Network Scheduling by *PERT/CPM*

"Every modern corporation has channels of communication along which lines, from foot to crown, reports flow up and vetoes down"

25:1. INTRODUCTION

Network Scheduling is a technique used for planning and scheduling large projects in the fields of construction, maintenance, fabrication, purchasing, computer system installation, research and development designs, etc. The technique is a method of minimizing trouble spots, such as, production bottlenecks, delays and interruptions, by determining critical factors and coordinating various parts of overall job.

There are two basic planning and control techniques that utilize a network to complete a pre-determined project or schedule. These are : *Program Evaluation and Review Technique (PERT)*; and the *Critical Path Method (CPM)*. Several variations of these have also been developed, one such important variation being the *Review Analysis of Multiple Projects (RAMP)* which is useful for guiding the 'activities' of several projects at one time.

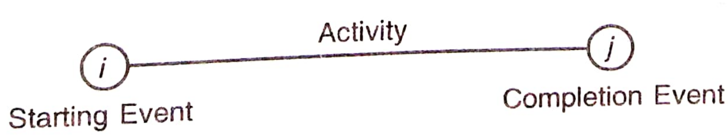
25:2. NETWORK : BASIC COMPONENTS

A *network* is a graphic representation of a project's operations and is composed of activities and events that must be completed to reach the end objective of a project, showing the planning sequence of their accomplishments, their dependence and inter-relationships. The basic components of a network are :

Activity. An *activity* is a task, or item of work to be done, that consumes time, effort, money or other resources. It lies between two events, called the 'preceding' and 'succeeding' ones. An activity is represented by an arrow with its head indicating the sequence in which the events are to occur.

Event. An *event* represent the start (beginning) or completion (end) of some activity and as such it consumes no time. It has no time duration and does not consume any resources. An event is nothing but a *node* and is generally represented on the network by a circle, rectangle, hexagon or some other geometric shape. An event is not complete until all the activities flowing into it are completed.

Activities are identified by the numbers of their starting (tail or initial) event and ending (head, or terminal) event. An arrow (i, j) extended between two events; the tail event i represents the start of the activity and the head event j, represents the completion of the activity as shown below :



An event representing the joint completion of more than one activity is called a *merge event*. If an event represents the joint initiation of more than one activity, it is called a *burst event*.

The activities can be further classified into the following three categories :

1. *Predecessor activity.* An activity which must be completed before one or more other activities start is known as predecessor activity.

2. *Successor activity.* An activity which started immediately after one or more of other activities are completed is known as successor activity.

3. *Dummy activity.* An activity which does not consume either any resource and time is known as dummy activity. A dummy activity is depicted by dotted line in the network diagram.

Remark. A dummy activity in the network is added only to represent the given precedence relationships among activities of the project and is needed when (*a*) two or more parallel activities in a project have same head and tail events, or (*b*) two or more activities have some (but not all) of their immediate predecessor activities in common.

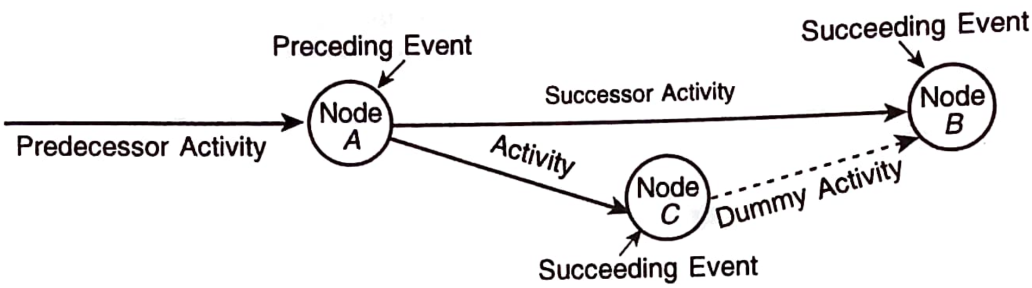


Fig. 25.1

A dummy activity is depicted by a dotted line in the network diagram as shown above.

25:3. LOGICAL SEQUENCING

All the projects consist of certain activities that can begin only after certain others are completed. In fact, the entire project may be considered as a series of activities which may begin only after another activity or activities are completed. In a network schedule, these types of relationships are called *constraints* and are represented by inequalities. For example, $A < B$ will indicate that the activity A must be completed before the start of the activity B.

For example, a project of laying a pipeline consists of the the activities : trenching, laying pipe, and welding of pipe.

In logical sequencing, following three types of errors are most common while drawing a network diagram :

1. *Looping.* If an activity were represented as going back in time, a *closed* loop would occur as shown in Fig. 25.2 :

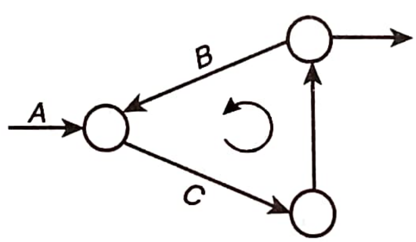


Fig. 25.2. Closed loop

A closed loop would produce an endless cycle in computer programmes without a built-in routine for detection or identification of the cycle. This situation can be avoided by checking the precedence relationship of the activities and by numbering them in a logical order. Thus one property of a correctly constructed network diagram is that it is "non-cyclic".

2. *Dangling*. No activity should end without being joined to the end event. If it is not so, a dummy activity is introduced in order to maintain the continuity of the system. Such end-events other than the end of the project as a whole are called *dangling* events.

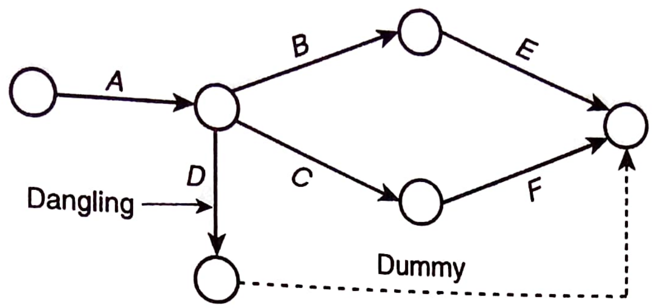


Fig. 25.3. Dangling

In the above network, activity D leads to dangling. A dummy activity is therefore introduced to avoid this dangling.

3. *Dependency relationships.* When two chains of activities have a common event, wholly or partly independent of each other, a dummy activity is used to establish proper logical relationships. Given below are the two cases of dependency relationships :

(*i*) Activity C is dependent upon both A and B.

(*ii*) Activity D is dependent upon A alone.

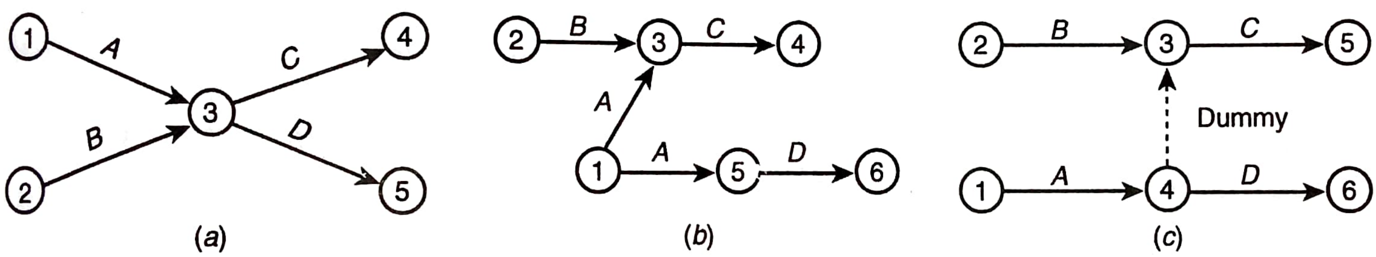


Fig. 25.4

From the above three figures, we observe that Fig. 25.4(*a*) is clearly wrong since it shows D as dependent upon not only A but also B which is not desired. Fig. 25.4(*b*) is also wrong since A has been shown twice and thus contravenes the fundamental axioms of network that there must be one arrow for each activity. The way out to this dilema is the representation by means of a dummy activity. Fig. 25.4(*c*) shows that C is dependent upon both A and B (*via* dummy) whereas D is dependent upon just A.

25:4. RULES OF NETWORK CONSTRUCTION

For the construction of a network, generally, the following rules are followed :

1. Each activity is represented by one and only one arrow.
2. Each activity must be identified by its *starting* and *end* node which implies that

(*i*) two activities should not be identified by the same completion events, and

(*ii*) activities must be represented either by their symbols or by the corresponding ordered pair of starting-completion events.

3. Nodes are numbered to identify an activity *uniquely*. Tail node (starting point) should be lower than the head node (end point) of an activity.
4. Between any pair of nodes, there should be one and only one activity; however more than one activity may emanate from and terminate to a node.

5. Arrows should be kept straight and not curved or bent.
6. The logical sequence (or inter-relationship) between activities must follow the following rules :
   (*i*) An event cannot occur until all the incoming activities into it have been completed.
   (*ii*) An activity cannot start unless all the preceding activities on which it depends, have been completed.
   (*iii*) Dummy activities should only be introduced if absolutely necessary.

Numbering the Events

After the network is drawn in a logical sequence, every event is assigned a number. The number sequence must be such so as to reflect the flow of the network. In event numbering, the following rules should be observed :

(*a*) Event numbers should be unique.

(*b*) Event numbering should be carried out on a sequential basis from left to right.

(*c*) The initial event which has all outgoing arrows with no incoming arrow is numbered 0 or 1.

(*d*) The head of an arrow should always bear a number higher than the one assigned at the tail of the arrow.

(*e*) Gaps should be left in the sequence of event numbering to accommodate subsequent inclusion of activities, if necessary.

Remark. The above procedure of assigning numbers to various events of a network is known as *Fulkerson's Rule.*

Illustration. *A television is manufactured in six steps, labelled A through F. Because of its size and complexity, the television is produced one at a time. The production control manager thinks that network scheduling techniques might be useful in planning future production. He recorded the following information :*

A is the first step and precedes B and C,
C precedes D and E,
F follows E and D is successor of F,
B follows D and precedes E.

(a) Draw an activity-on-node diagram for the production manager.

(b) On checking with the records, the production manager corrects his last note to read, "D is a predecessor of F". Draw a new diagram for the revised network incorporating this new change.

(c) After pondering over the network and rechecking from the records, it was found that B was really a predecessor of D rather than vice versa. Draw a revised network incorporating this new change.

(d) Draw an arrow-diagram representation of the network in part (c). How many dummy activities did you use in your network?

Solution. (*a*) (*i*) *A* is the first step which precedes *B* and *C*.

(*ii*) *C* precedes *D* and *E*.

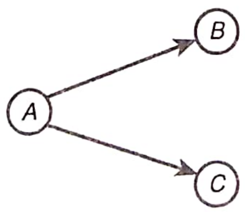


Fig. 25.5

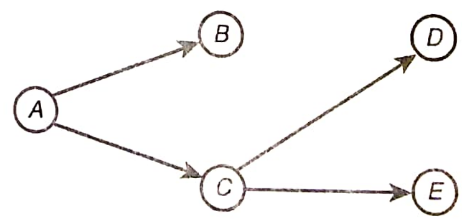


Fig. 25.6

(*iii*) F follows E and D and is the successor of F :

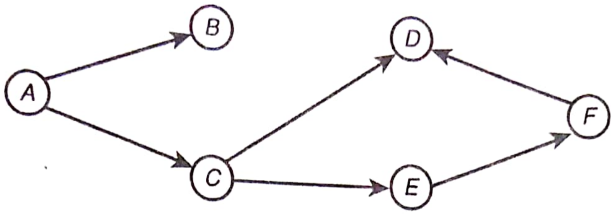


Fig. 25.7

(*iv*) Now, since B follows D and precedes E, the complete network drawing is shown in Fig. 25.8 below :

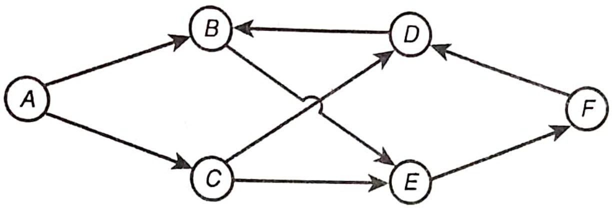


Fig. 25.8

Evidently, this network contains a cycle as shown in Fig. 25.9 below :

Fig. 25.9

(*b*) Revised network when D is a predecessor of F is as follows :

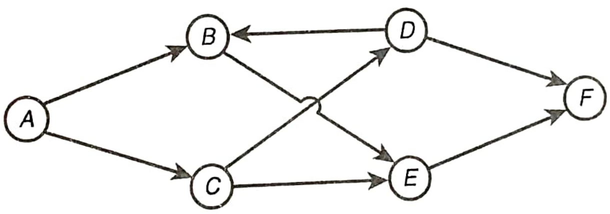


Fig. 25.10

(*c*) Revised network when D is a predecessor of F and B is a predecessor of D is given below :

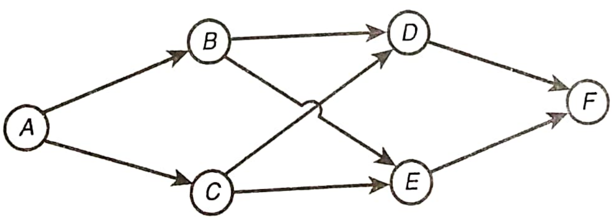


Fig. 25.11

(*d*) The arrow diagram of the network presented in part (*c*) can be represented as :

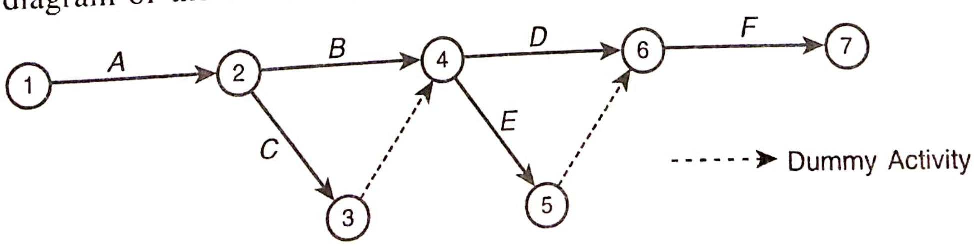


Fig. 25.12

The events of the diagram are numbered such that their ascending order indicates the direction of progress in the project.

25:5. CONCURRENT ACTIVITIES

Activities discussed so far are discrete in nature. But in practice, they may not always be discrete, *i.e.*, they may be done in part allowing the subsequent activities to commence before the preceding activity is fully completed. Activities of this kind are to be frequently encountered in batch production. If, for example, a batch of 50 spindles is to be processed on two machines, obviously it is not necessary to process all the items of the batch on the first machine and then transfer these to the next machine. A few items processed on the first machine may be transferred to the second machine before completion of the entire batch on the first machine. Such *simultaneous* or *concurrent activities* are to be encountered in sewage work; *e.g.*, trenching, laying pipe, welding pipe, and back filling; all going on simultaneously with suitable lags on construction work.

Illustration. *A batch of four axles is to be processed on the following three machines in this sequence : Lathe (L), Milling (M), Grinding (G).*

Instead of first working these four axles on Lathe, then on Milling and finally on Grinding in sequence, it is desired to process the first axle on the Lathe and as and when it is processed, it is taken up on Milling and the second axle on the Lathe, and so on. In other words, each of the three activities L, M and G have been quartered for the sake of concurrent operations.

You are required to draw the network.

Solution. The dependency relationships are sorted out hereunder :

Quartered activity	*Preceded by*	*Quartered activity*	*Preceded by*
L_1	*None*	M_3	L_3, M_2
L_2	L_1	M_4	L_4, M_3
L_3	L_2	G_1	M_1
L_4	L_3	G_2	M_2, G_1
M_1	L_1	G_3	M_3, G_2
M_2	L_2, M_1	G_4	M_4, G_3

Using the above relationships, the resulting network is shown in Fig. 25.13 :

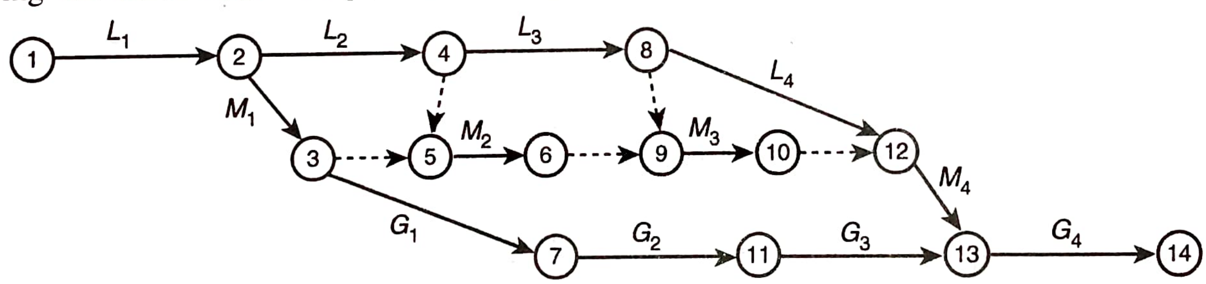


Fig. 25.13

Note. The concurrent activities so drawn are known as ladders in the network jargon.

SAMPLE PROBLEMS

2501. *Draw a network diagram for the following data :*

Activity :	A	B	C	D	E	F	G	H	I	J
Preceding activities :	None	A	A	B	A	B, E	C	D, F	G	H, I

Solution. Using the precedence relationships, the required network is given below :

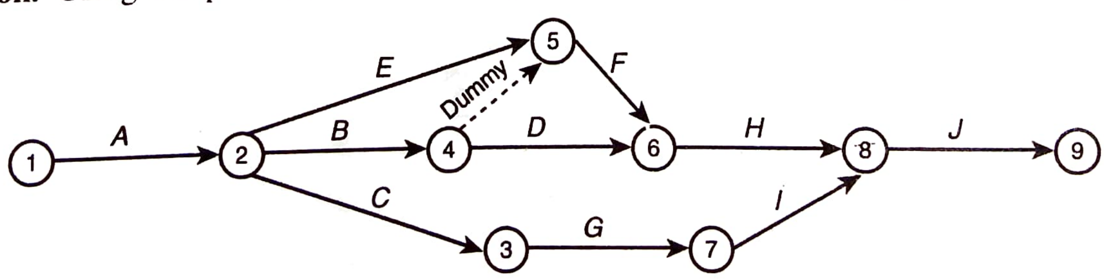


Fig. 25.14

It may be noted that a dummy activity is used to identify the activities B and F with unique end nodes. The nodes of the project are numbered such that their ascending order indicates the direction of progress in the project.

2502. *In a boiler overhauling project following activities are to be performed :*

A. Inspection of boiler by boiler engineer and preparation of list of parts to be replaced/ repaired.
B. Collecting quotations for the parts to be purchased.
C. Placing the orders and purchasing.
D. Dismantling of the defective parts from the boiler.
E. Preparation of necessary instructions for repairs.
F. Repair of parts in the workshop.
G. Cleaning of the various mountings and fittings.
H. Installation of the repaired parts.
I. Installation of the purchased parts.
J. Inspection.
K. Trial run.

Assuming that the work is assigned to the boiler engineer who has one boiler mechanic and one boiler attendant at his disposal, draw a network showing the precedence relationships.

Solution. Looking at the list of activities, we note that activity A (inspection of boiler) is to be followed by dismantling of defective parts (D) and only after that it can be decided which parts can be repaired and which will have to be replaced. Now the repairing and purchasing can go side by side. But the instructions for repairs may be prepared after sending the letters for quotations. Note that it becomes a partial constraint, also started after activity D. Now we assume that repairing will take less time than purchasing. But the installation of repaired parts can be started only when the cleaning is completed. This results in the use of a dummy activity. After the installation of repaired parts, installation of purchased parts can be taken up. This will be followed by inspection and trial run.

The network showing the precedence relationships is given below :

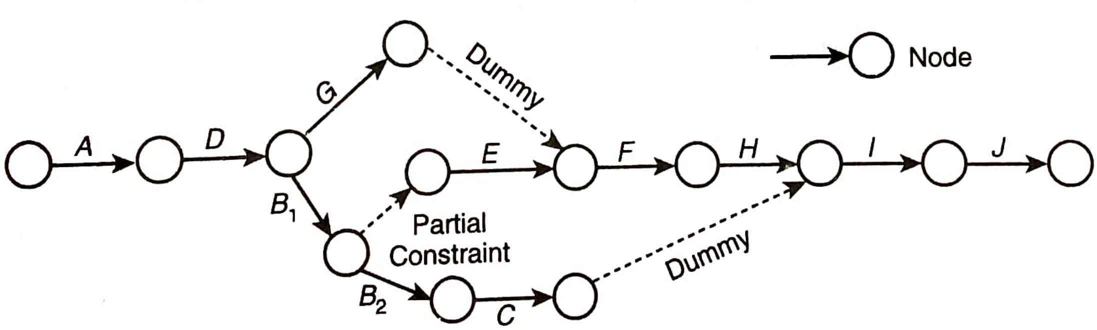


Fig. 25.15

The dummy activities are used to identify the activities C, H and E, G with unique end nodes.

2503. *Construct the network diagram comprising activities B, C, ..., Q and N such that the following constraints are satisfied :*

$B < E, F;\ C < G, L;\ E, G < H;\ L, H < I;\ L < M;\ H < N;\ H < J;\ I, J < P;\ P < Q.$

The notation $X < Y$ means that the activity X must be finished before Y can begin.

[Jammu Univ. B.E. (Mech.) 2004]

Solution. The resulting network is shown in Fig. 25.16. The dummy activities D_1, D_2 and D_3 are used to establish the correct precedence relationships. D_4 is used to identify the activities I and J with unique end nodes. The nodes of the project are numbered such that their ascending order indicates the direction of progress in the project :

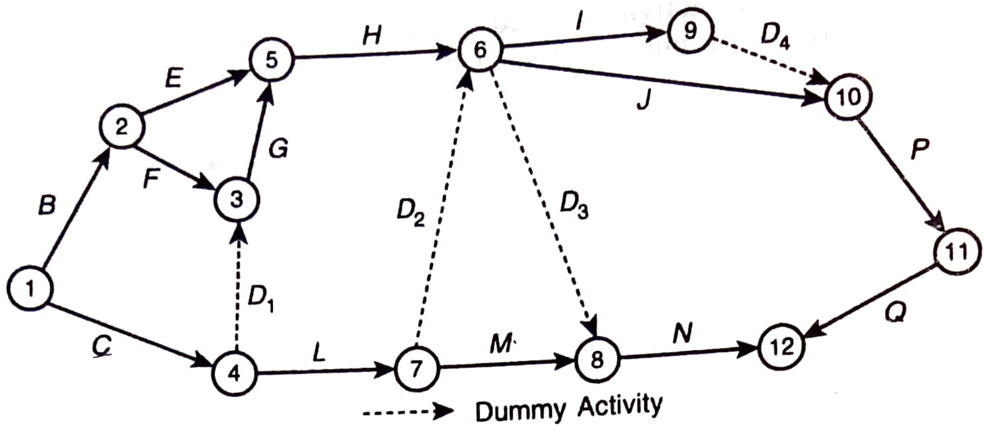


Fig. 25.16

PROBLEMS

2504. Construct the arrow diagram comprising activities *A*, *B*, ... and *L* such that the following relationships are satisfied :

(*i*) *A*, *B* and *C*, the first activities of the project, can start simultaneously,
(*ii*) *A* and *B* precede *D*,
(*iii*) *B* precedes *E*, *F* and *H*,
(*iv*) *F* and *C* precede *G*,
(*v*) *E* and *H* precede *I* and *J*,
(*vi*) *C*, *D*, *F* and *J* precede *K*,
(*vii*) *K* precedes *L*,
(*viii*) *I*, *G* and *L* are the terminal activities of the project.

[Bharathidasan B.Sc. (Allied) 1995]

2505. Draw a network diagram for the following data :

Task :	A	B	C	D	E	F	G
Immediate predecessor :	None	None	B	B	B	E	A, D, C

2506. Draw an arrow diagram showing the following relationships :

Activity	*Immediate predecessor*	*Activity*	*Immediate predecessor*
A	None	*H*	*D, E, F*
B	None	*I*	*D*
C	None	*J*	*G*
D	*A*	*K*	*G*
E	*B, C*	*L*	*H, J*
F	*A*	*M*	*K*
G	*C*	*N*	*I, L*

2507. Following are the activities which are to be performed for a building site preparation. Determine the precedence relationship and draw the network :

1. Clear the site.
2. Survey and layout.
3. Rough grade.
4. Excavate for sewer.
5. Excavate for electrical manholes.
6. Install sewer and baskfill.
7. Install electrical manholes.
8. Construct the boundary wall.

[Calicut M.Com. 1995]

2508. Listed in the table are the activities and sequencing necessary for a maintenance job on the heat exchangers in a refinery :

Activity	*Description*	*Predecessor activity*
A	Dismantle pipe connections	—
B	Dismantle header, closure, and floating front	*A*
C	Remove tube bundle	*B*
D	Clean bolts	*B*
E	Clean header and floating head front	*B*
F	Clean tube bundle	*C*
G	Clean shell	*C*
H	Replace tube bundle	*F, G*
I	Prepare shell pressure test	*D, E, H*
J	Prepare tube pressure test and reassemble	*I*

Draw a network diagram for the project. ***[Delhi M.B.A. (PT.) 2008]***

2509. *XYZ* company has decided to redesign its electronic components. The project involves several activities which are listed in the following table. *First, the electronic engineering staff must finish the design of the component. Second*, the marketing programme for promoting the component must be developed. *Third*, a new manufacturing process must be designed. *Fourth*, advertising media must be selected. *Fifth*, an initial production run must be successfully completed. And *sixth*, the component must be released to the market.

Activity	*Description of activity*	*Predecessor activity*
A	Finish component development	—
B	Design marketing programme	*A*
C	Design production system	*A*
D	Select advertising media	*B*
E	Initial production run	*C*
F	Release component to market	*D, E*

[Gurunanak Dev B.B.A. 2008]

2510. A publisher has a contract with an author to publish a textbook. The (simplified) activities associated with the production of the textbook are given subsequently. Develop the associated network for the project :

Activity	*Activity description*	*Predecessor activity*	*Duration (weeks)*
A	Manuscript reading by the editor	—	5
B	Sample pages prepared by the typesetter	—	3
C	Book cover design	—	5
D	Preparation of diagrams used in the book	—	4
E	Author's approval of sample pages	*A, B*	3
F	Book typesetting	*E*	5
G	Author's proof reading of typeset pages	*F*	3
H	Author checks art-work	*D*	2
I	Production of printing plates	*G, H*	3
J	Book production and binding	*C, I*	7

25:6. CRITICAL PATH ANALYSIS

The purpose of analysis is to find the critical path, *i.e.*, the sequence of activities with the longest duration, and to find the float associated with each non-critical activity. This helps in checking actual progress against the scheduled duration of the project. To achieve this objective, we carry out the special computations that produce the following information :

(*a*) Total duration needed for the completion of the project.

(*b*) Categorization of the activities of the project as being *critical* or *non-critical*.

An activity in a network diagram is said to be *critical*, if the delay in its start will further delay the project completion time. A *non-critical* activity allows some scheduling slack, so that the start time of the activity may be advanced or delayed within limits without affecting the completion date of the entire project.

To accomplish the above-mentioned objectives, the following factors should be known to prepare project scheduling :

(*i*) Time schedule for each activity, *i.e.*, the time by which an activity must begin and the time before which it is completed.

(*ii*) Earlier and latest start time as well as earlier finish and latest finish of each activity.

(*iii*) Float for each activity, *i.e.*, the spare time associated with each activity.

(*iv*) Critical activities and critical path for the network.

For the purpose of calculating various times of events and activities, the following terms shall be used in critical path calculations :

E_i = Earliest occurrence time of event i

L_j = Latest occurrence time of event j

t_{ij} = Duration of activity (i, j)

The critical path calculations are done in the following two ways :

(*a*) Forward Pass Calculations, and (*b*) Backward Pass Calculations.

Forward Pass Calculations. We start from the initial node *1* (event 1) with starting time of the project as zero. Proceed through the network visiting nodes in an increasing order of node number and end at the final node of the network. At each node, we calculate earliest start and finish times for each activity by considering F_i as the earliest occurrence of node i. The method may be summarized as follows :

Step 1. Set $E_1 = 0$; $i = 1$ (initial node)

Step 2. Set the earliest start time for each activity that begins at node i as

$$ES_{ij} = E_i \text{; for all activities } (i, j) \text{ that start at node } i.$$

Step 3. Compute the earliest finish time of each activity that begins at node i by adding the earliest start time of the activity to the duration of the activity. Thus

$$EF_{ij} = ES_{ij} + t_{ij} = E_i + t_{ij}$$

Step 4. Move on to next node, say node j $(j > i)$ and compute the earliest occurrence for node j, using

$$E_j = \max_i \{EF_{ij}\} = \max_i \{E_i + t_{ij}\},$$

for all immediate predecessor activities.

Step 5. If $j = n$ (final node), then the earliest finish time for the project is given by

$$E_n = \max \{EF_{ij}\} = \max \{E_{n-1} + t_{ij}\}.$$

Backward Pass Calculations. We start from the final (last) node (event) n of the network, proceed through the network visiting nodes in the decreasing order of node numbers and end at the initial node *1*. At each node, we calculate the least finish and start times for each activity by considering L_j as the latest occurrence of node j. The method may be summarized below :

Step 1. $L_n = E_n$; for $j = n$.

Step 2. Set the latest finish time of each activity that ends at node j as

$$LF_{ij} = L_j.$$

Step 3. Compute the latest occurrence times of all activities ending at j by subtracting the duration of each activity from the latest finish time of the activity. Thus

$$LS_{ij} = LF_{ij} - t_{ij} = L_j - t_{ij}.$$

Step 4. Proceed backward to the node in the sequence, that decrease j by 1. Also compute the latest occurrence time of node i ($i < j$) using

$$L_i = \min_j \{LS_{ij}\} = \min_j \{L_j - t_{ij}\},$$

for all immediate successor activities.

Step 5. If $j = 1$ (initial node), then

$$L_1 = \min\{LS_{ij}\} = \min\{L_2 - t_{ij}\}.$$

Based on the above calculations, an activity (i, j) will be critical if it satisfies the following conditions :

(*i*) $E_i = L_i$ and $E_j = L_j$

(*ii*) $E_j - E_i = L_j - L_i = t_{ij}$.

An activity that does not satisfy the above conditions is termed as *non-critical.*

Critical Path. The critical activities of a network that constitute an uninterrupted path which spans the entire network from start to finish is known as *critical path.*

Float (or Slack) of an Activity and Event

The float of an activity is the amount of time by which it is possible to delay its completion time without affecting the total project completion time.

1. **Event float.** The *float* (also called 'slack') of an event is the difference between its latest time (L_i) and its earliest time (E_i). That is

$$\text{Event float} = L_i - E_i$$

It is a measure of how much later than expected a particular event could occur without delaying the completion of the entire project.

2. **Activity float.** As mentioned earlier, it is the float (or slack) in the activity time estimates. There are mainly three types of activity floats as discussed below :

(*i*) ***Total float.*** The *total float* of an activity represents the amount of time by which an activity can be delayed without delay in the project completion date. In other words, it refers to the amount of the free time associated with an activity which can be used before, during or after the performance of this activity. Total float is the positive difference between the earliest finish time and the latest finish time, or the positive difference between the earliest start time and the latest start time of an activity depending upon which way it is defined.

Thus, for each activity (i, j), the total float values are computed as follows :

$$\text{Total float } (TF_{ij}) = L_j - (E_i + t_{ij}) = LF_{ij} - EF_{ij}$$
$$= (L_j - t_{ij}) - E_i = LS_{ij} - ES_{ij}.$$

(*ii*) ***Free float.*** *Free float is that portion of the total float within which an activity can be manipulated without affecting the float of subsequent activities.* It is computed for an activity by subtracting the head event slack from its total float. The free float indicates the value by which an activity in question can be delayed without causing any delay in its immediate successor activities.

Free float values for each activity (i, j) are computed as follows :

$$\text{Free float } (FF_{ij}) = (E_j - E_i) - t_{ij}$$
$$= E_j - (E_1 + t_{ij}) = \text{min. } \{ES_{ij}\} - EF_{ij}, \quad (i < j).$$

(*iii*) ***Independent float.*** It is that portion of the total float within which an activity can be delayed for start without affecting floats of the preceding activities. It is computed by subtracting the tail event slack from the free float of the activity. If the result is negative, it is taken as zero.

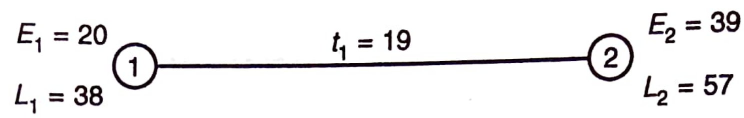
Independent float values for each activity (i, j) are computed as follows :

$$\text{Independent float } (IF_{ij}) = (E_j - L_i) - t_{ij}$$

or

$$IF_{ij} = FF_{ij} - (\text{slack of event } i)$$

Illustration. Consider the following activity of certain network :



Here

Total Float $= L_2 - (E_1 + t_{12}) = 57 - (20 + 19) = 18$

Free Float $= E_2 - (E_1 + t_{12}) = 39 - (20 + 19) = 0$

Independent Float $= E_2 - (L_1 + t_{12}) = 39 - (38 + 19) = -18$

(*iv*) ***Interfering float.*** Utilization of the float of an activity can affect the float of subsequent activities in the network. Thus, interfering float can be defined as that part of the total float which causes a reduction in the float of the successor activities. In other words, it can be defined as the *difference between the latest finish time of the activity under consideration and the earliest start time of the following activity, or zero, whichever is larger.* Thus, interfering float refers to that portion of the activity float which cannot be consumed without affecting adversely the float of the subsequent activity or activities.

Remarks 1. The basic difference between slack and float times is that slack is used for events only, whereas float is applied for activities.

2. Latest occurrent time of an event is always greater than or equal to its earliest occurrence time, *i.e.*, $L_i \geq E_i$. This implies that

$$\text{Independent float} \leq \text{Free float} \leq \text{Total float.}$$

3. An activity is critical if its total float is zero, otherwise it is non-critical.

4. Once the float of an activity is disturbed, float of all other activities of the project is changed and should be re-calculated.

5. The calculation of various floats can help the decision-maker in identifying the underutilized resources, flexibility in the total schedule and possibilities of redeployment of resources.

SAMPLE PROBLEMS

2511. *A project consists of a series of tasks labelled A, B, ..., H, I with the following relationships (W < X, Y means X and Y cannot start until W is completed; X, Y < W means W cannot start until both X and Y are completed). With this notation construct the network diagram having the following constraints :*

$$A < D, E;\ B, D < F;\ C < G;\ B, G < H;\ F, G < I.$$

Find also the minimum time of completion of the project, when the time (in days) of completion of each task is as follows :

Task :	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*
Time :	23	8	20	16	24	18	19	4	10

[Delhi M.Com. 2007; Madras M.B.A., 2010]

Solution. Using the given constraints, the resulting network is shown in Fig. 25.17. The dummy activities D_1 and D_2 are introduced to establish the correct precedence relationships. The events of the projects are numbered in such a way that their ascending order indicates the direction of progress in the project :

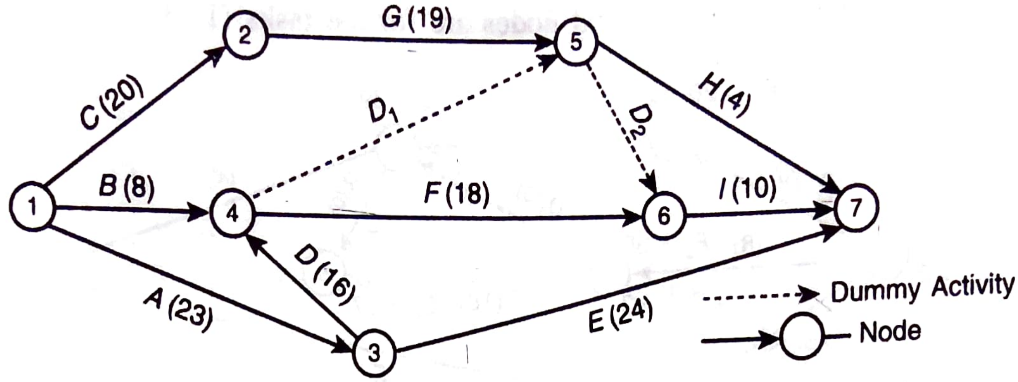


Fig. 25.17

To determine the minimum time of completion of the project (critical path), we compute E_j and E_i for each of the task (i, j) of the project. The critical path calculations as applied to Fig. 25.17 are as follows :

Forward calculations

Node 1 : Set $E_1 = 0$

Node 2 : $E_2 = E_1 + t_{12} = 0 + 20 = 20$

Node 3 : $E_3 = E_1 + t_{13} = 0 + 23 = 23$

Node 4 : $E_4 = \max_{i=1,3} \{E_i + t_{i4}\} = \max \{0+8,\ 23+16\} = 39$

Node 5 : $E_5 = \max_{i=2,4} \{E_i + t_{i5}\} = \max \{20+19,\ 39+0\} = 39$

Node 6 : $E_6 = \max_{i=4,5} \{E_i + t_{i6}\} = \max \{39+18,\ 39+0\} = 57$

Node 7 : $E_7 = \max_{i=3,5,6} \{E_i + t_{i6}\} = \max \{23+24,\ 39+4,\ 57+10\} = 67$

Backward calculations

Node 7 : Set $L_7 = E_7 = 67$

Node 6 : $L_6 = \min_{j=7} \{L_j - t_{6j}\} = L_7 - t_{67} = 57$

Node 5 : $L_5 = \min_{j=6,7} \{L_j - t_{5j}\} = \min \{57-0,\ 67-4\} = 57$

Node 4 : $L_4 = \min_{j=5,6} \{L_j - t_{4j}\} = \min \{57-0,\ 57-18\} = 39$

Node 3 : $L_3 = \min_{j=4,7} \{L_j - t_{3j}\} = \min \{39-16,\ 67-24\} = 23$

Node 2 : $L_2 = L_5 - t_{25} - 57 - 19 = 38$

Node 1 : $L_1 = \min_{j=2,3,4} \{L_j - t_{ij}\} = \min \{38-20,\ 23-23,\ 39-8\} = 0$

To evaluate the critical nodes, all these calculations are displayed in the flowing table :

Task	*Normal time*	*Earliest time*		*Latest time*		*Float*		
i, j	*(days)*	*Start (E_i)*	*Finish (E_j)*	*Start (L_i)*	*Finish (L_j)*	*Total*	*Free*	*Independent*
(1, 2)	20	0	20	18	38	18	0	0
(1, 3)	23	0	23	0	23	0	0	0
(1, 4)	8	0	8	31	39	31	31	31
(2, 5)	19	20	39	38	57	18	0	–18
(3, 4)	16	23	39	23	39	0	0	0
(3, 7)	24	23	47	43	67	20	20	20
(4, 5)	0	39	39	57	57	18	0	0
(4, 6)	18	39	57	39	57	0	0	0
(5, 6)	0	39	39	57	57	18	18	0
(5, 7)	4	39	43	63	67	24	24	6
(6, 7)	10	57	67	57	67	0	0	0

The above table shows that the critical nodes are for the tasks (1, 3), (3, 4), (4, 6) and (6, 7).

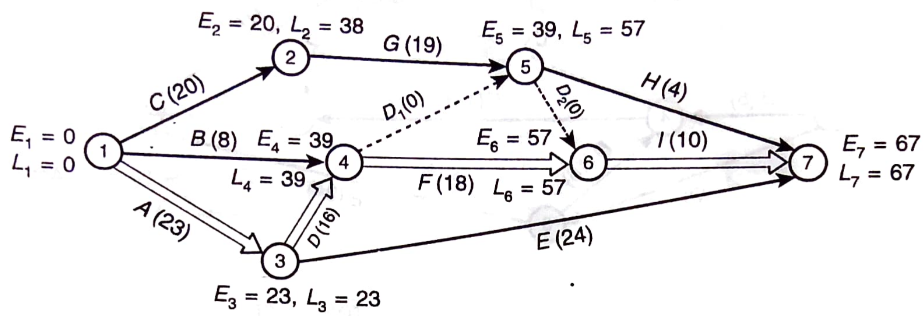


Fig. 25.18

It is apparent from Fig. 25.18 that the critical path comprises the tasks (1, 3), (3, 4), (4, 6) and (6, 7). This path represents the shortest time to complete the entire project.

2512. *A small project consists of seven activities for which the relevant data are given below :*

Activity	*Preceding Activities*	*Activity Duration (Days)*
A	—	4
B	—	7
C	—	6
D	*A, B*	5
E	*A, B*	7
F	*C, D, E*	6
G	*C, D, E*	5

(*i*) *Draw the network and find the project completion time.*

(*ii*) *Calculate total float for each of the activities and highlight the critical path.*

(*iii*) *Draw the time scaled diagram.* ***[C.A. Final (Nov.) 2006; Madras B.Com. 2005]***

Solution. (*i*) Using the precedence relationship among the activities, the resulting network is shown in Fig. 25.19 below :

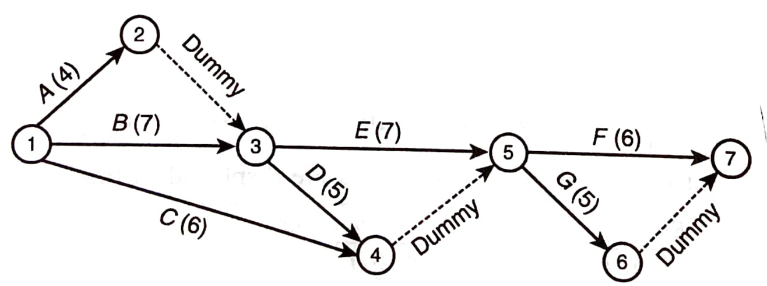


Fig. 25.19

(*ii*) To determine the project completion time, we compute the earliest time, E_i, and latest finish time, L_j, for each activity of the project. For it we proceed as follows :

Forward calculations

Set $E_1 = 0$

$E_2 = E_1 + t_{12} = 0 + 4 = 4$

$E_3 = \max(E_1 + t_{13},\ E_2 + t_{23})$

$= \max(0 + 7,\ 4 + 0) = 7$

Backward calculations

Set $L_7 = E_7 = 20$

$L_6 = L_7 - t_{67} = 20 - 0 = 20$

$L_5 = \min(L_6 - t_{56},\ L_7 - t_{57})$

$= \min(20 - 5,\ 20 - 6) = 14$

$$E_4 = \max\ (E_1 + t_{14},\ E_3 + t_{34})$$
$$= \max\ (0+6,\ 7+5) = 12$$
$$E_5 = \max\ (E_3 + t_{35},\ E_4 + t_{45})$$
$$= \max\ (7+7,\ 12+0) = 14$$
$$E_6 = E_5 + t_{56} = 14+5 = 19$$
$$E_7 = \max\ (E_5 + t_{57},\ E_6 + t_{67})$$
$$= \max\ (14+6,\ 19+0) = 20$$

$$L_4 = L_5 - t_{45} = 14-0 = 14$$
$$L_3 = \min\ (L_4 - t_{34},\ L_5 - t_{35})$$
$$= \min\ (14-5,\ 14-7) = 7$$
$$L_2 = L_3 - t_{23} = 7-0 = 7$$
$$L_1 = \min\ (L_2 - t_{12},\ L_3 - t_{13},\ L_4 - t_{14})$$
$$= \max\ (7-4,\ 7-7,\ 14-6) = 0.$$

To evaluate the critical nodes and total float, the above calculations are displayed in the following table :

Activity (*i, j*)	*Normal time* (*days*)	*Earliest time*		*Latest time*		*Total float*
		E_i	$E_i + t_{ij}$	$L_j - t_{ij}$	L_j	
1—2	4	0	4	3	7	3
1—3	7	0	7	0	7	0
1—4	6	0	6	8	14	8
3—4	5	7	12	9	14	2
3—5	7	7	14	7	14	0
5—7	6	14	20	14	20	0
5—6	5	14	19	20	20	1

From the above table, we observe that the critical nodes (events) are : (1, 3), (3, 5) and (5, 7). Thus, we have

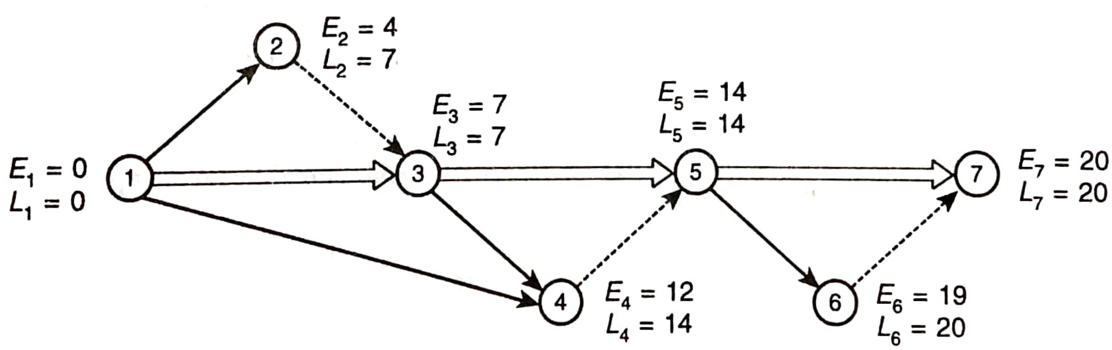


Fig. 25.20

CPM is *1* → *3* → *5* → *7* and Project completion time is 20 days.

(*iii*) Time scale diagram is given below :

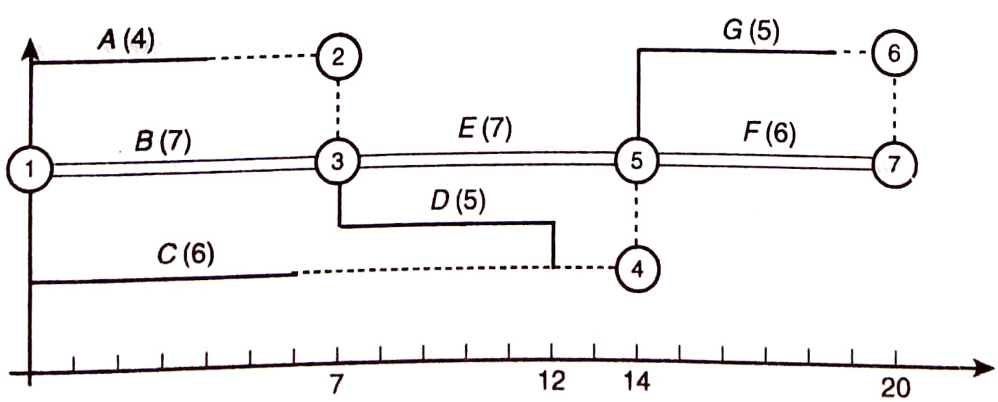


Fig. 25.21

25:7. PROBABILITY CONSIDERATIONS IN *PERT*

The network methods discussed so far may be termed as deterministic, since estimated activity times are assumed to be the expected values. But no recognition is given to the fact that expected activity time is the mean of a distribution of possible values which could occur.

Under the conditions of uncertainty, the estimated time for each activity for *PERT* network is represented by a probability distribution. This probability distribution of activity time is based upon three different time estimates made for each activity. These are as follows :

t_o = the *optimistic time*, is the shortest possible time to complete the activity if all goes well.

t_p = the *pessimistic time*, is the longest time that an activity could take if every thing goes wrong.

t_m = the *most likely time*, is the estimate of the normal time an activity would take. If only one time were available, this would be it. Otherwise it is the mode of the probability distribution.

The range specified by the optimistic time (t_o) and pessimistic time (t_p) estimates supposedly must enclose every possible estimate of the duration of the activity. The most likely time (t_m) estimate may not coincide with the midpoint $(t_o + t_p)/2$ and may occur to its left or to its right as shown in Fig. 25.22 :

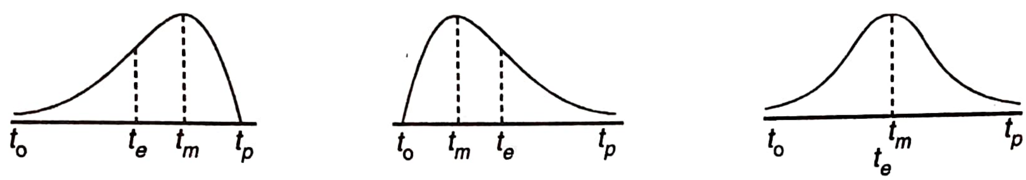


Fig. 25.22

Keeping in view of the above-mentioned properties, it may be justified to assume that the duration of each activity may follow a Beta (β) distribution with its unimodal point occurring at t_m and its end points t_o and t_p.

In Beta distribution the mid-point $(t_o + t_p)/2$ is assumed to weigh half as much as the most likely point (t_m). Thus, the expected value of the activity duration can be approximated as the arithmetic mean of $(t_o + t_p)/2$ and $2\,t_m$. Thus, we have

$$t_e = \frac{1}{3}\left[2t_m + (t_o + t_p)/2\right] = (t_o + 4t_m + t_p)/6$$

Since almost (99%) all values of random variables fall within ± 3 standard deviation from the mean or fall within the range approximately 6 standard deviation in length, therefore the interval (t_o, t_p) is assumed to enclose about 6 standard deviation of a symmetric distribution. Thus, if σ denotes the standard deviation, then

$$6\sigma \cong t_p - t_o \quad \text{or} \quad \sigma = (t_p - t_o)/6.$$

The variance, therefore, is : $\sigma^2 = \{(t_p - t_o)/6\}^2$.

Remark. In *PERT* analysis, a Beta distribution is assumed because it is unimodal, has non-negative end points, and is approximately symmetric.

Probability of Meeting the Schedule Time

With *PERT*, it is possible to determine the probability of completing a contract on schedule. The scheduled dates are expressed as a number of time units from the present time. Initially they may be the latest times, T_L, for each event, but after a project is started we shall know how far it has progressed at any given date, and the scheduled time will be the latest time if the project is to be completed on its original schedule.

The probability distribution of times for completing an event can be approximated by the normal distribution due to the central limit theorem. Thus the probability of completing the project by scheduled time (T_s) is given by

$$\text{Prob}\left(Z < \frac{T_s - T_e}{\sigma_e}\right)$$

The standard normal variate (SNV) is given by,

$$Z = \frac{T_s - T_e}{\sigma_e}$$

where, T_e = expected completion time of the project and

σ_e = number of standard deviations the scheduled time lies from the expected (mean) time, *i.e.*, the standard deviation of the scheduled time.

Using the cumulative normal distribution tables (given at the end of book), the corresponding value of the standard normal variate is read off. This will give the required probability of completing the project on scheduled time.

SAMPLE PROBLEMS

2527. *A project consists of eight activities with the following relevant information :*

Activity	*Immediate predecessor*	*Estimated duration (days)*		
		Optimistic	*Most likely*	*Pessimistic*
A	—	1	1	7
B	—	1	4	7
C	—	2	2	8
D	*A*	1	1	1
E	*B*	2	5	14
F	*C*	2	5	8
G	*D, E*	3	6	15
H	*F, G*	1	2	3

(i) Draw the PERT network and find out the expected project completion time.

(ii) What duration will have 95% confidence for project completion?

(iii) If the average duration for activity F increases to 14 days, what will be its effect on the expected project completion time which will have 95% confidence?

(For standard normal Z = 1.645, area under the standard normal curve from 0 to Z is 0.45)

[Panjab M.B.A. 2007; Jodhpur M.B.A. 1997; IGNOU M.B.A. 2008]

Solution. The expected time and variance of each activity is computed in table below :

Activity	t_o	t_m	t_p	$t_e = (t_o + 4t_m + t_p)/6$	$[(t_p - t_o)/6]^2$
A	1	1	7	2	1
B	1	4	7	4	1
C	2	2	8	3	1
D	1	1	1	1	0
E	2	5	14	6	4
F	2	5	8	5	1
G	3	6	15	7	4
H	1	2	3	2	$(2/6)^2$

(*i*) Using the precedence relationship among the activities, the resulting network is shown in Fig. 25.23 :

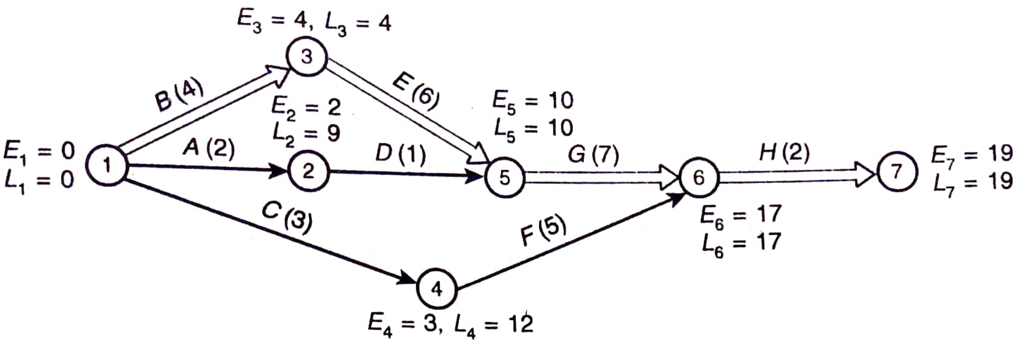


Fig. 25.23

From the above network diagram, we observe :

$$CPM : 1 \rightarrow 3 \rightarrow 5 \rightarrow 6 \rightarrow 7, \text{ i.e., } B \rightarrow E \rightarrow G \rightarrow H.$$

Expected duration of the project is 19 days.

The Variance of the project length is : $\sigma^2 = 1 + 4 + 4 + 0.108$

Thus $\sigma_e^2 = 1 + 4 + 4 + 0.108 = 9.108$ or $\sigma_e = \sqrt{9.108} = 3.02$.

(*ii*) Since $P(z \leq 1.645) = 0.5 + 0.45$, *i.e.*, 0.95 ;

$\therefore$ We get $\dfrac{T_s - T_e}{\sigma_e} = 1.645.$

or $T_s = 19 + 3.02 \times 1.645 = 24$ days.

Hence, 24 days of project completion time will have 95% confidence of completion in the scheduled time.

(*iii*) When the average duration of activity F increases to 14, the path $C \rightarrow F \rightarrow H$ also becomes critical. The standard deviation of new critical path is : $\sigma_e = \sqrt{9.108 + 2} = 3.36$

$\therefore$ $P(z < 1.645) = 0.95$ gives us

$$\frac{T_s - 19}{3.36} = 1.645, \text{ i.e., } T_s = 19 + 3.36 \times 1.645 = 24.52 \text{ days.}$$

Hence, the project completion duration of 24.52 days will have 95% confidence.

2528. *A small project is composed of seven activities whose time estimates are listed in the table as follows :*

Activity		*Estimated duration (weeks)*		
i	*j*	*Optimistic*	*Most likely*	*Pessimistic*
1	2	1	1	7
1	3	1	4	7
1	4	2	2	8
2	5	1	1	1
3	5	2	5	14
4	6	2	5	8
5	6	3	6	15

(*a*) *Draw the project network.*

(*b*) *Find the expected duration and variance of each activity. What is the expected project length?*

(*c*) *Calculate the variance and standard deviation of project length. What is the probability that the project will be completed :*

(*i*) *at least 4 weeks earlier than expected?*

(*ii*) *no more than 4 weeks later than expected?*

(*d*) *If the project due date is 19 weeks, what is the probability of meeting the due date?*

Given :						
	z	*0.50*	*0.67*	*1.00*	*1.33*	2.00
	p	*0.3085*	*0.2514*	*0.1587*	*0.0918*	0.0228

[*Delhi M.Com. 2006; Panjab Tech. Univ. M.B.A. 2008; Madras M.B.A. 2010*]

Solution. (*a*) The problem is same as in sample *problem 2527*, except that activity *6—7* has been deleted. As such the network diagram remains the same as given in *Fig. 25.22* after deleting activity *6—7*.

Thus, we have

(*b*) *CPM* : *1* → *3* → *5* → *6*, Duration of project = 17 days.

Variance of project length is given by

$$\sigma_e^2 = 1 + 4 + 4 = 9 \quad \text{or} \quad \sigma_e = 3.$$

(*c*) The standard normal deviate is :

$$z = \frac{\text{Due date} - \text{Expected date of completion}}{\sqrt{\text{Variance}}}.$$

∴ We compute

(*i*) $$z = \frac{13 - 17}{3} = -\frac{4}{3} = -1.33$$

This implies that the probability of meeting the due date (4 weeks earlier than expected) is : 0.0918 or 9.18%.

(*ii*) $$z = \frac{21 - 17}{3} = \frac{4}{3} = 1.33.$$

This implies that the probability of meeting the due date (weeks later than expected) is : 1 – 0.0918, *i.e.*, 0.9082 or 90.82%.

(*d*) When due date is 19 weeks,

$$z = \frac{19 - 17}{3} = \frac{2}{3} = 0.67$$

This implies that the probability of meeting the due date is : 1 –0.2514, *i.e.*, 0.7486 or 74.86%.

Alternative calculations (when values of area under standard normal curve are not given) :

(*c*) When $$z = \frac{13 - 17}{3} = -\frac{4}{3} = -1.33,$$

$$P(z \le -1.33) = 0.5 - \Phi(1.33) = 0.5 - 0.4082 \quad \text{(from Normal Tables)}$$
$$= 0.0918 = 9.18\%$$

When $$z = \frac{21 - 17}{3} = \frac{4}{3} = 1.33,$$

The probability of meeting the due date (4 weeks later than expected) is

$$P(z \le 1.33) = 0.5 + \Phi(1.33) = 0.5 + 0.4082 = 0.9082$$
$$= 0.9082 \approx 90.82\%.$$

(*d*) When the due date is 19 weeks,

$$z = \frac{19 - 17}{3} = \frac{2}{3} = 0.67.$$

∴ Probability of meeting the due date is

$$P(z \le 0.67) = 0.5 + \Phi(0.67) = 0.7486 \approx 74.86\%.$$

Thus the probability of not meeting the due date is 1 – 0.7486, *i.e.*, 0.2514 or 25.14%.

(iii) What are the critical activities, and what is the expected project completion time?

(iv) If the club manager plans to start the project on May 1, 2010, What is the probability the swimming pool will be ready by the scheduled August 1 (within 13 weeks)? *[Delhi M.Com. 2010]*

2544. A nationalised bank wishes to plan and schedule the development and installation of a new computerized cheque processing system. The changeover in cheque-processing procedures requires employment of additional personnel to operate the new system, development of new systems (computer software), and modification of existing cheque sorting equipment. The activities required to complete the project along with three time estimates and the precedence relationship among the activities have been determined by bank management and are given in the following table :

Activity	*Description*	*Predecessor*	*Time Estimates (days)*		
			Optimistic	*Most likely*	*Pessimistic*
A	Position recruiting	—	5	8	17
B	System development	—	3	12	15
C	System training	A	4	7	10
D	Equipment training	A	5	8	23
E	Manual system test	B, C	1	1	1
F	Preliminary system changeover	B, C	1	4	13
G	Computer-personnel interface	D, E	3	6	9
H	Equipment modification	D, E	1	2.5	7
I	Equipment testing	H	1	1	1
J	System debugging and installation	F, G	2	2	2
K	Equipment changeover	G, I	5	8	11

(a) Draw a network diagram for this project and find the critical path and its length.

(b) Calculate the total and free floats for non-critical activities.

(c) What is the probability that the length of the critical path does not exceed 40 days?

[Delhi M.B.A. (Nov.) 2005; B.H.U. M.B.A. (Oct.) 2009]

2545. A publisher is preparing to produce the second edition of a textbook. The activities required and their estimated times are as follows :

Activity	*Description*	*Immediate Predecessor*	*Activity Times (in days)*		
			Optimistic	*Pessimistic*	*Most likely*
A	Assess market	—	1	3	2
B	Get reviews from users	A	1	2	1.5
C	Revamp old material and add new material	B	3	9	5
D	Obtain review and prepare final draft	C	4	12	6
E	Revise and expand problems	B	2	7	4
F	Copy edit final draft	D	1	2.5	1.5
G	Copy edit problems	E	0.5	1.5	1
H	Set type, proof and print book	F, G	5	9	6
I	Prepare instructor's manual	E	2	4	3
J	Produce instructor's manual	I	1	2	1.5
K	Complete book and instructor's manual	H, J	0	0	0

(a) Draw an arrow diagram and identify the critical path.

(b) What is probability that the project will be completed within 21 months? 24 months? 27 months?

[Delhi M.B.A. 2009]

25:8. DISTINCTION BETWEEN *PERT* AND *CPM*

Both *PERT* and *CPM* are managerial techniques for planning and control of large complex projects. Both are techniques to network analysis wherein a network is prepared to analyse inter-relationships between different activities of a project. However, there are several differences between the two techniques :

1. *CPM* is used for *repetitive jobs* like planning the construction of a house. On the other hand, *PERT* is used for *non-repetitive jobs* like planning the assembly of the space platform.

2. *PERT* is a *probabilistic model* with uncertainty in activity duration. Multiple time estimates are made to calculate the probability of completing the project within scheduled time. On the contrary, *CPM* is a *deterministic model* with well-known activity (single) times based upon past experience. It, therefore, does not deal with uncertainty in project duration.

3. *PERT* is said to be *event-oriented* as the results of analysis are expressed in terms of events or distinct points in time indicative of progress. *CPM* is, on the other hand, *activity-oriented* as the results of calculations are considered in terms of activities or operations of the project.

4. *PERT* is applied mainly for planning and scheduling research programmes. On the other hand, *CPM* is employed in construction and business problems.

5. *PERT* incorporates *statistical analysis* and thereby enables the determination of probabilities concerning the time by which each activity and the entire project would be completed. On the other hand, *CPM* does not incorporate statistical analysis in determining time estimates because time is precise and known.

6. *PERT* serves a useful *control device* as it assists the management in controlling a project by calling attention through constant review to such delays in activities which might lead to a delay in the project completion date. But, it is difficult to use *CPM* as a controlling device for the simple reason that one must repeat the entire evaluation of the project each time the changes are incorporated into the network.