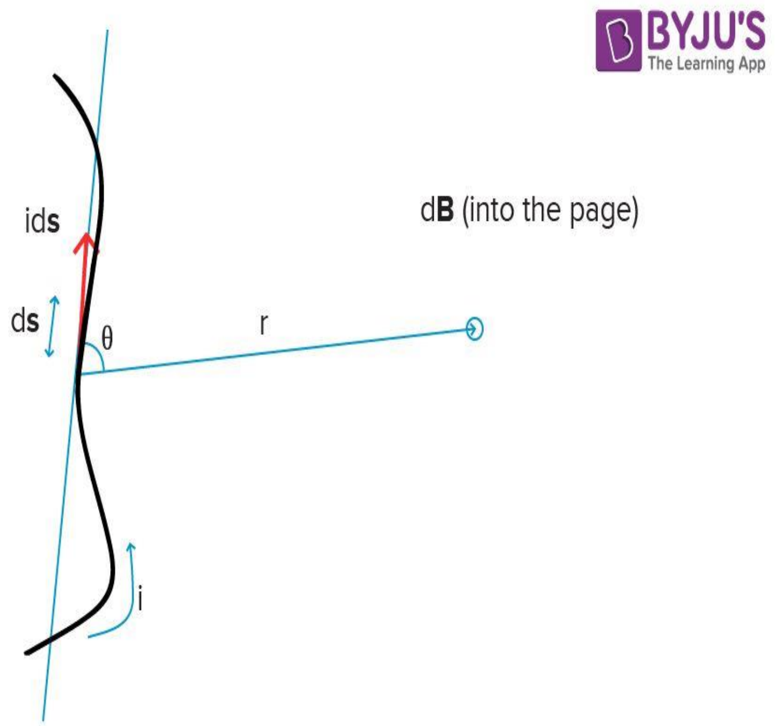
BIOT SAVART LAW

- **Biot–Savart law** is an equation describing the magnetic field generated by a constant electric current. It relates the magnetic field to the magnitude, direction, length, and proximity of the electric current.
- The **Biot-Savart law** states how the value of the magnetic field at a specific point in space from one short segment of current-carrying conductor depends on each factor that influences the field.

Biot-Savart’s law is an equation that gives the magnetic field produced due to a current carrying segment.
This segment is taken as a vector quantity known as the current element



Consider a current carrying wire ‘i’ in a specific direction
as shown in the above figure.
Take a small element of the wire of length ds
. The direction of this element is along that of the current
so that it forms a vector i ds.
To know the magnetic field produced at a point
due to this small element, one can apply Biot-Savart’s Law
. Let the position vector of the point in question drawn from
the current element be **r** and the angle between the two be θ. Then,

$$|dB|=(\mu 0/4\pi)(Idl\sin\Theta/r2)$$

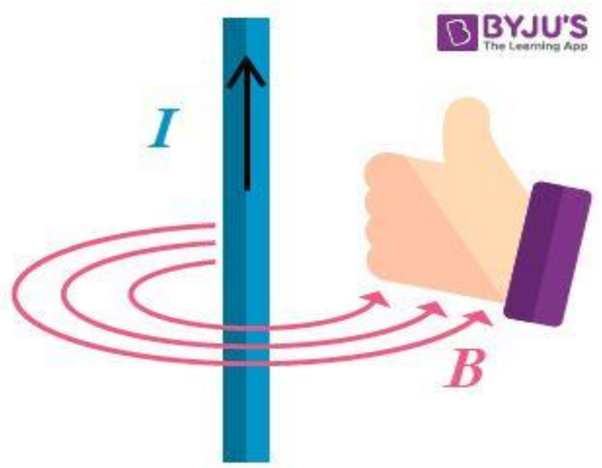
Where

• μ_0 is the permeability of free space and is equal to $4\pi \times 10^{-7}$ H/m

The direction of the magnetic field is always in a plane perpendicular to the line of element and position vector.

It is given by the right-hand thumb rule

where the thumb points to the direction of conventional current and the other fingers show the magnetic field’s direction.



Applications of Biot-Savart’s Law

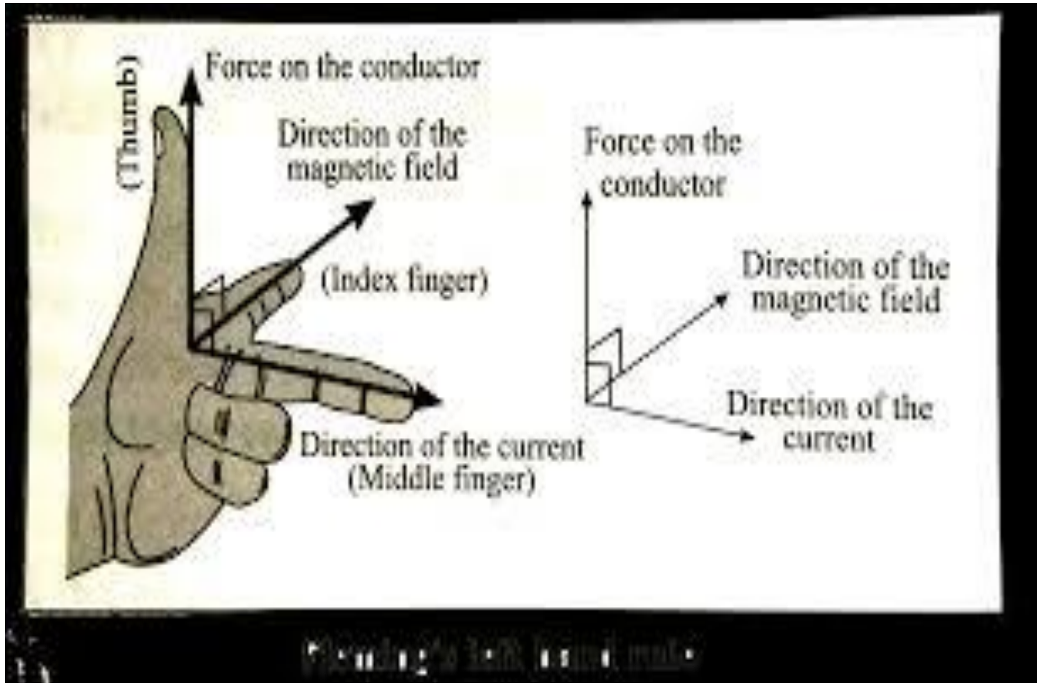
Some of Biot-Savart’s Law applications are given below.

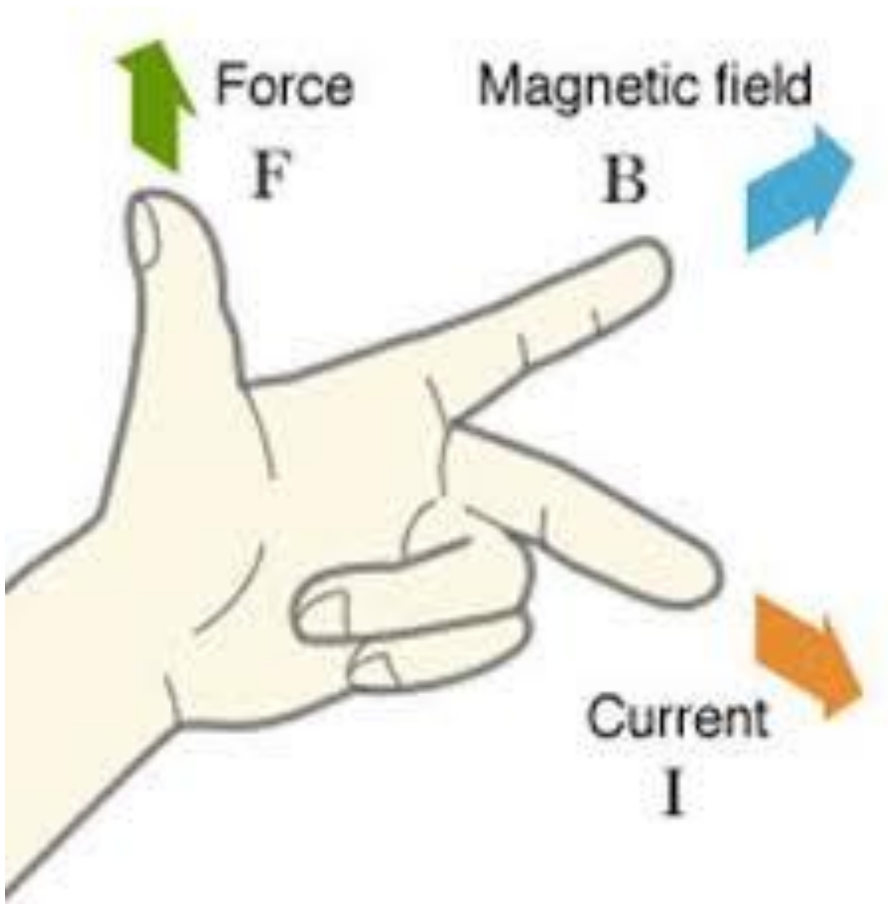
•We can use Biot–Savart law to calculate magnetic responses even at the atomic or molecular level.

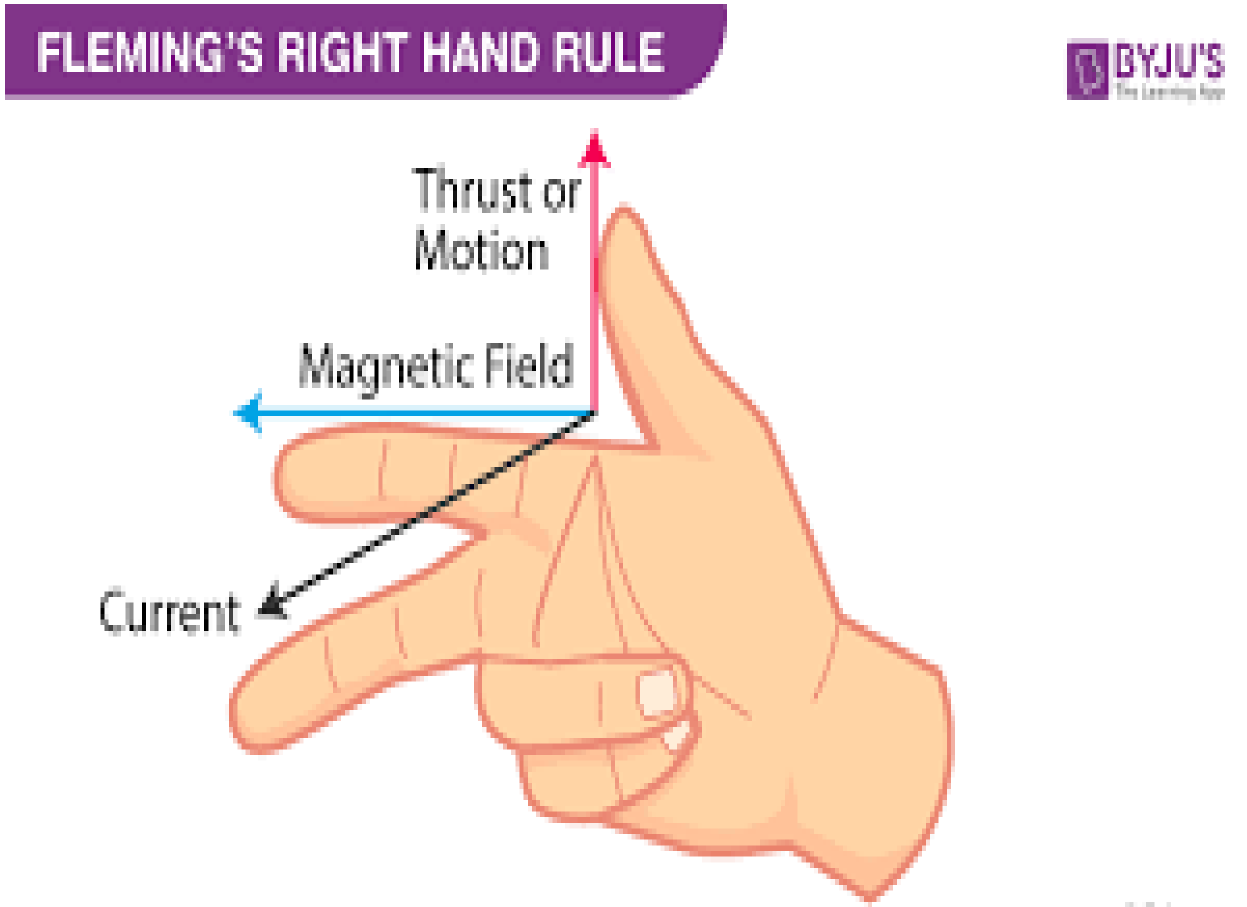
•It is also used in aerodynamic theory to calculate the velocity induced by vortex lines

FLEMINGS LEFT HAND RULE

Fleming's Left Hand Rule. Whenever a current carrying conductor is placed in a magnetic field, it experiences a force due to the magnetic field. ... The middle finger points in the direction of the current. The thumb gives the direction of force or motion acting on the conductor.




Force
F
Magnetic field
B
Current
I


FLEMING'S RIGHT HAND RULE
BYJU'S
Thrust or
Motion
Magnetic Field
Current
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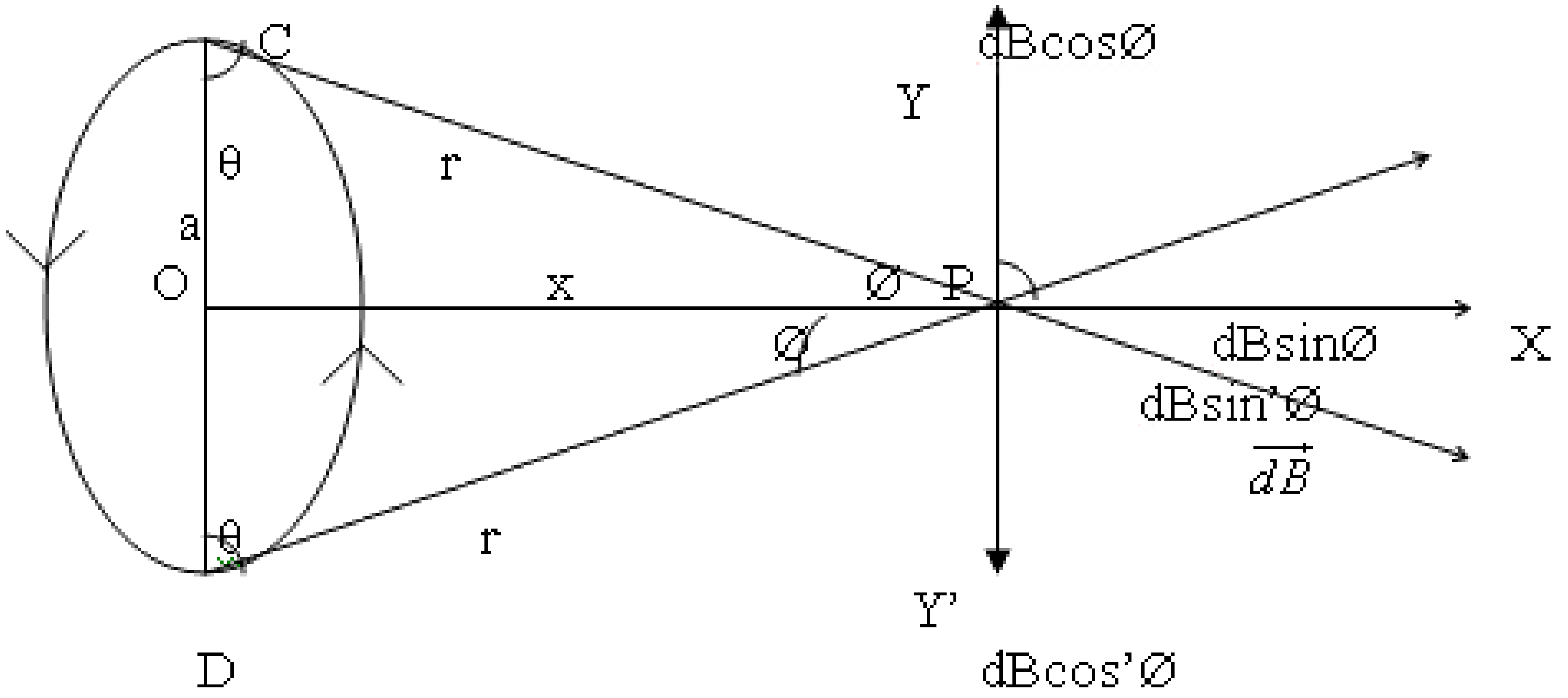
Fleming's right-hand rule (for generators) shows the direction of induced current when a conductor attached to a circuit moves in a magnetic field. It can be used to determine the direction of current in a generator's windings. When a conductor such as a wire attached to a circuit moves through a magnetic field, an electric current is induced in the wire due to Faraday's law of induction. The current in the wire can have two possible directions. Fleming's right-hand rule gives which direction the current flows.

The right hand is held with the thumb, index finger and middle finger mutually perpendicular to each other (at right angles), as shown in the diagram.
The thumb is pointed in the direction of the **m**otion of the conductor relative to the magnetic field.
•The **f**irst finger is pointed in the direction of the magnetic **f**ield. (north to south)
•Then the se**c**ond finger represents the direction of the induced or generated **c**urrent within the conductor (from + to -, the terminal with lower electric potential to the terminal with higher electric potential.

The magnetic field induction at a point on the axis of a circular coil carrying current.



Consider a circular coil of radius (a) with centre O.
Let current I be flowing in the coil.
Suppose P is any point on the axis of the circular coil at s distance x from its centre O.
Consider two small elements of the coil each of length dl at C and D which are situated at diametrically opposite edges as shown in figure b.
As per the figure b, PC = PD = $\sqrt{a^2 + x^2}$
CPO = Ø = DPO
According to Biot Savart''s law, the magnetic field induction at P due to current element at C is

$dB = \frac{\mu_0}{4\pi} \frac{Idl \sin 90^\circ}{r^2}$ (Because θ = 90° , as a is small)

$$B = \int dB \sin\phi = \int \frac{\mu_0}{4\pi} \frac{Idl \sin\phi}{a^2 + x^2}$$

$$B = \frac{\mu_0}{4\pi} \frac{I \sin\phi}{a^2 + x^2} \int dl$$

$$\int dl = 2\pi a \qquad \sin\phi = \frac{a}{\sqrt{a^2 + x^2}}$$

$$B = \frac{\mu_0}{4\pi} \frac{I}{(a^2 + x^2)} \frac{a}{\sqrt{(a^2 + x^2)}} \times 2\pi a$$

$$\int dl = 2\pi n a \qquad B = \frac{\mu_0}{4\pi} \frac{2\pi I a^2}{(a^2 + x^2)^{3/2}} = \frac{\mu_0 I a^2}{2(a^2 + x^2)^{3/2}}$$

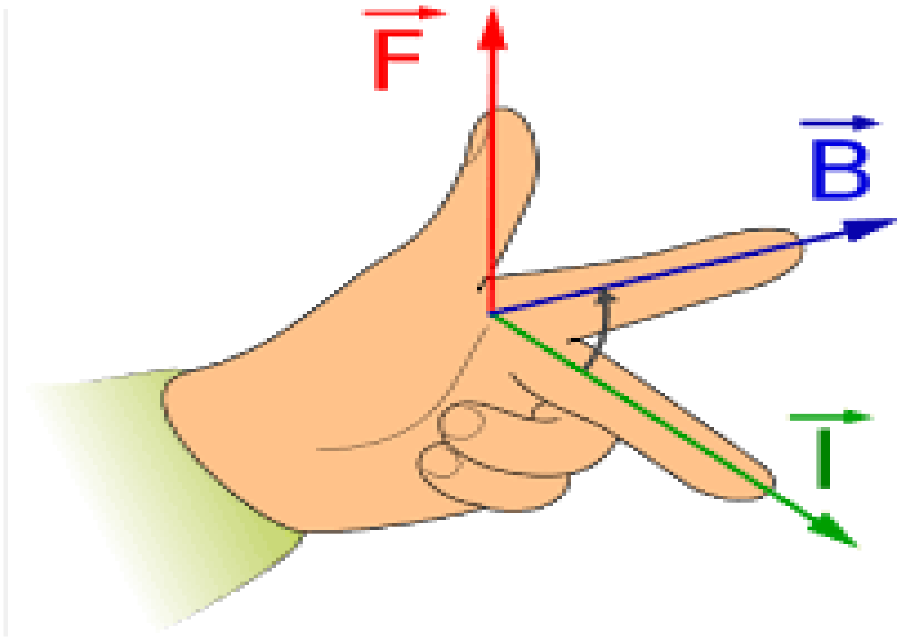
$$B = \frac{\mu_0}{4\pi} \frac{2\pi n I a^2}{(a^2 + x^2)^{3/2}} = \frac{\mu_0 n I a^2}{2(a^2 + x^2)^{3/2}}$$

Similarly, the magnitude
of magnetic field at P due to current element at D
dB = dB"

Resolving and into two rectangular components:
i. dBcosØ acts along PY and dBsinØ acts along PX.
ii. dB"cosØ acts along PY" and dB"sinØ acts along PX.
Since the component of the magnetic field along PY and PY" are equal and opposite,
they cancel each other and the components of the magnetic field along PX and PX" (along the axis of the coil)
being in the same direction are added up. Therefore the total magnetic field induction at P due to current through the whole circular coil is given by

Force on a Current-Carrying Conductor

Because charges ordinarily cannot escape a conductor,
the magnetic force on charges
moving in a conductor is transmitted to the conductor itself.



We can derive an expression for the magnetic force on a current by taking a sum of the magnetic forces on individual charges. (The forces add because they are in the same direction.)

The force on an individual charge moving at the drift velocity v_d is given by F = $qv_dB \sin \vartheta$.

Taking B to be uniform over a length of wire l and zero elsewhere, the total magnetic force on the wire is then F = $(qv_dB \sin \vartheta)(N)$, where N is the number of charge carriers in the section of wire of length l.

Now, $N = nV$, where n is the number of charge carriers per unit volume and V is the volume of wire in the field.

Noting that $V = Al$, where A is the cross-sectional area of the wire, then the force on the wire is F = $(qv_dB \sin \vartheta)$ (nAl). Gathering terms,

F=(nqAvd)lBsinθ

F=(nqAvd)lBsinθ.

Because *nqAv*$_d$ = *I* (see Current)

F=IlBsinθ

The magnetic force on current-carrying conductors is given by **F=IlBsinθ**

F=IlBsinθ

where *I* is the current, *l* is the length of a straight conductor in a uniform magnetic field *B*, and *ϑ* is the angle between *I* and *B*.

- We have learned about the existence of a magnetic field due to a current carrying conductor and the Biot – Savart’s law.
- We have also learned that an external magnetic field exerts a force on a current-carrying conductor and the Lorentz force formula that governs this principle.
- Thus, from the two studies, we can say that any two current carrying conductors when placed near each other, will exert a magnetic force on each other.

FORCE BETWEEN TWO PARALLEL CURRENT CARRYING CONDUCTORS

- Consider the system shown in the figure above. Here, we have two parallel Current carrying conductor, separated by a distance ‘d’, such that one of the conductors is carrying a current I_1 and the other is carrying I_2, as shown in the figure.
- we can say that the conductor 2 experiences the same magnetic field at every point along its length due to the conductor 1. The direction of magnetic force is indicated in the figure and is found using the right-hand thumb rule.
- The direction of magnetic field, as we can see, is downwards due to the first conductor.

Similarly, we can calculate the force exerted by the conductor 2
on the conductor 1.
We see that, the conductor 1experiences the same force due to the conductor 2
but the direction is opposite.
F12 = F21
We also observe that, the currents flowing in the same direction
make the conductors attract each other and that showing in the opposite direction
makes the conductors repel each other.
The magnitude of force acting per unit length can be given as,

$$\Delta F = B_1 I_2 \Delta L \sin 90^\circ$$

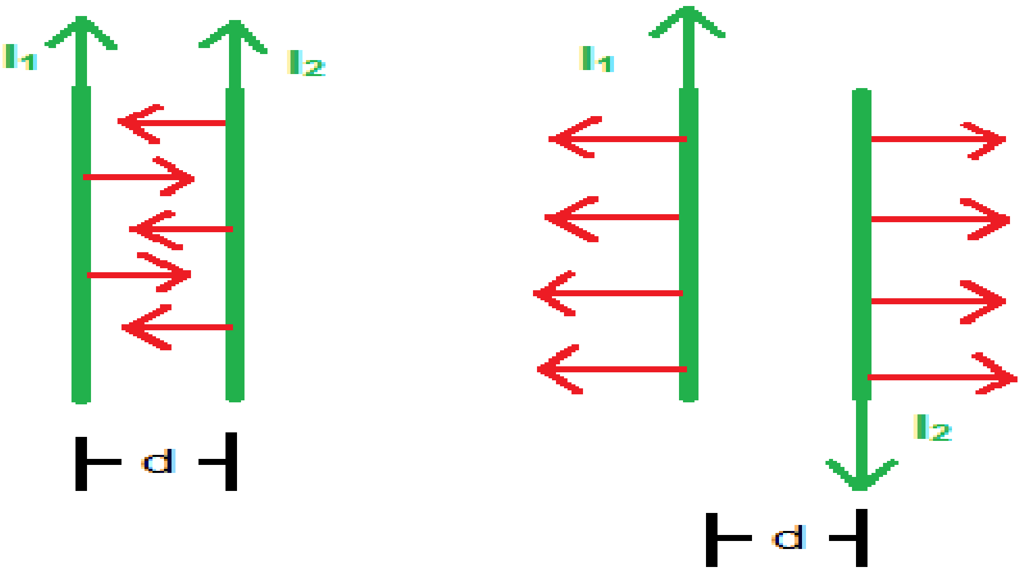
$$\therefore \quad = \frac{\mu_0 I_1}{2\pi r} I_2 \Delta L$$

$\therefore$ The total force on conductor of length L will be

$$F = \frac{\mu_0 I_1 I_2}{2\pi r} \Sigma \Delta L = \frac{\mu_0 I_1 I_2}{2\pi r} L$$

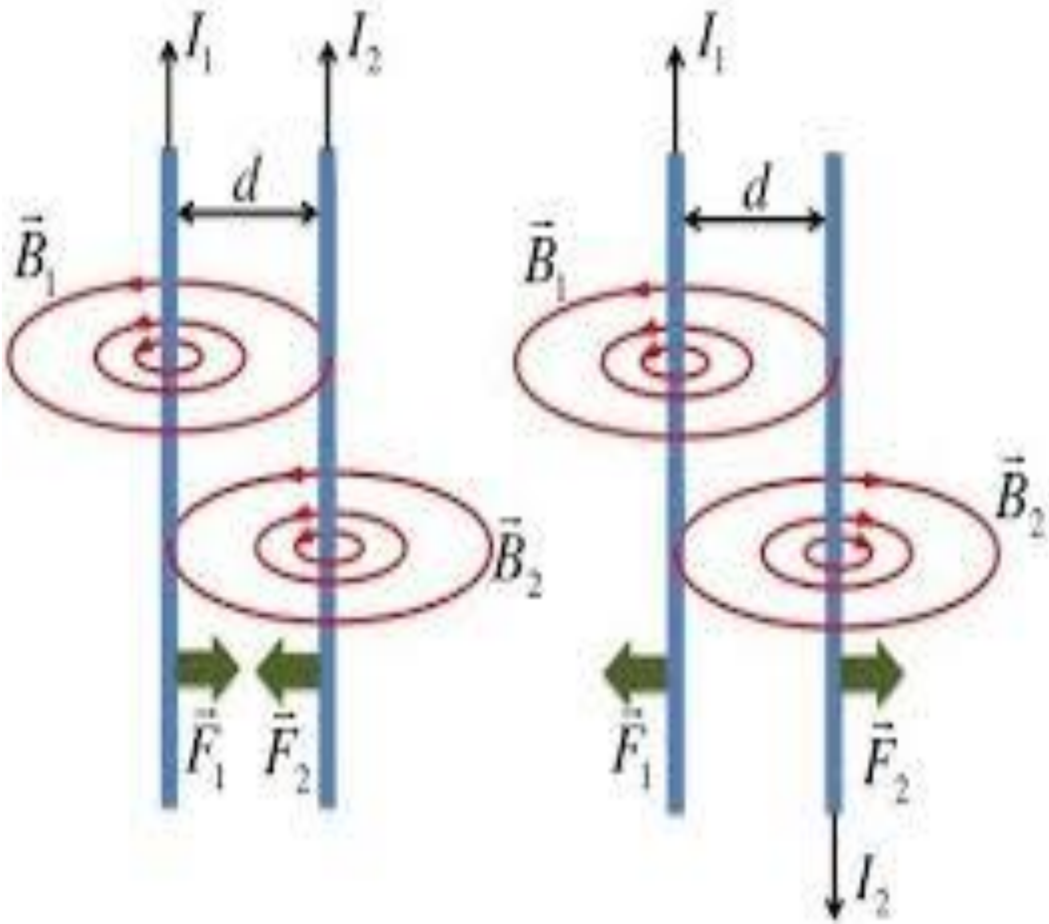
$\therefore$ Force acting on per unit length of conductor

$$f = \frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi r} \text{ N/m} \quad \ldots(2)$$

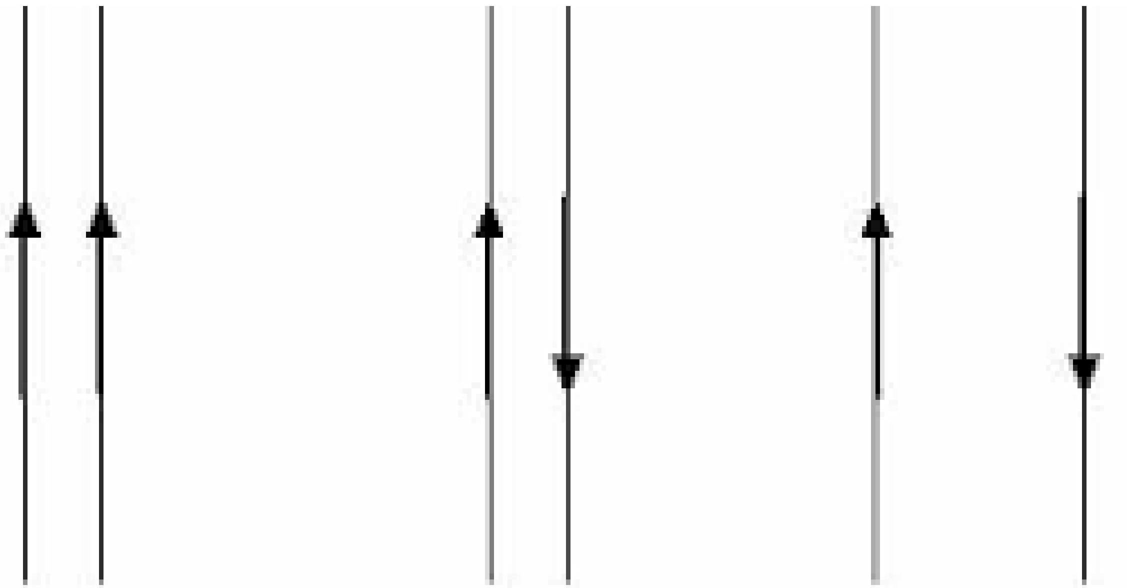


$$\boxed{F_{parallel\,wires} = \frac{\mu_0 l I_1 I_2}{2\pi d}}$$

$\mu_0 = 1.26 \times 10^{-6}\ T\ m/A$



I_1
I_2
d
$\vec{B}_1$
$\vec{B}_2$
$\vec{F}_1$
$\vec{F}_2$
I_1
d
$\vec{B}_1$
$\vec{B}_2$
$\vec{F}_1$
$\vec{F}_2$
I_2



Currents in same direction: wires attracted to each other

Currents in opposite direction: wires repelled by each other

Greater distance d means less force F

ge is moving
velocity, *v* in
netic field, *B*
t will
ience a
etic force

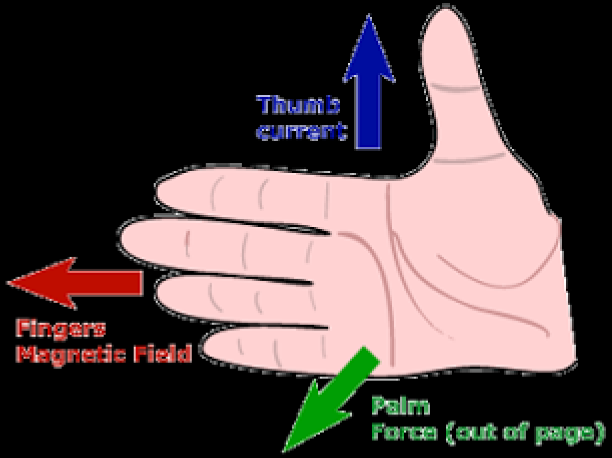
$$\vec{F} = q\left(\vec{v} \times \vec{B}\right)$$

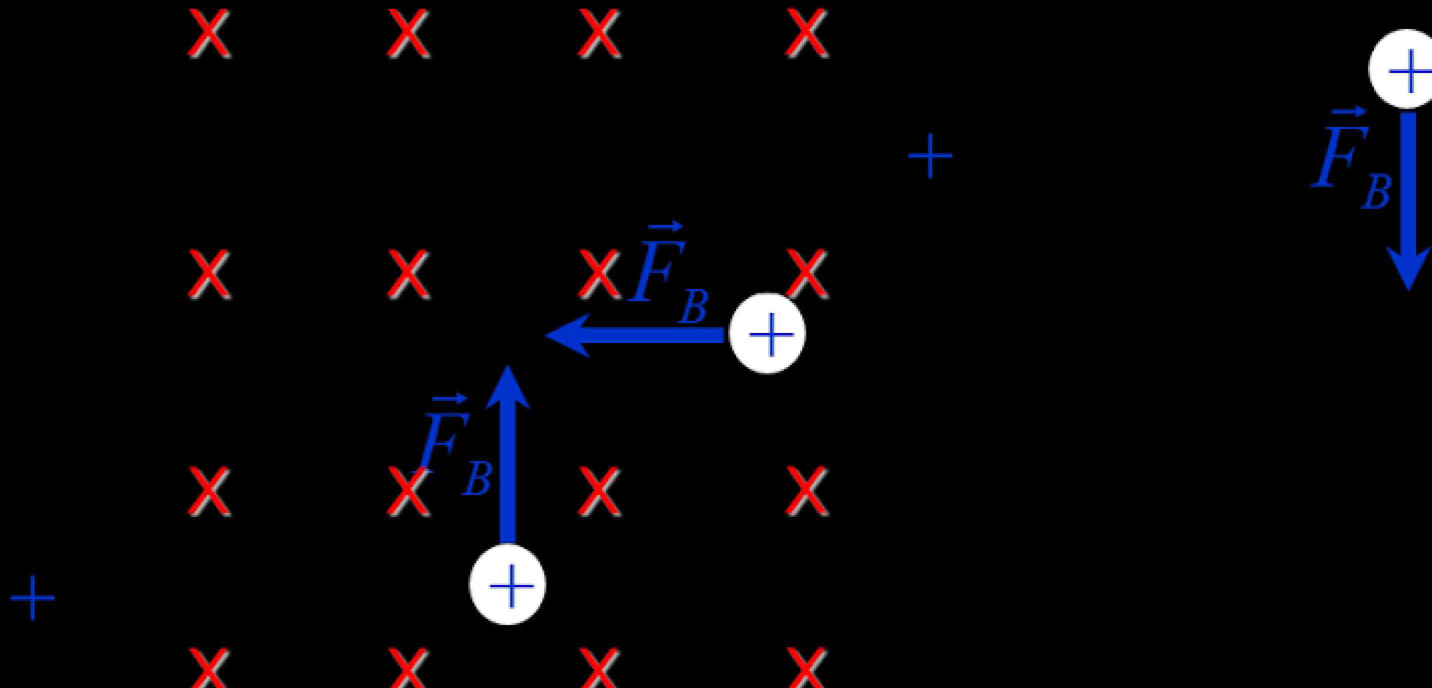
$$F = qvB\sin\theta$$

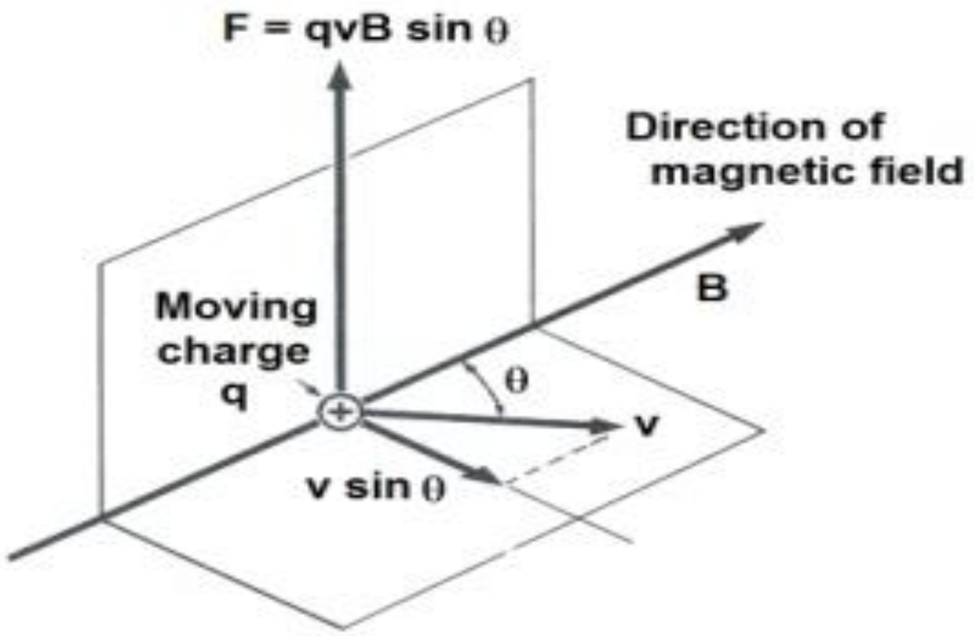
$$r = \frac{mv}{Bq}$$

$$T = \frac{2\pi m}{Bq}$$

direction of
be
rmine by
g **right**
d rule







The magnetic force F acting on a charge q moving with velocity v

$$\overline{F} = q\overline{v} \times \overline{B}$$

Fig. 1

MOVING COIL BALLISTIC GALVANOMETER

- A galvanometer is a device that is used to detect small electric current or measure its magnitude. The current and its intensity is usually indicated by a magnetic needle’s movement or that of a coil in a magnetic field that is an important part of a galvanometer.
- A moving coil galvanometer is an instrument which is used to measure electric currents.
- It is a sensitive electromagnetic device which can measure low currents even of the order of a few microamperes.

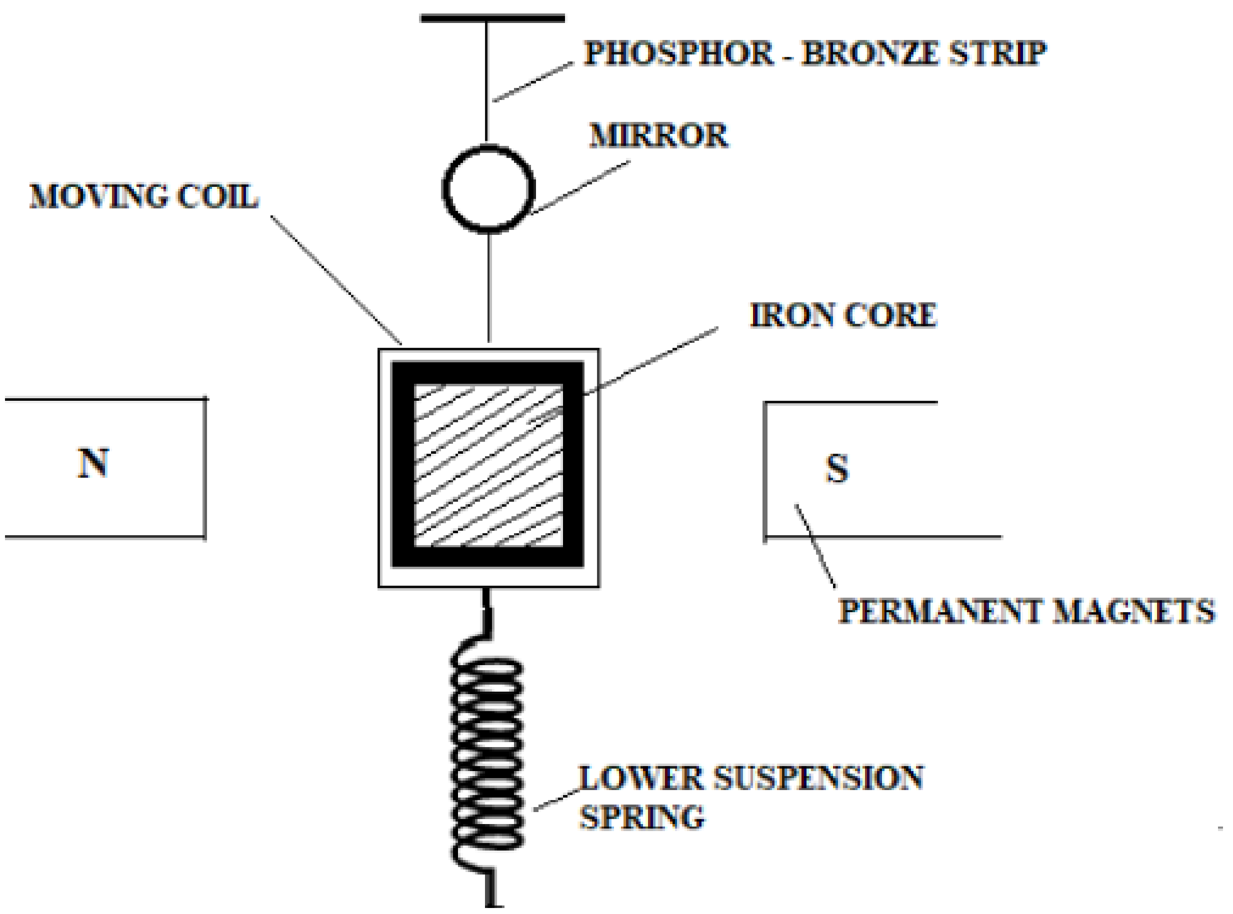
Moving Coil Galvanometer Principle

A current-carrying coil when placed in an external magnetic field experiences magnetic torque.

The angle through which the coil is deflected due to the effect of the magnetic torque is proportional to the magnitude of current in the coil.

Principle:

A rectangular coil suspended in a magnetic field experiences a torque when a current flows through it is the basic principle of a moving coil galvanometer. The BG is used to measure electric charge so that the current is always momentary. This produces only an impulse on the coil and a throw is registered. The oscillations of the coil are practically undamped and the period of oscillation is fairly large.


PHOSPHOR - BRONZE STRIP
MIRROR
MOVING COIL
IRON CORE
N
S
PERMANENT MAGNETS
LOWER SUSPENSION
SPRING

Theory of moving coil ballistic galvanometer

Moving coil ballistic galvanometer consists of a rectangular coil of thin copper wire wound on a non-metallic frame of ivory.
It is suspended by means of a phosphor bronze wire between the pole pieces of a powerful horse-shoe magnet.
A small circular mirror is attached to the suspension wire.
The lower end of the rectangular coil is connected to a hair-spring.
The upper end of the suspension wire and the lower end of the spring are connected to the terminals T1 and T2.
C is a cylindrical soft iron core placed inside the coil symmetrically with the grooves of the pole pieces.
This iron core concentrates the magnetic field and helps in producing the radial field.

Working of Moving Coil Galvanometer

Let a current I flow through the rectangular coil of n number of turns and a cross-sectional area A. When this coil is placed in a uniform radial magnetic field B, the coil experiences a torque τ.

Let us first consider a single turn ABCD of the rectangular coil having a length *l* and breadth ***b.*** This is suspended in a magnetic field of strength B such that the plane of the coil is parallel to the magnetic field.

Since the sides AB and DC are parallel to the direction of the magnetic field, they do not experience any effective force due to the magnetic field.

The sides AD and BC being perpendicular to the direction of field experience an effective force F given by $F = BIl$

Using **Fleming’s left-hand rule** we can determine that the forces on AD and BC are in opposite direction to each other. When equal and opposite forces F called couple acts on the coil, it produces a torque.

This **torque** causes the coil to deflect.

We know that torque τ = force x perpendicular distance between the forces

$\tau = F \times b$

Substituting the value of F we already know,

Torque τ acting on single-loop ABCD of the coil = $BIl \times b$

Where lx b is the area A of the coil,

Hence the torque acting on n turns of the coil is given by

$\tau = nIAB$

The magnetic torque thus produced causes the coil to rotate, and the phosphor bronze strip twists. In turn, the spring S attached to the coil produces a counter torque or restoring torque $k\theta$ which results in a steady angular deflection.
Under equilibrium condition:
$k\theta = nIAB$
Here k is called the torsional constant of the spring (restoring couple per unit twist).

The deflection or twist θ is measured as the value indicated on a scale by a pointer which is connected to the suspension wire.
$\theta = (nAB / k)I$
Therefore $\theta \propto I$
The quantity nAB / k is a constant for a given galvanometer. Hence it is understood that the deflection that occurs the galvanometer is directly proportional to the current that flows through it.

Figure of Merit of a Galvanometer

It is the ratio of the full-scale deflection current and the number of graduations on the scale of the instrument. It also the reciprocal of the current sensitivity of a galvanometer.

- **Current Sensitivity**

The deflection θ per unit current I is known as current sensitivity θ/I

$\theta/I = nAB/k$

- **Voltage Sensitivity**

The deflection θ per unit voltage is known as Voltage sensitivity θ/V. Dividing both sides by V in the equation $\theta = (nAB / k)I$;

$\theta/V = (nAB / V k)I = (nAB / k)(I/V) = (nAB / k)(1/R)$

The **differential form of Ampere's Circuital Law** the current density at any point in space is proportional to the spatial rate of change of the magnetic field and is perpendicular to the magnetic field at that point.

$\oint B.dl = \mu_0 \int_s j.dA$ ---------(3)

By divergence theorem

$\oint B.dl = curl \int_s B.dA$ ---------------(4)

Comparing equation (3) and (4) we get:

curl$\int_s B.dA$ =$\mu_0 \int_s j.dA$

curl B=$\mu_0 J$

$$\nabla \times B = \mu_0 J$$

A **solenoid** is a **long** coil of wire wrapped in many turns.

When a current passes through it, it creates a nearly uniform **magnetic field inside**. ...

The energy density of the **magnetic field** depends on the strength of the **field**, squared, and also upon the **magnetic** permeability of the material it fills.

Magnetic Field of a Solenoid

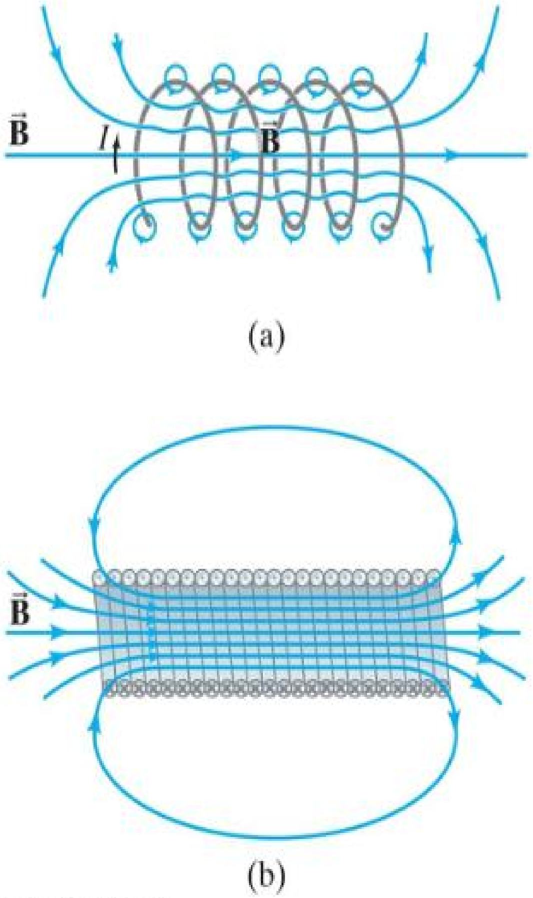
- A *solenoid* is a long coil of wire made many (N) loops, each producing a magnetic field

- Inside the solenoid, the magnetic field is parallel to the long axis

- Outside the solenoid, the magnetic field is zero

- The magnetic field on-axis is:

$$\boxed{B = \frac{\mu_0 NI}{l}}$$



MAGNETIC FIELD INSIDE A SOLENOID

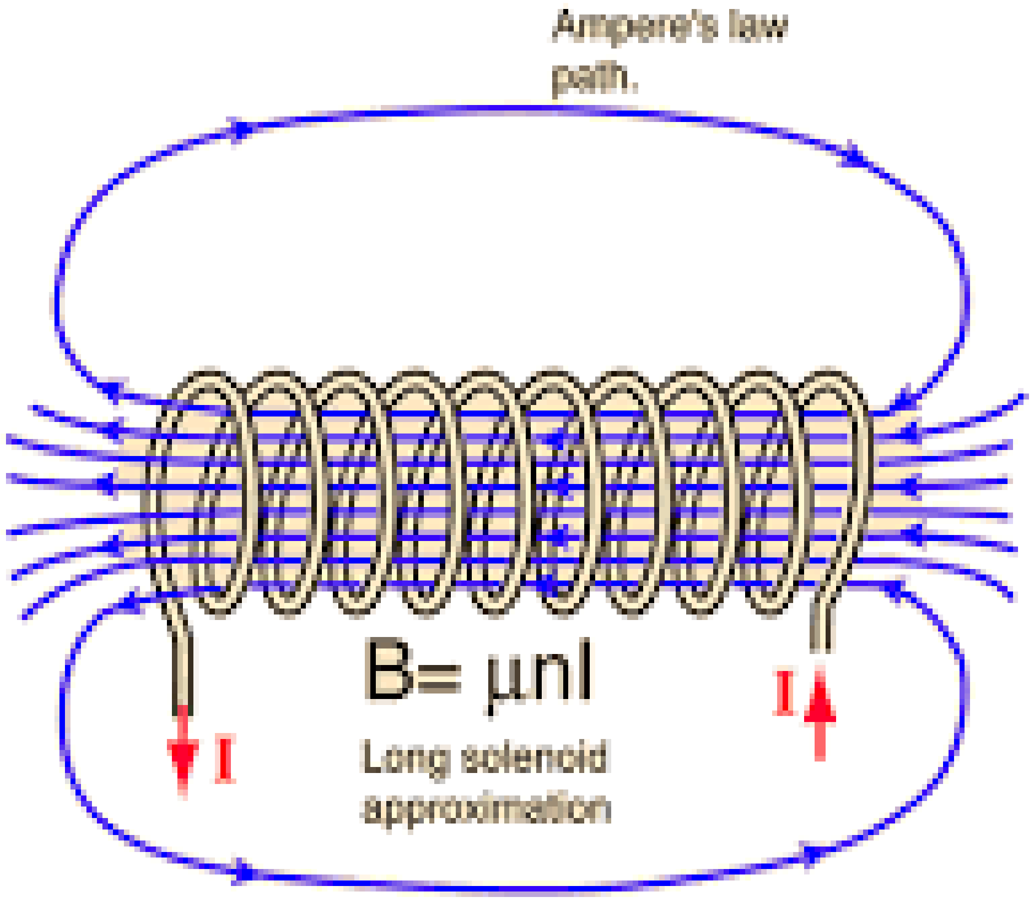
Magnetic field inside a long, straight solenoid

Number of windings per meter

$$B = \mu_0 n i$$

Permeability of free space

Current in the solenoid windings



The magnetic field is concentrated into a nearly uniform field in the center of a long solenoid. The field outside is weak and divergent.

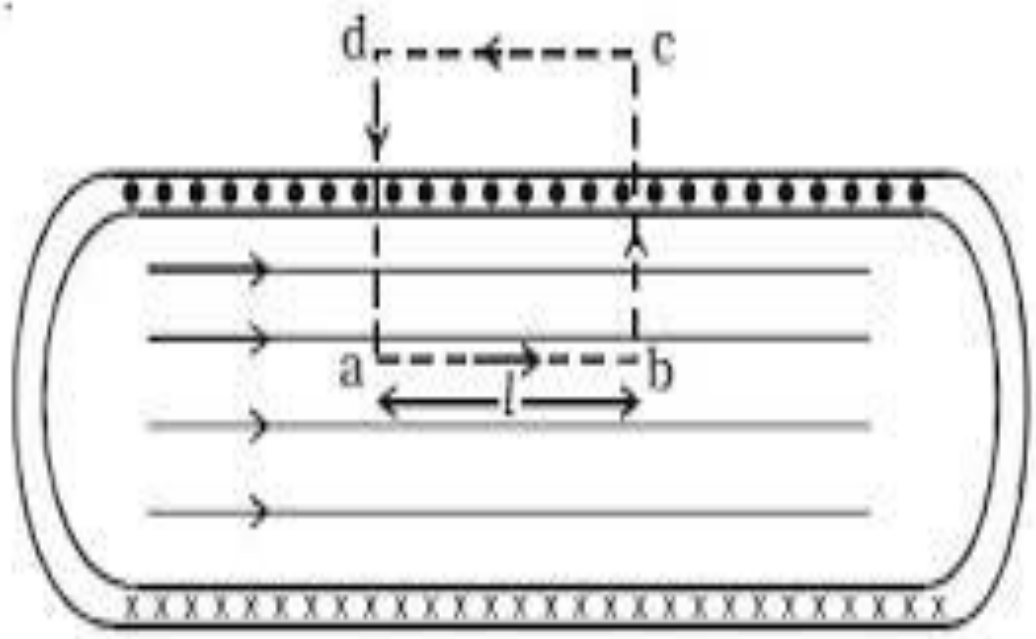


Fig 3.17 Magnetic field due to a long solenoid.

The line integral $\oint \vec{B}.\vec{dl}$ for the loop *abcd* is the sum of four integrals.

$$\therefore \oint \vec{B}.\vec{dl} = \int_a^b \vec{B}.\vec{dl} + \int_b^c \vec{B}.\vec{dl} + \int_c^d \vec{B}.\vec{dl} + \int_d^a \vec{B}.\vec{dl}$$

If l is the length of the loop, the first integral on the right side is Bl. The second and fourth integrals are equal to zero because $\vec{B}$ is at right angles for every element $\vec{dl}$ along the path. The third integral is zero since the magnetic field at points outside the solenoid is zero.

$$\therefore \quad \oint \vec{B}.\vec{dl} = Bl \qquad ...(1)$$

Magnetic Field Inside a Long Solenoid

$$\oint \vec{B}\cdot d\vec{s} = \int_1 \vec{B}\cdot d\vec{s} + \int_2 \vec{B}\cdot d\vec{s} + \int_3 \vec{B}\cdot d\vec{s} + \int_4 \vec{B}\cdot d\vec{s}$$

$$\oint \vec{B}\cdot d\vec{s} = B\ell + 0 + 0 + 0 = \mu_0 I_{enclosed}$$

-The total current through the rectangular path equals the current through each turn multiplied by the number of turns

$$B\ell = \mu_0 N I$$

$$\boxed{B = \mu_o \frac{N}{\ell} I = \mu_o n I}$$