,}000} \times 40 + \frac{2{,}000}{2} \times 0.28 \times 9.90,$$

since the order quantity = 2,000 units, and C_1 = Re. 0.28 × 9.90.

Thus, $TC = 79{,}200 + 2{,}932 = \text{Rs. } 82{,}132.$

Since, the total cost comes out to be higher when one per cent discount is offered, the company should not accept the offer.

Now, let x be the percentage minimum discount acceptable to the company. Then, it can be determined by setting the total cost equal to the total cost associated with the policy of ordering EOQ. Accordingly,

$$8{,}000 \times \left(\frac{100 - x}{100}\right) \times 10 + \frac{8{,}000}{2{,}000} \times 40 + \frac{1}{2} \times 2{,}000 \times 0.28 \times \left(\frac{100 - x}{100}\right) = \text{Rs. } 81{,}338.66.$$

or
$$\left(\frac{100 - x}{100}\right) \times 82{,}800 = 81{,}338.66 - 160 = 81{,}178.66$$

or
$$100 - x = 100 \times \frac{81{,}178.66}{82{,}800} = 98.04$$

This gives $x = 100 - 98.04 = 1.96$ or 2 (approx.)

Hence, the minimum discount acceptable to the company is 2%.

PROBLEMS

1947. The annual demand for an item of inventory whose price is Rs. 10 per unit is 2,400 units, ordering cost per order is Rs. 350 and inventory holding costs are 2% per month. The supplier offers a quantity discount of 7.5% if the quantity ordered is 400 units or more. The rate of discount will increase to 12.5%, if the order is for 3,000 units or more. Find out the economic order quantity. *[Delhi M.Com. 2006]*

1948. A retailer gets discount for large orders. The discount is 4 per cent if the quantity ordered is 500 or more, and an additional 1 per cent discount is received if the quantity ordered is 1,000 or more. The product costs Rs. 50 and sells uniformly at the rate of 25,000 units a year. The fixed order cost is Rs. 50, while the inventory carrying cost is 20% of the value of the average inventory held.

(*i*) What is the best order quantity?

(*ii*) What is the minimum total annual cost, including the purchasing cost?

[Delhi M.Com. 2004]

1949. For one of the boughtout items, the following are the relevant data :

Ordering cost = Rs. 500, Holding cost = 40%, Cost per item = Rs. 100, Annual demand = 1,000.

The purchase manager placed five orders of equal quantity in one year, in order to avail the discount of 5% on the cost of items. Work out the gain or loss to the organization due to his ordering policy for this item. *[ICWA (Dec.) 1990]*

1950. A Purchase Manager has decided to place order for a minimum quantity of 500 numbers of a particular item in order to get a discount of 10%. From the past records, it was found out that in the last year, 8 orders each of size 200 units were placed. Given the ordering cost = Rs. 500 per order, inventory carrying cost of 40% of the inventory value and the price of the item of Rs. 400 per unit. Is the Purchase Manager justified in his decision? What is the effect of his decision to the company? *[Delhi M.Com. 2006]*

1951. The annual demand of a product is 1,000 units. Each unit costs Rs. 20 if orders placed in quantities of 250 and inventory carrying cost is 40%. The following three strategies are available for the procurement :

(*i*) Place four orders of equal size every year.

(*ii*) Place the order for 500 units at a time and avail a discount of 10% on the cost of items.

(*iii*) Follow *EOQ* policy. ***[Delhi PG Dip. in Glob. Bus. Oper. 2010]***

1952. A soft drinks manufacturing company buys a large number of pallets every year which it uses in the warehousing of its bottled products. A local vendor has offered the following discount schedule for pallets :

Order Quantity	*Unit Price (Rs.)*
Up to 699	10.00
700 and above	9.25

The average yearly replacement is 2,400 pallets. The carrying cost is 12% of the average inventory and ordering cost per order is Rs. 100. Find out the optimum order quantity.

Case 2. Problem of EOQ with more than One Price Break

When there are n price breaks, the situation may be illustrated as follows :

Range of quantity	*Purchase cost per unit*
$0 \le Q_1 < b_1$	K_{11}
$b_1 \le Q_2 < b_2$	K_{12}
$\vdots$	$\vdots$
$b_{n-1} \le Q_n$	K_{1n}

where $b_1, b_2, \ldots, b_{n-1}$ are those quantities which determine the price breaks.

The computational procedure for obtaining the optimum order quantity in this case will be as follows :

Step 1. Compute Q_n°. If $Q_n^{\circ} \ge b_{n-1}$, the optimum order quantity is reached, *i.e.*, Q_n°.

Step 2. If $Q_n^{\circ} < b_{n-1}$, compute Q_{n-1}°. If $Q_{n-1}^{\circ} \ge b_{n-2}$, proceed as in the case of one price break; that is, the optimum order quantity is determined by comparing $TC(Q_{n-1}^{\circ})$ with $TC(b_{n-1})$.

Step 3. If $Q_n^{\circ} < b_{n-2}$, compute Q_{n-2}. If $Q_{n-2}^{\circ} \ge b_{n-3}$, proceed as in the case of two price breaks; that is, the optimum order quantity is determined by comparing $TC(Q_{n-2}^{\circ})$ with $TC(b_{n-2})$ and $TC(b_{n-1})$.

Step 4. If $Q_{n-2}^{\circ} < b_{n-2}$, compute Q_{n-3}°. If $Q_{n-3}^{\circ} \ge b_{n-4}$, then compare $TC(Q_{n-3}^{\circ})$ with $TC(b_{n-3})$, $TC(b_{n-2})$ and $TC(b_{n-1})$.

Step 5. Compute in this way, until $Q_{n-j}^{\circ} \ge b_{n-(j+1)}$; $0 \le j \le n-1$; and then compare $TC(Q_{n-j}^{\circ})$ with $TC(b_{n-j-2}), \ldots, TC(b_{n-1})$.

SAMPLE PROBLEMS

1953. *Find the optimal order quantity for a product for which the price breaks are as follows :*

Quantity	*Unit cost (Rs.)*
$0 \le Q_1 < 500$	10.00
$500 \le Q_2 < 750$	9.25
$750 \le Q_3$	8.75

The monthly demand for the product is 200 units, the cost of storage is 2% of the unit cost and the cost of ordering is Rs. 350. ***[Bharthidasan M.Com. 2007]***

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Solution. We are given :

C_o = Rs. 350, I = Re. 0.02, D = 200 units per month

$$\therefore \qquad Q_3^o = \sqrt{\frac{2 \times 350 \times 200}{(8.75)(0.02)}} = 894 \text{ units.}$$

This shows that $Q_3^o > b_2$, since $b_2 = 750$.

Hence, the optimum order quantity is given by

$$Q^o \equiv Q_3^o = 894 \text{ units.}$$

1954. *Find the optimum order quantity for a product for which the price breaks are as follows :*

Quantity	*Purchasing cost per unit (Rs.)*
$0 \le Q_1 < 100$	*20*
$100 \le Q_2 < 200$	*18*
$200 \le Q_3$	*16*

The monthly demand for the product is 400 units. The storage cost is 20% of the unit cost of the product and the cost of ordering is Rs. 25.00 per month. **[Calicut M.Com. 2009]**

Solution. Here D = 400 units, I = Re. 0.20 and C_o = Rs. 25.

$$\therefore \qquad Q_3^o = \sqrt{\frac{2 \times 25 \times 400}{16 \times 0.20}} = 79 \text{ units.}$$

Since $Q_3^o < b_2$ (= 200), we next compute Q_2^o, we have

$$Q_2^o = \sqrt{\frac{2 \times 25 \times 400}{18 \times 0.20}} = 75 \text{ units.}$$

Again, since $Q_2^o < b_1$ (= 100), we next compute Q_1^o. We have

$$Q_1^o = \sqrt{\frac{2 \times 25 \times 400}{20 \times 0.20}} = 70 \text{ units (approx.)}$$

Now $$TC(Q_1^o) = 25 \times \frac{400}{70} + 400 \times 20 + 20 \times (0.20) \times \frac{70}{2} = \text{Rs. } 8{,}283.00.$$

$$TC(b_1) = 25 \times \frac{400}{100} + 400 \times 18 + 18 \times (0.20) \times \frac{100}{2} = \text{Rs. } 7{,}480.00.$$

and $$TC(b_2) = 25 \times \frac{400}{200} + 400 \times 16 + 16 \times (0.20) \times \frac{200}{2} = \text{Rs. } 6{,}770.00.$$

Since $TC(b_2) < TC(b_1) < TC(Q_1^o)$, the optimum order quantity is $Q^o \equiv b_1 = 200$ units.

1955. *A manufacturing concern requires 2,000 units of a material per year. The ordering costs are Rs. 10 per order. While carrying costs are Re. 0.16 per year per unit of average inventory, the purchase price is Re. 1 per unit. Find the economic order quantity, and the total inventory cost. If a discount of 5 per cent is available for orders of 1,000 units or more but less than 2,000 units, should the manufacturer accept this offer? Also, if he purchases a single lot of 2,000 units or more he has to pay Re. 0.93 per unit. What purchase quantity would you recommend?* **[Annamalai M.B.A. (Nov.) 2002]**

Solution. We are given

D = 2,000 units per year, C_o = Rs. 10 per order, C_1 = Re. 1 × 0.16 = Re. 0.16.

When no discount is offered, the optimum order quantity is given by

$$Q^o = \sqrt{\frac{2 \times 2{,}000 \times 10}{0.16}} = 500 \text{ units.}$$

Total inventory cost is given by

$$TC = 2{,}000 \times 1 + \frac{2{,}000}{500} \times 10 + \frac{1}{2} \times 500 \times 0.16 = \text{Rs. } 2{,}440.$$

When quantity discounts are offered, the following information is available :

Quantity	*Price per unit (Re.)*
$0 < Q_1 < 1{,}000$	1.00
$1{,}000 \le Q_2 < 2{,}000$	0.95
$2{,}000 \le Q_3$	0.93

The optimum order quantity, Q_3^o, based on Re. 0.93 per unit is given by

$$Q_3^o = \sqrt{\frac{2 \times 2{,}000 \times 10}{0.93 \times 0.16}} = 518 \text{ units (approx.)}$$

Since $Q_3^o < b_2$ (= 2,000), we next compute Q_2^o. We have

$$Q_2^o = \sqrt{\frac{2 \times 2{,}000 \times 10}{0.95 \times 0.16}} = 513 \text{ units (approx.)}$$

Again, since $Q_2^o < b_1$ (= 1,000), we next compute Q_1^o.

$$Q_1^o = \sqrt{\frac{2 \times 2{,}000 \times 10}{1.00 \times 0.16}} = 500 \text{ units.}$$

Now,

$$TC(Q_1^o) = 2{,}000 \times 10 + \frac{2{,}000}{500} \times 10 + \tfrac{1}{2} \times 500 \times 0.16 = \text{Rs. } 2{,}440$$

$$TC(b_1) = 2{,}000 \times 0.95 + \frac{2{,}000}{1{,}000} \times 10 + \tfrac{1}{2} \times 1{,}000 \times 0.95 \times 0.16 = \text{Rs. } 1{,}996.00$$

$$TC(b_2) = 2{,}000 \times 0.93 + \frac{2{,}000}{2{,}000} \times 10 + \tfrac{1}{2} \times 2{,}000 \times 0.93 \times 0.16 = \text{Rs. } 2{,}018.80$$

Since, $TC(b_1) < TC(b_2) < TC(Q_1^o)$, the optimum order quantity is $Q^o = b_1 = 500$ units.

Hence, the manufacturer should accept the offer of 5 per cent discount only, because in this case his net saving per year would be Rs. (2,440 – 1,996), *i.e.* Rs. 444.

PROBLEMS

1956. Find the optimal order quantity for a product for which the price breaks are as follows :

Quantity	*Unit cost (Rs.)*
$0 \le Q_1 < 50$	10.00
$50 \le Q_2 < 100$	9.00
$100 \le Q_3$	8.00

The monthly demand for the product is 200 units, the cost of storage is 25% of the unit cost and ordering cost is Rs. 20.00 per order. ***[Meerut M.Sc. (Math.) 2003]***

1957. A shopkeeper has a uniform demand of an item @50 items per month. He buys from a supplier at a cost of Rs. 6 per item and the cost of ordering is Rs. 10 each time. If the stock holding costs are 20% per year of stock value, how frequently should he replenish his stock? Suppose the supplier offers a 5% discount on orders between 200 and 999 items and a 10% discount on orders exceeding or equal to 1,000 can the shopkeeper reduce his cost by taking advantage of either of these discounts? ***[Nagpur M.B.A. (Nov.) 2008; Delhi MIB 2011]***

1958. The annual demand for a product is 64,000 units (or 1,280 units per week). The buying cost per order is Rs. 10 and the estimated cost of carrying one unit in stock for a year is 20%. The normal price of the product is Rs. 10 per unit. However, the supplier offers a quantity discount of 2% on an order of at least 1,000 units at a time, and a discount of 5% if the order is for at least 5,000 units.

Suggest the most economic purchase quantity per order. ***[Pune M.B.A. 2007]***

1959. A manufacturer of engines is required to purchase 4,800 castings per year. The requirement is assumed to be known as fixed. These castings are subject to quantity discounts. The price schedule is as follows :

Quantity	*Cost per unit*
Less than 500 units	Rs. 150.00
500 or more but less than 750 units	Rs. 138.75
750 or more units	Rs. 131.25