Three Dimensional Display Methods

Parallel Projection

- This method generates view from solid object by projecting parallel lines onto the display plane.
- By changing viewing position we can get different views of 3D object onto 2D display screen.

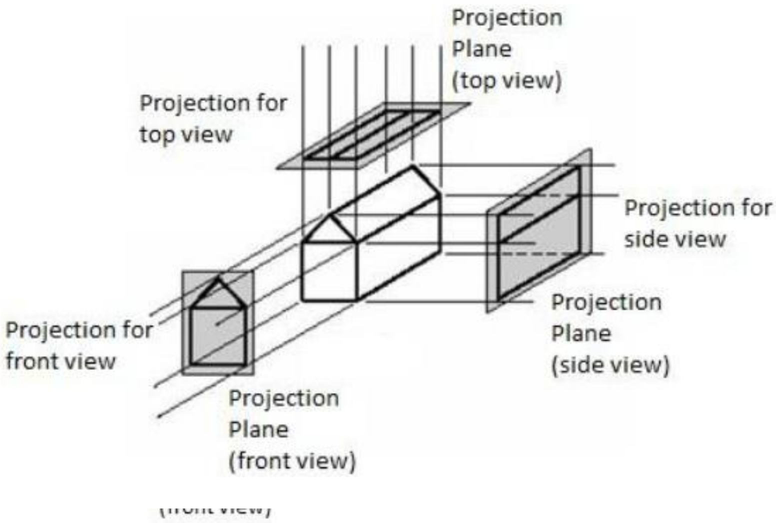

Fig. 4.1: - different views object by changing viewing plane position.

- Above figure shows different views of objects.
- This technique is used in Engineering & Architecture drawing to represent an object with a set of views that maintain relative properties of the object e.g.:- orthographic projection.

Perspective projection

- This method generating view of 3D object by projecting point on the display plane along converging paths.

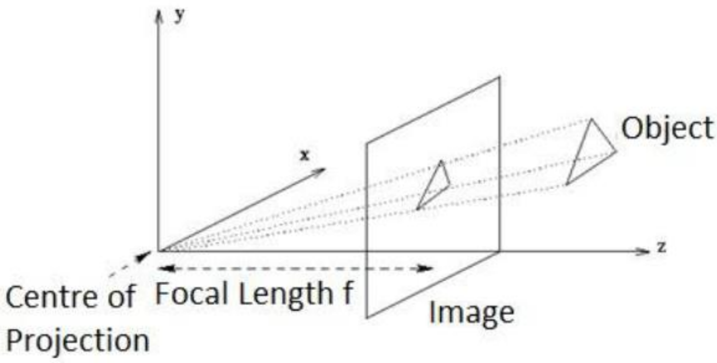

Fig. 4.2: - perspective projection

- This will display object smaller when it is away from the view plane and of nearly same size when closer to view plane.

- It will produce more realistic view as it is the way our eye is forming image.

Depth cueing

- Many times depth information is important so that we can identify for a particular viewing direction which are the front surfaces and which are the back surfaces of display object.

Simple method to do this is depth cueing in which assign higher intensity to closer object & lower intensity to the far objects.

Depth cuing is applied by choosing maximum and minimum intensity values and a range of distance over which the intensities are to vary.

Another application is to modeling effect of atmosphere.

Projections

- Once world-coordinate descriptions of the objects in a scene are converted to viewing coordinates, we can project the three-dimensional objects onto the two-dimensional view plane.
- Process of converting three-dimensional coordinates into two-dimensional scene is known as **projection**.
- There are two projection methods namely.
  1. Parallel Projection.
  2. Perspective Projection.
- Lets discuss each one.

Parallel Projections

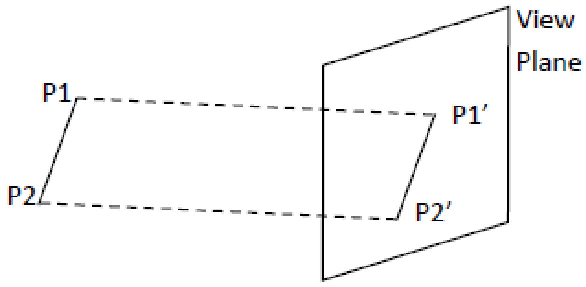

Fig. 5.12: - Parallel projection.

- In a parallel projection, coordinate positions are transformed to the view plane along parallel lines, as shown in the, example of above Figure.
- We can specify a parallel projection with a projection vector that defines the direction for the projection lines.
- It is further divide into two types.
  1. Orthographic parallel projection.
  2. Oblique parallel projection.

Orthographic parallel projection

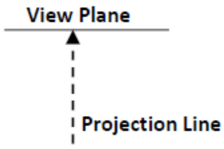

Fig. 5.13: - Orthographic parallel projection.

- When the projection lines are perpendicular to the view plane, we have an orthographic parallel projection.
- Orthographic projections are most often used to produce the front, side, and top views of an object, as shown in Fig.

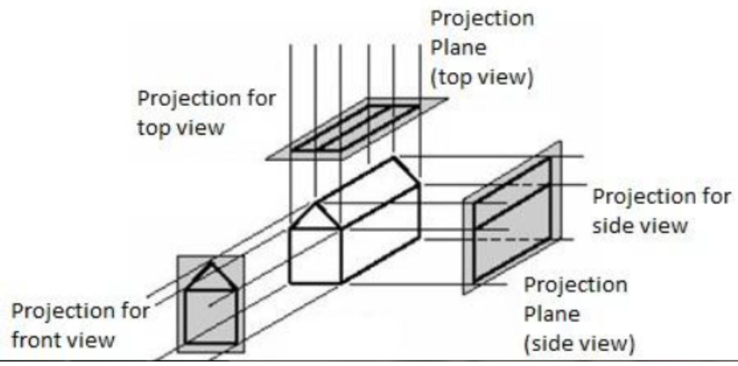

Fig. 5.14: - Orthographic parallel projection.

- Engineering and architectural drawings commonly use orthographic projections, because lengths and angles are accurately depicted and can be measure from the drawings.
- We can also form orthographic projections that display more than one face of an object. Such view are called **axonometric orthographic projections**. Very good example of it is **Isometric** projection.
- Transformation equations for an orthographic parallel projection are straight forward.
- If the view plane is placed at position z_{vp} along the z_v axis, then any point (x, y, z) in viewing coordinates is transformed to projection coordinates as

  $x_p = x, \qquad y_p = y$

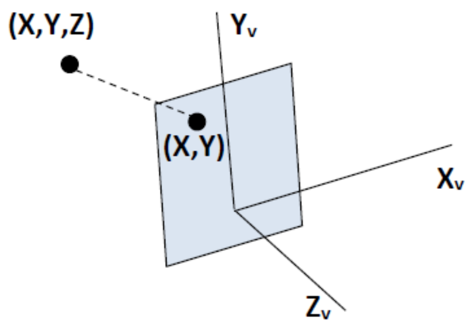

Fig. 5.15: - Orthographic parallel projection.

- Where the original z-coordinate value is preserved for the depth information needed in depth cueing and visible-surface determination procedures.

Oblique parallel projection.

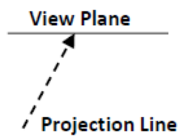

Fig. 5.16: - Oblique parallel projection.

- An oblique projection is obtained by projecting points along parallel lines that are not perpendicular to the projection plane.
- Coordinate of oblique parallel projection can be obtained as below.

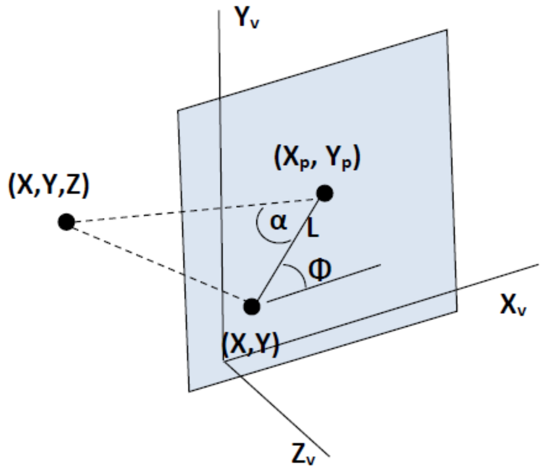

Fig. 5.17: - Oblique parallel projection.

- As shown in the figure (X,Y,Z) is a point of which we are taking oblique projection (Xp,Yp) on the view plane and point (X,Y) on view plane is orthographic projection of (X,Y,Z).

- Now from figure using trigonometric rules we can write

$$x_p = x + L\cos\emptyset$$
$$y_p = y + L\sin\emptyset$$

- Length L depends on the angle α and the z coordinate of the point to be projected:

$$\tan\alpha = \frac{Z}{L}$$
$$L = \frac{Z}{\tan\alpha}$$
$$L = ZL_1, \quad Where\ L_1 = \frac{1}{\tan\alpha}$$

- Now put the value of L in projection equation.

$$x_p = x + ZL_1\cos\emptyset$$
$$y_p = y + ZL_1\sin\emptyset$$

- Now we will write transformation matrix for this equation.

$$M_{parallel} = \begin{bmatrix} 1 & 0 & L_1\cos\emptyset & 0 \\ 0 & 1 & L_1\sin\emptyset & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- This equation can be used for any parallel projection. For orthographic projection L_1=0 and so whole term which is multiply with z component is zero.
- When value of $\tan\alpha = 1$ projection is known as **Cavalier projection**.
- When value of $\tan\alpha = 2$ projection is known as **Cabinet projection**.

Perspective Projection

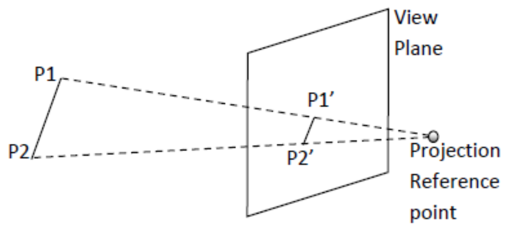

Fig. 5.18: - Perspective projection.

- In perspective projection object positions are transformed to the view plane along lines that converge to a point called the **projection reference point** (or **center of projection** or **vanishing point**).

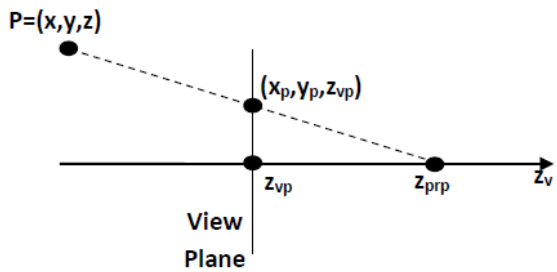

Fig. 5.19: - Perspective projection.

- Suppose we set the projection reference point at position z_{prp} along the z_v axis, and we place the view plane at z_{vp} as shown in Figure above. We can write equations describing coordinate positions along this perspective projection line in parametric form as

$$x' = x - xu$$
$$y' = y - yu$$
$$z' = z - (z - z_{prp})u$$

- Here parameter u takes the value from 0 to 1, which is depends on the position of object, view plane, and projection reference point.
- For obtaining value of u we will put $z' = z_{vp}$ and solve equation of z'.

$$z' = z - (z - z_{prp})u$$
$$z_{vp} = z - (z - z_{prp})u$$
$$u = \frac{z_{vp} - z}{z_{prp} - z}$$

- Now substituting value of u in equation of x' and y' we will obtain.

$$x_p = x\left(\frac{z_{prp} - z_{vp}}{z_{prp} - z}\right) = x\left(\frac{d_p}{z_{prp} - z}\right)$$
$$y_p = y\left(\frac{z_{prp} - z_{vp}}{z_{prp} - z}\right) = y\left(\frac{d_p}{z_{prp} - z}\right), \quad \textit{Where } d_p = z_{prp} - z_{vp}$$

- Using three dimensional homogeneous-coordinate representations, we can write the perspective projection transformation matrix form as.

$$\begin{bmatrix} x_h \\ y_h \\ z_h \\ h \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -z_{vp}/d_p & z_{vp}(z_{prp}/d_p) \\ 0 & 0 & -1/d_p & z_{prp}/d_p \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

- In this representation, the homogeneous factor is.

$$h = \frac{z_{prp} - z}{d_p} \textit{ and}$$
$$x_p = x_h/h \textit{ and } y_p = y_h/h$$

- There are number of special cases for the perspective transformation equations.
- If view plane is taken to be uv plane, then $z_{vp} = 0$ and the projection coordinates are.

$$x_p = x\left(\frac{z_{prp}}{z_{prp} - z}\right) = x\left(\frac{1}{1 - z/z_{prp}}\right)$$
$$y_p = y\left(\frac{z_{prp}}{z_{prp} - z}\right) = y\left(\frac{1}{1 - z/z_{prp}}\right)$$

- If we take projection reference point at origin than $z_{prp} = 0$ and the projection coordinates are.

$$x_p = x\left(\frac{z_{vp}}{z}\right) = x\left(\frac{1}{z/z_{vp}}\right)$$
$$y_p = y\left(\frac{z_{vp}}{z}\right) = y\left(\frac{1}{z/z_{vp}}\right)$$

- The vanishing point for any set of lines that are parallel to one of the principal axes of an object is referred to as a principal vanishing point

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- We control the number of principal vanishing points (one, two, or three) with the orientation of the projection plane, and perspective projections are accordingly classified as one-point, two-point, or three-point projections.
- The number of principal vanishing points in a projection is determined by the number of principal axes intersecting the view plane.

3D Transformation

3D Translation

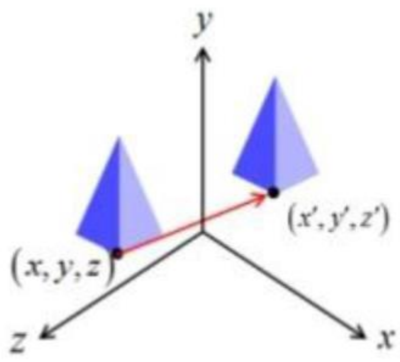

Fig. 5.1: - 3D Translation.

- Similar to 2D translation, which used 3x3 matrices, 3D translation use 4X4 matrices (X, Y, Z, h).
- In 3D translation point (X, Y, Z) is to be translated by amount tx, ty and tz to location (X', Y', Z').

$$x' = x + tx$$
$$y' = y + ty$$
$$z' = z + tz$$

- Let's see matrix equation

$$P' = T \cdot P$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

- Example : - Translate the given point P (10,10,10) into 3D space with translation factor T (10,20,5).

$$P' = T \cdot P$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 10 \\ 0 & 1 & 0 & 20 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 10 \\ 10 \\ 10 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 20 \\ 30 \\ 15 \\ 1 \end{bmatrix}$$

Final coordinate after translation is P′ (20, 30, 15).

Rotation

- For 3D rotation we need to pick an axis to rotate about.
- The most common choices are the X-axis, the Y-axis, and the Z-axis

Coordinate-Axes Rotations

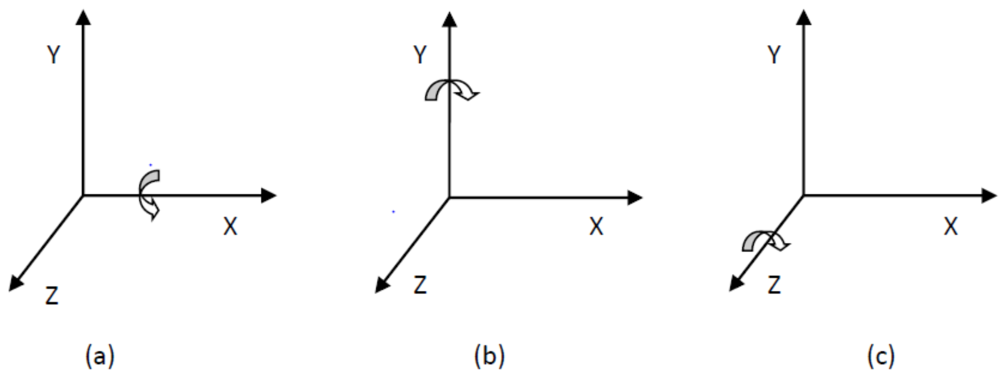

Fig. 5.2: - 3D Rotations.

Z-Axis Rotation

- Two dimension rotation equations can be easily convert into 3D Z-axis rotation equations.
- Rotation about z axis we leave z coordinate unchanged.

$$x' = x\cos\theta - y\sin\theta$$

$z' = z$

Where Parameter θ specify rotation angle.

- Matrix equation is written as:

$P' = R_z(\theta) \cdot P$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

X-Axis Rotation

- Transformation equation for x-axis is obtain from equation of z-axis rotation by replacing cyclically as shown here

$x \rightarrow y \rightarrow z \rightarrow x$

- Rotation about x axis we leave x coordinate unchanged.

$y' = y\cos\theta - z\sin\theta$

$z' = y\sin\theta + z\cos\theta$

$x' = x$

Where Parameter θ specify rotation angle.

- Matrix equation is written as:

$P' = R_x(\theta) \cdot P$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Y-Axis Rotation

- Transformation equation for y-axis is obtain from equation of x-axis rotation by replacing cyclically as shown here

$x \rightarrow y \rightarrow z \rightarrow x$

- Rotation about y axis we leave y coordinate unchanged.

  $z' = z\cos\theta - x\sin\theta$

  $x' = z\sin\theta + x\cos\theta$

  $y' = y$

  Where Parameter θ specify rotation angle.

- Matrix equation is written as:

  $P' = R_y(\theta) \cdot P$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

- Example: - Rotate the point P(5,5,5) 90° about Z axis.

  $P' = R_z(\theta) \cdot P$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

- Example: - Rotate the point P(5,5,5) 90° about Z axis.

$$P' = R_z(\theta) \cdot P$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos 90 & -\sin 90 & 0 & 0 \\ \sin 90 & \cos 90 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ 5 \\ 5 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} -5 \\ 5 \\ 5 \\ 1 \end{bmatrix}$$

Final coordinate after rotation is P' (-5, 5, 5).

Scaling

- It is used to resize the object in 3D space.
- We can apply uniform as well as non uniform scaling by selecting proper scaling factor.
- Scaling in 3D is similar to scaling in 2D. Only one extra coordinate need to consider into it.

Coordinate Axes Scaling

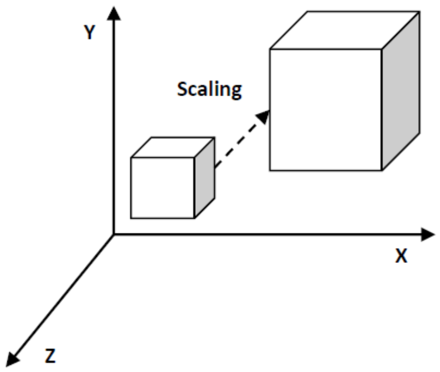

Fig. 5.6: - 3D Scaling.

- Simple coordinate axis scaling can be performed as below.

$$\boldsymbol{P' = S \cdot P}$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

- Example: - Scale the line AB with coordinates (10,20,10) and (20,30,30) respectively with scale factor S(3,2,4).

$P' = S \cdot P$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} A_x' & B_x' \\ A_y' & B_y' \\ A_z' & B_z' \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 10 & 20 \\ 20 & 30 \\ 10 & 30 \\ 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} A_x' & B_x' \\ A_y' & B_y' \\ A_z' & B_z' \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 30 & 60 \\ 40 & 60 \\ 40 & 120 \\ 1 & 1 \end{bmatrix}$$

Final coordinates after scaling are A′ (30, 40, 40) and B′ (60, 60, 120).

Other Transformations

Reflections

- Reflection means mirror image produced when mirror is placed at require position.
- When mirror is placed in XY-plane we obtain coordinates of image by just changing the sign of z coordinate.
- Transformation matrix for reflection about XY-plane is given below.

$$RF_z = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Similarly Transformation matrix for reflection about YZ-plane is.

$$RF_x = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Similarly Transformation matrix for reflection about XZ-plane is.

$$RF_y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Shears

- Shearing transformation can be used to modify object shapes.
- They are also useful in 3D viewing for obtaining general projection transformations.
- Here we use shear parameter '**a**' and '**b**'
- Shear matrix for Z-axis is given below

$$SH_z = \begin{bmatrix} 1 & 0 & a & 0 \\ 0 & 1 & b & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Similarly Shear matrix for X-axis is.

$$SH_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ a & 1 & 0 & 0 \\ b & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Similarly Shear matrix for X-axis is.

$$SH_y = \begin{bmatrix} 1 & a & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & b & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$