SUB CODE
18BMA25A

DISCRETE MATHEMATICAL STRUCTURES

objectives:

1. To study the fundamentals of Discrete Mathematical Structures

2. To obtain knowledge about finite theorectical Machines and theorectical languages.

Unit - I

MATHEMATICAL LOGIC: Connectives - negation - Conjuction - disconjuction - conditional and Biconditional - well formed formulas - Tautologies - Equivalence of formulas - Duality law - Tautological Implication Functionally complete set of connectives - other connectives - Normal form - disjuntive and conjuctive normal forms - Principle Disjunctive and principle conjuctive normal forms.

(Chapter 1 - Sections: 1.1 to 1.3)

Unit - II

THEORY OF INFERENCE AND PREDICATE CALCULUS: Rules of Inference consistency of premises and Indirect method of proof - The predicates calculus - predicates The statement function variable and Quantities free and sounds variable. Informic Theory of the predicate calculus

(Chapter - 1 - Sections 1.4 to 1.6)

UNIT- III

GRAPH THEORY: Basic definitions - Degree of vertex Some Special simple graphs - Matrix Representation of Graphs - Trees - Spanning trees - Minimum Spanning trees Rooted and Binary trees - Binary trees - Tree Traversal. Expression Trees - Problems.

(chapter - 7)

Unit - IV

LATTICES: Lattices as partially ordered sets Definition and example - Some properties of Lattices - Lattices as Algebraic system - Sub lattices Direct product and homomorphism some Special Lattices

(Chapter : 4 section : 4.1)

Unit - V

FORMAL LANGUAGE AND AUTOMATE: Phrase Structure Grammar - Types of Phrase Structure Grammar - Backus - Naur form Finite state machine - input and output strings for FSM - Finite state Automate - Problems.

(chapter 8 - page No 448 to 490)

Textbook 1

Mathematical Logic: ①

Logic is concern with studying arguments and conclusion. It is a decipline that deals with the method of reasoning. It Provides rules and Techniques for determining whether a given argument is Valid or not.

Statement or proposition

A statement or a proposition is a Sentence which is either true or false but not.

Eg: Chennai is the capital of Tamilnadu - True
The sun is a planet

Truth value

The Truth or false qual

If statement is true we say that its truth value is capital T. If it is false, its truth value is capital F

1. Chennai is the capital of Tamilnadu - T

Simple Statement

A Statement is said to be simple if it cannot be broken into two or more statement

Eg: given in ① are simple statement

Compound statement:

If a statement is a combination of two or more simple statement, there it is

Said to be compound statement Composite statement or Moliculai Statement

Eg: It is raining and It is cold.

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note:

English alphabet P, Q, R, S are used to represent simple statement and are called Propositional variables.

The truth value T, F are called propositional Constants.

Eg:

P: Chennai is capital of Tamilnadu - T

Q: 3+4 = 8 - F

Atomic Proposition:

A proposition consisting of only one propositional variable. is called Atomic Propostion. (or) Primary (or) Primitive Proposition.

Connectives:

The words (or) phrases (or) symbles that are used to form a compound proposition are called connectives.

1. CONJUCTION : (and, $\wedge$)

If two simple statement P and Q are connected by the word "and" then the resulting compound statement "P and Q" is called the conjuction of P and Q and is return in the symbolic form as "P and Q": $P \wedge Q$

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Rules:

(i) The statement P and Q has truth value T whenever both P and Q have truth value T

(ii) The statement P and Q has truth value F whenever either P (or) Q (or) both have truth value F

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

2. Dijunction: (or, $\vee$)

If two simple statement P & Q are connected by the word "or" then the resulting compound statement "P or Q" is called the disjunction of P & Q and is return in the symbolic form as $P \vee Q$

Rules

(i) The statement P or Q has truth value F only when both P and Q have the truth value F

(ii) The statement P (or) Q has truth value T whenever either P (or) Q (or) both have truth value T

P	Q	P∨Q
T	T	T
T	F	T
F	T	T
F	F	F

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Negation: (Unary Operator)

The Negation of a Statement is formed by introducing the word "not" at some proper place in the statement. or by prefixing in the Statement with "It is not the case that" (Or) "It is false that"

If P denotes a statement then the negation of P is ~~return~~ Written as $\neg P$ (or) $\sim P$

Rule:

(i) If the truth value of P is T then the truth value of $\neg P$ is F

(ii) If the truth value of P is F then the truth value of $\neg P$ is T

Negate the following statement

P : Seetha is beautiful.

$\neg P$: Seetha is not beautiful

$\neg P$: It is not the case that Seetha is beautiful.

Problems:

Construct the truth table formula

1. $P \vee \neg Q$

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Soln.

P	Q	$\neg Q$	$P \vee \neg Q$
T	T	F	T
T	F	T	T
F	T	F	F
F	F	T	T

2. $P \wedge \neg P$

~~P Q~~ $\neg P$ $P \wedge \neg P$

~~T T~~

~~T T~~

~~F T~~

~~F T~~

2. $P \wedge \neg P$

P	$\neg P$	$P \wedge \neg P$
T	F	F
F	T	F

3. $(P \vee Q) \vee \neg P$

P	Q	$\neg P$	$P \vee Q$	$(P \vee Q) \vee \neg P$
T	T	F	T	T
T	F	F	T	T
F	T	T	T	T
F	F	T	F	T

4. $\neg(\neg P \vee \neg Q)$

P	Q	$\neg P$	$\neg Q$	$\neg P \vee \neg Q$	$\neg(\neg P \vee \neg Q)$
T	T	F	F	F	T
T	F	F	T	T	F
F	T	T	F	T	F
F	F	T	T	T	F

5. $\neg(\neg P \wedge \neg Q)$

P	Q	$\neg P$	$\neg Q$	$\neg P \wedge \neg Q$	$\neg(\neg P \wedge \neg Q)$
T	T	F	F	F	T
T	F	F	T	F	T
F	T	T	F	F	T
F	F	T	T	T	F

6. $P \wedge (P \vee Q)$

P	Q	$P \vee Q$	$P \wedge (P \vee Q)$
T	T	T	T
T	F	T	T
F	T	T	F
F	F	F	F

7. $P \wedge (P \wedge Q)$

P	Q	$P \wedge Q$	$P \wedge (P \wedge Q)$
T	T	T	T
T	F	F	F
F	T	F	F
F	F	F	F

8. $[\neg P \wedge (\neg Q \wedge R)] \vee (Q \wedge R) \vee (P \wedge R)$

P	Q	R	$\neg P$	$\neg Q$	$\neg Q \wedge R$	$\neg P \wedge (\neg Q \wedge R)$
T	T	T	F	F	F	F
T	T	F	F	F	F	F
T	F	T	F	T	T	F
T	F	F	F	T	F	F
F	T	T	T	F	F	F
F	T	F	T	F	F	F
F	F	T	T	T	T	T
F	F	F	T	T	F	F

$Q \wedge R$	$P \wedge R$	$(Q \wedge R) \vee (P \wedge R)$	$[\neg P \wedge (\neg Q \wedge R)] \vee (Q \wedge R) \vee (P \wedge R)$
T	T	T	T
F	F	F	F
F	T	T	T
F	F	F	F
T	F	T	T
F	F	F	F
F	F	F	T
F	F	F	F

CONDITIONAL STATEMENT

If P and Q are any two statement then the statement $P \to Q$ which is read has "If ~~w~~ P then Q" is called a conditional statement.

Rule:

The statement $P \to Q$ has a truth value F when Q has truth value F, P has truth value T " otherwise it has truth value T '

TRUTH TABLE

P	Q	P—> Q
T	T	T
T	F	F
F	T	T
F	F	T

BICONDITIONAL STATEMENT.

If P and Q are any two statement then the statement P<=>Q which is read has "P if and only if Q" is called a Biconditional statement.

Rule:

The statement P<=>Q has the truth value T whenever both P and Q have identical truth value otherwise false.

TRUTH TABLE

P	Q	P<=>Q
T	T	T
T	F	F
F	T	F
F	F	T

1. Express in english the statement P—> Q where

P: The sun is shining today

Q: 2+7 > 4

If the sun is shining today then 2+7>4

2. Symbolic the following statement using the given Statement.

R : Mark is rich.

H : Mark is happy.

i) Mark is poor but happy.

$\neg R \wedge H$

ii) Mark is rich or unhappy.

$R \vee \neg H$

(iii) Mark is neither rich nor happy.

$\neg R \vee \neg H$

$\neg R \vee [R \wedge \neg H]$

(iv) Mark is poor or he is both rich and unhappy

$\neg R \vee [R \wedge \neg H]$

3. Write the following statement in Symbolic form.

"If either Ram takes calculus or Raja takes Algebra then Seetha will have English"

Soln:

denoting the Statement as

P : Ram takes calculus

Q : Raja takes Algebra

R : Seetha takes English.

Symbolic representation of the given statement is

$(P \vee Q) \rightarrow R$.

Write the following statement in Symbolic form. 10.

"The crop will be destroyed if there is a flood."

Soln:

Rewrite the given statement has "If there is flood then crop will be destroyed"

Let the statement will be denoted as

P : If there is flood

Q : the crop will be destroyed.

The Symbolic representation of given Statement.

$$P \rightarrow Q$$

Construct the truth table form

5(i) $(P \rightarrow Q) \wedge (Q \rightarrow P)$ (ii) $\neg(P \wedge Q) \Leftrightarrow (\neg P \vee \neg Q)$

(iii) $(P \wedge (P \rightarrow Q)) \rightarrow Q$ (iv) $(P \rightarrow Q) \Leftrightarrow (\neg P \vee Q)$

(v) $[(P \rightarrow Q) \wedge (Q \rightarrow R)] \rightarrow (P \rightarrow R)$

(vi) $[(P \rightarrow Q) \vee (Q \rightarrow R)] \rightarrow (P \rightarrow R)$

(vii) $(P \Leftrightarrow Q) \Leftrightarrow [(P \wedge Q) \vee (\neg P \wedge \neg Q)]$

(viii) $[Q \wedge (P \rightarrow Q)] \rightarrow P$

(ix) $\neg(P \vee (Q \wedge R)) \Leftrightarrow [(P \vee Q) \wedge (P \vee R)]$

(i) $(P \to Q) \wedge (Q \to P)$

P	Q	$P \to Q$	$Q \to P$	$(P \to Q) \wedge (Q \to P)$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

(ii) $\neg(P \wedge Q) \Leftrightarrow (\neg P \vee \neg Q)$

$\neg P$	P	$\neg Q$	Q	$P \wedge Q$	$\neg(P \wedge Q)$	$\neg P \vee \neg Q$	$\neg(P \wedge Q) \Leftrightarrow (\neg P \vee \neg Q)$
F	T	F	T	T	F	F	T
F	T	T	F	F	T	T	T
T	F	F	T	F	T	T	T
T	F	T	F	F	T	T	T

(iii)

P	Q	$P \to Q$	$P \wedge (P \to Q)$	$(P \wedge (P \to Q)) \to Q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

(iv)

P	$\neg P$	Q	$P \to Q$	$\neg P \vee Q$	$(P \to Q) \Leftrightarrow (\neg P \vee Q)$
T	F	T	T	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	T	F	T	T	T

(v)

P	Q	R	$P \to Q$	$Q \to R$	$(P \to Q) \wedge (Q \to R)$	$P \to R$
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	F	T	F	T
T	F	F	F	T	F	F
F	T	T	T	T	T	T
F	T	F	T	F	F	T
F	F	T	T	T	T	T
F	F	F	T	T	T	T

$[(P \to Q) \wedge (Q \to R)] \to (P \to R)$
T
T
T
T
T
T
T
T

(vi)

P	Q	R	$P \to Q$	$Q \to R$	$(P \to Q) \vee (Q \to R)$	$P \to R$
T	T	T	T	T	T	T
T	T	F	T	F	T	F
T	F	T	F	T	T	T
T	F	F	F	T	T	F
F	T	T	T	T	T	T
F	T	F	T	F	T	T
F	F	T	T	T	T	T
F	F	F	T	T	T	T

$[(P\to Q)\vee(Q\to R)] \to P\to R$
T
F
T
F
T
T
T
T

(vii) $(P \Leftrightarrow Q) \Leftrightarrow [(P\wedge Q)\vee(\neg P\wedge\neg Q)]$

P	Q	$\neg P$	$\neg Q$	$P\Leftrightarrow Q$	$P\wedge Q$	$\neg P\wedge\neg Q$
T	T	F	F	T	T	F
T	F	F	T	F	F	F
F	T	T	F	F	F	F
F	F	T	T	T	F	T

$[(P\wedge Q)\vee(\neg P\wedge\neg Q)]$	$(P\Leftrightarrow Q)\Leftrightarrow[(P\wedge Q)\vee(\neg P\wedge\neg Q)]$
T	T
F	T
F	T
T	T

(viii)

P	Q	$P\to Q$	$[Q\wedge(P\to Q)]$	$[Q\wedge(P\to Q)]\to P$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

1(x)

P	Q	R	Q∧R	P∨(Q∧R)	¬(P∨(Q∧R))
T	T	T	T	T	F
T	T	F	F	T	F
T	F	T	F	T	F
T	F	F	F	T	F
F	T	T	T	T	F
F	T	F	F	F	T
F	F	T	F	F	T
F	F	F	F	F	T

P∨Q	P∨R	[(P∨Q) ∧ (P∨R)]	¬(P∨(Q∧R)) ⇔ [(P∨Q)∧(P∨R)
T	T	T	F
T	T	T	F
T	T	T	F
T	T	T	F
T	T	T	F
T	F	F	F
F	T	F	F
F	F	F	F

Well-Formed Formula's

A well-formed formula's can be generated by the following rules:-

1) A statement variable standing alone is a well formed formula

2) If A is a well formed formula, Then $\neg A$ is a well-formed formula.

3) If A and B are well-formed formula then $(A \wedge B)$, $(A \vee B)$, $(A \rightarrow B)$ and $(A \rightleftharpoons B)$ are well-formed formula.

4) A String of symbols containing the statement variables, connectives and parenthesis is a well-formed formula, iff it can be obtained by finitely many applications of the rules 1, 2 and 3

Ex: $\neg(P \wedge Q)$, $\neg(P \vee Q)$, $(P \rightarrow (P \vee Q))$, $(P \rightarrow (Q \rightarrow R))$, $[(P \rightarrow Q) \wedge (Q \rightarrow R)] \rightleftharpoons (P \rightarrow R)$

The following are not well-formed formula's

1. $\neg P \wedge Q$ obviously P and Q are well-formed Formula's. A wff would be either $(\neg P \wedge Q)$ or $\neg(P \wedge Q)$.

2) $(P \rightarrow Q) \rightarrow$ This is not a wff because $\wedge Q$ is not.

3. $(P \to Q)$

4. $((P \wedge Q) \to Q)$

TATAULOGIES:

A statement formula which is truth regardless of the truth value of the statements which replace the variables in it is called a universally valid or a tautaulogy or a logical truth.

Ex:

P	$\neg P$	$P \vee \neg P$
T	F	T
F	T	T

CONTRODUCTION:

A statement formula which is false regardless of the truth values of the statements which replace the variable in it is called a controduction.

Ex:

P	$\neg P$	$P \wedge \neg P$
T	F	F
F	T	F

1. $[P \to (P \vee Q)]$

P	Q	$P \vee Q$	$P \to (P \vee Q)$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

2. $[(P \to \neg P) \to \neg P]$

P	$\neg P$	$P \to \neg P$	$(P \to \neg P) \to \neg P$
T	F	F	T
F	T	T	T

3. $[(\neg Q \wedge P) \wedge Q]$

P	Q	$\neg Q$	$\neg Q \wedge P$	$(\neg Q \wedge P) \wedge Q$
T	T	F	F	F
T	F	T	T	F
F	T	F	F	F
F	F	T	F	F

4. $(P \to (Q \to R)) \to [(P \to Q) \to (P \to R)]$

P

P	Q	R	Q → R	P → Q	P → R	P → (Q → R)
T	T	T	T	T	T	T
T	T	F	F	T	F	F
T	F	T	T	F	T	T
T	F	F	T	F	F	T
F	T	T	T	T	T	T
F	T	F	F	T	T	T
F	F	T	T	T	T	T
F	F	F	T	T	T	T

[(P → Q) → (P → R)]	(P → (Q → R)) → [(P → Q) → (P → R)]
T	T
F	T
T	T
T	T
T	T
T	T
T	T
T	T

5. $(\neg P \to Q) \to (Q \to P)$

P	Q	¬P	¬P → Q	Q → P	(¬P → Q) → (Q → P)
T	T	F	T	T	T
T	F	F	T	T	T
F	T	T	T	F	F
F	F	T	F	T	T

6. $((P \wedge Q) \Rightarrow P)$

P	Q	$P \wedge Q$	$(P \wedge Q) \Rightarrow P$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

Equalence of Formulas

Let A & B be two statement formulas.
$P_1, P_2, P_3, \ldots P_n$, denote all the variables accuring in both A & B.

If the truth value of A is equal to the truth value of B for everyone of the 2^n possible sets of truth values assigned to $P_1, P_2, P_3 \ldots P_n$ then A and B are set to be equalent

1) $\neg\neg P$ is equivalent to P
2) $P \vee P$ is equivalent to P
3) $(P \wedge \neg P) \vee Q$ is equivalent to Q
4) $P \vee \neg P$ is equivalent to $Q \vee \neg P$

Problems:

1. Prove: $P \rightarrow Q \Leftrightarrow (\neg P \vee Q)$
2. Prove: $P \rightarrow (Q \rightarrow R) \Leftrightarrow P \rightarrow (\neg Q \vee R) \Leftrightarrow (P \wedge Q) \rightarrow R$

1)

P	¬P	Q	P → Q	(¬P ∨ Q)	P → Q ⇔ (¬P ∨ Q)
T	F	T	T	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	T	F	T	T	T

∴ $P \to Q \Leftrightarrow (\neg P \vee Q)$ is Equivalent

2)

P	Q	¬Q	R	Q → R	P → (Q → R)	¬Q ∨ R	P → (¬Q ∨ R)
T	T	F	T	T	T	T	T
T	T	F	F	F	F	F	F
T	F	T	T	T	T	T	T
T	F	T	F	T	T	T	T
F	T	F	T	T	T	T	T
F	T	F	F	F	T	F	T
F	F	T	T	T	T	T	T
F	F	T	F	T	T	T	T

P ∧ Q	(P ∧ Q) → R
T	T
T	F
F	T
F	T
F	T
F	T
F	T
F	T

∴ $P \to (Q \to R) \Leftrightarrow P \to (\neg Q \vee R) \Leftrightarrow (P \wedge Q) \to R$ is Equivalent

EQUIVALENT FORMULAS:

1) Idempotent law:

$P \vee P \Leftrightarrow P$

$P \wedge P \Leftrightarrow P$

2. Commutative Law:

$P \vee Q \Leftrightarrow Q \vee P$

$P \wedge Q \Leftrightarrow Q \wedge P$

3. Associative law:

$\underline{(P \vee Q) \vee R} \Leftrightarrow \underline{P \vee (Q \vee R)}$

$(P \wedge Q) \wedge R \Leftrightarrow P \wedge (Q \wedge R)$

4. Distibutive Law:

$\underline{P \vee (Q \wedge R) \Leftrightarrow (P \vee Q) \wedge (P \vee R)}$

$P \wedge (Q \vee R) \Leftrightarrow (P \wedge Q) \vee (P \wedge R)$

5. Apsonption Law:

$P \vee (P \wedge Q) \Leftrightarrow P$

$P \wedge (P \vee Q) \Leftrightarrow P$

6. Demorgan's law:

$\neg(P \vee Q) \Leftrightarrow \neg P \wedge \neg Q$

$\neg(P \wedge Q) \Leftrightarrow \neg P \vee \neg Q$

7. Identity Law:

$P \wedge T \Leftrightarrow P$

$\cancel{P \vee T \Leftrightarrow T}$

$P \vee F \Leftrightarrow P$

$\cancel{P \wedge F \Leftrightarrow F}$

Dominent Law

1. $P \vee T \Leftrightarrow T$

2. $P \wedge F \Leftrightarrow F$

8. Involution law:

$\neg\neg P \Leftrightarrow P$

$P \rightarrow Q \Leftrightarrow \neg P \vee Q$

$(P \Leftrightarrow Q) \Leftrightarrow (P \rightarrow Q) \wedge (Q \rightarrow P)$ } H.W

$(P \Leftrightarrow Q) \Leftrightarrow (P \wedge Q) \vee (\neg P \wedge \neg Q)$

$(P \vee \neg P) \Leftrightarrow T$

$(P \wedge \neg P) \Leftrightarrow F$

$(P \rightarrow Q) \Leftrightarrow (\neg Q \rightarrow \neg P)$ H.W

Problems H.W

i) $(P \rightarrow (Q \rightarrow P)) \Leftrightarrow (\neg P \rightarrow (P \rightarrow Q))$

ii) $P \rightarrow (Q \vee R) \Leftrightarrow (P \rightarrow Q) \vee (P \rightarrow R)$

Problems:

1. Show that $(\neg P \wedge (\neg Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R$

Soln:

$(\neg P \wedge (\neg Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R)$

$\Leftrightarrow ((\neg P \wedge \neg Q) \wedge R) \vee (Q \vee P) \wedge R$

[∴ By Associative & Distributive law]

$\Leftrightarrow (\neg (P \vee Q) \wedge R) \vee ((P \vee Q) \wedge R)$

[∵ using Demorgan's & commutative law]

$\Leftrightarrow ((\neg (P \vee Q) \vee (P \vee Q)) \wedge R$

[Distributive law]

$\Leftrightarrow (T \wedge R)$ [∵ $\neg P \vee P \Leftrightarrow T$]

$\Leftrightarrow R.$

2. Show that $[(P \vee Q) \wedge \neg(\neg P \wedge (\neg Q \vee \neg R)) \vee (\neg P \wedge \neg Q) \vee (\neg P \wedge \neg R)$ is a Tautology

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Soln:

$[(P \vee Q) \wedge \neg(\neg P \wedge (\neg Q \vee \neg R))] \vee (\neg P \wedge \neg Q) \vee (\neg P \wedge \neg R)$

$\Leftrightarrow [(P \vee Q) \wedge \neg(\neg P \wedge \neg(Q \wedge R))] \vee (\neg P \wedge (\neg Q \vee \neg R))$

[Demorgan's & Distributive Law]

$\Leftrightarrow [(P \vee Q) \wedge \neg\neg(P \vee (Q \wedge R))] \vee (\neg P \wedge \neg(Q \wedge R))$

[Demorgan's law]

$\Leftrightarrow [(P \vee Q) \wedge (P \vee (Q \wedge R))] \vee \neg(P \vee (Q \wedge R))$

[Involution & Demorgan's Law]

$\Leftrightarrow [(P \vee Q) \wedge (P \vee Q) \wedge (P \vee R)] \vee \neg(P \vee (Q \wedge R))$

[$\because$ By Distributive Law]

$\Leftrightarrow [(P \vee Q) \wedge (P \vee R)] \vee \neg(P \vee (Q \wedge R))$

[$\because$ by Idempotent Law]

$\Leftrightarrow [P \vee (Q \wedge R)] \vee \neg(P \vee (Q \wedge R))$

[$\because P \vee \neg P \Leftrightarrow T$]

$\Leftrightarrow T$

3. Show that the following equivalence without constructing truth table

i) $\neg(P \Leftrightarrow Q) \Leftrightarrow (P \wedge \neg Q) \vee (\neg P \wedge Q)$

iii) $P \to (Q \to P) \Leftrightarrow \neg P \to (P \to Q)$

ii) $(P \to Q) \wedge (R \to Q) \Leftrightarrow (P \vee R) \to Q$

iv) $P \to (Q \to R) \Leftrightarrow (P \to Q) \vee (P \to R)$

v) $\neg(P \Leftrightarrow Q) \Leftrightarrow (P \vee Q) \wedge \neg(P \wedge Q)$

vi) $\neg(P \to Q) \Leftrightarrow P \wedge \neg Q$

1. $(P \Leftrightarrow Q) \Leftrightarrow (P \to Q) \wedge (Q \to P)$

P	Q	$P \Leftrightarrow Q$	$P \to Q$	$Q \to P$	$(P \to Q) \wedge (Q \to P)$
T	T	T	T	T	T
T	F	F	F	T	F
F	T	F	T	F	F
F	F	T	T	T	T

$\therefore (P \Leftrightarrow Q) \Leftrightarrow (P \to Q) \wedge (Q \to P)$ is Equivalent

2. $(P \Leftrightarrow Q) \Leftrightarrow (P \wedge Q) \vee (\neg P \cap \neg Q)$

P	Q	$\neg P$	$\neg Q$	$P \Leftrightarrow Q$	$P \wedge Q$	$\neg P \wedge \neg Q$	$(P \wedge Q) \vee (\neg P \wedge \neg Q)$
T	T	F	F	T	T	F	T
T	F	F	T	F	F	F	F
F	T	T	F	F	F	F	F
F	F	T	T	T	F	T	T

$\therefore (P \Leftrightarrow Q) \Leftrightarrow (P \to Q)$

3. $(P \to Q) \Leftrightarrow (\neg Q \to \neg P)$

P	Q	$P \to Q$	$\neg P$	$\neg Q$	$\neg Q \to \neg P$
T	T	T	F	F	T
T	F	F	F	T	F
F	T	T	T	F	T
F	F	T	T	T	T

4. $(P \to (Q \to P)) \Leftrightarrow (\neg P \to (P \to Q))$

P	Q	$\neg P$	$Q \to P$	$P \to (Q \to P)$	$P \to Q$	$\neg P \to (P \to Q)$
T	T	F	T	T	T	T
T	F	F	T	T	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T

5. $P \to (Q \vee R) \Leftrightarrow (P \to Q) \vee (P \to R)$

P	Q	R	$Q \vee R$	$P \to (Q \vee R)$	$P \to Q$	$P \to R$	$(P \to Q) \vee (P \to R)$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T
T	F	T	T	T	F	T	T
T	F	F	F	F	F	F	F
F	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T
F	F	T	T	T	T	T	T
F	F	F	F	T	T	T	T

DUALITY LAW

Two formulas A and A^* are said to be duals of each other if either one can be obtained from the other by replacing $\wedge$ by $\vee$ and $\vee$ by $\wedge$ are called duals of other. If the formula A contains T(or) F then A^* is obtained by replacing T by F and F by T

Problems:

1. Find the dual of $P \vee (Q \wedge R) \wedge F$

Soln:

Given A: $P \vee (Q \wedge R) \wedge F$

Dual A^* : $P \wedge (Q \vee R) \vee T$

2. Find the dual of $\neg(P \vee Q) \wedge (P \vee \neg(Q \wedge \neg S)) \wedge T$

Given A : $\neg(P \vee Q) \wedge (P \vee \neg(Q \wedge \neg S)) \wedge T$

Dual A^* : $\neg(P \wedge Q) \vee (P \wedge \neg(Q \vee \neg S)) \vee F$

3. Find the dual of $(P \vee Q) \wedge R$

Given A : $(P \vee Q) \wedge R$

Dual A^* : $(P \wedge Q) \vee R$

4. Find the dual of $(P \wedge Q) \vee T$

Given A : $(P \wedge Q) \vee T$

Dual A^* : $(P \vee Q) \wedge F$

5. Show that :

a) $\neg(P \wedge Q) \rightarrow (\neg P \vee (\neg P \vee Q)) \Leftrightarrow (\neg P \vee Q)$

b) $(P \vee Q) \wedge (\neg P \wedge (\neg P \wedge Q)) \Leftrightarrow (\neg P \wedge Q)$

Soln a) Consider

$\neg(P \wedge Q) \rightarrow (\neg P \vee (\neg P \vee Q))$

$\Leftrightarrow (P \wedge Q) \vee (\neg P \vee (\neg P \vee Q))$ — ①

[$\because P \rightarrow Q \Leftrightarrow \neg P \vee Q$]

$\Leftrightarrow (P \wedge Q) \vee ((\neg P \vee \neg P) \vee Q)$ [Associative Law]

$\Leftrightarrow (P \wedge Q) \vee (\neg P \vee Q)$ [Idempotent Law]

$\Leftrightarrow ((P \wedge Q) \vee \neg P) \vee Q$ [Associative Law]

$\Leftrightarrow ((P \vee \neg P) \wedge (Q \vee \neg P)) \vee Q$ [Distributive Law]

$\Leftrightarrow (T \wedge (Q \vee \neg P)) \vee Q$ [$P \vee \neg P \Leftrightarrow T$]

$\Leftrightarrow (Q \vee \neg P) \vee Q$ [$T \wedge P \Leftrightarrow P$]

$\Leftrightarrow ((\neg P \vee Q) \vee Q)$ [Commutative Law]

$\Leftrightarrow (\neg P \vee (Q \vee Q))$ [Associative Law]

$\Leftrightarrow (\neg P \vee Q)$ [Idempotent Law]

$\therefore \neg(P \wedge Q) \rightarrow (\neg P \vee (\neg P \vee Q)) \Leftrightarrow (\neg P \vee Q)$

b) From eqn ①

A : $(P \wedge Q) \vee (\neg P \vee (\neg P \vee Q)) \Leftrightarrow (\neg P \vee Q)$

Apply dual to A

A^* : $(P \vee Q) \wedge (\neg P \wedge (\neg P \wedge Q)) \Leftrightarrow (\neg P \wedge Q)$

(Duality Law)

TAUTOLOGICAL IMPLICATIONS:

A statement 'A' is said to be tautologically imply a statement B iff $A \to B$ is a tautology

ie) $A \Rightarrow B$ iff $A \to B$ is T

NOTE:

1. For any statement Formula $P \to Q$ the Statement formula $Q \to P$ is called its converse
2. $\neg P \to \neg Q$ is called its inverse
3. $\neg Q \to \neg P$ is called its contrapositive

Problems:

1. Show that $P \wedge (P \to Q) \Rightarrow Q$

Soln:

To prove $P \wedge (P \to Q) \Rightarrow Q$

It is enough to prove

$(P \wedge (P \to Q)) \to Q$ is T.

P	Q	$P \to Q$	$P \wedge (P \to Q)$	$(P \wedge (P \to Q)) \to Q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

2. Prove that : $\neg Q \wedge (P \to Q) \Rightarrow \neg P$
3. Prove that: $(P \to Q) \wedge (Q \to R) \Rightarrow (P \to R)$
4. $(P \wedge Q) \Rightarrow (P \to Q)$
5. $(P \to (Q \to R)) \Rightarrow (P \to Q) \to (P \to R)$
6. $[(P \to Q) \Rightarrow (P \to (P \wedge Q))]$
7. $(P \to Q) \to Q \Rightarrow (P \vee Q)$

} Without using truth table.

IMPLICATIONS:

1. $P \wedge Q \Rightarrow P$
2. $P \wedge Q \Rightarrow Q$
3. $P \Rightarrow P \vee Q$
4. $\neg P \Rightarrow P \to Q$
5. $Q \Rightarrow P \to Q$
6. $\neg(P \to Q) \Rightarrow P$
7. $\neg(P \to Q) \Rightarrow \neg Q$
8. $P \wedge (P \to Q) \Rightarrow Q$
9. $\neg Q \wedge (P \to Q) \Rightarrow \neg P$
10. $\neg P \wedge (P \vee Q) \Rightarrow Q$
11. $(P \to Q) \wedge (Q \to R) \Rightarrow (P \to R)$
12. $(P \to Q) \wedge (P \to R) \wedge (Q \to R) \Rightarrow R$

2. Prove that $\neg Q \wedge (P \to Q) \Rightarrow \neg P$ It is enough to prove $\neg Q \wedge (P \to Q) \to \neg P$

P	Q	$\neg Q$	$\neg P$	$P \to Q$	$\neg Q \wedge (P \to Q)$	$\neg Q \wedge (P \to Q) \to \neg P$
T	T	F	F	T	F	~~F~~ T
T	F	T	F	F	F	T
F	T	F	T	T	F	T
F	F	T	T	T	T	T

Hence it is proved $\neg Q \wedge (P \to Q) \to \neg P$ is Tautology

3. Prove that $(P \to Q) \wedge (Q \to R) \Rightarrow (P \to R)$

Soln: To prove $(P \to Q) \wedge (Q \to R) \Rightarrow (P \to R)$

Here is enough to prove $(P \to Q) \wedge (Q \to R) \to (P \to R)$