9.

UNIT-I

PROJECTILES

Projectile:

Particle

Motion of a projectile projected into the air in any direction and with any velocity such a particle is called a projectile.

Angle of projection:

The angle of projection is the angle that the direction in which the particle is initially projected makes with the horizontal plane through the point of projection.

Velocity of projection:

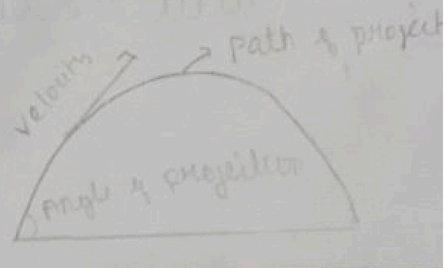
The velocity of projection is the velocity with which the particle is projected.

Trajectory:

The trajectory is the path which the particle describes.

Range:

The range on a plane through the point of projection is the distance between the point of projection and the point where the trajectory meets that plane.



Time of flight :

The time of flight is the interval of time that elapses from the instant of projection till the instant when the particle again meets the horizontal plane through the point of projection.

Two fundamental principles :

* Here we consider the horizontal and vertical component of the motion seperately.

* The only force acting on the projectile is gravity and this acts vertically downwards. Hence it has no effect on the horizontal motion of the particle so the horizontal velocity remain constant through out the motion.

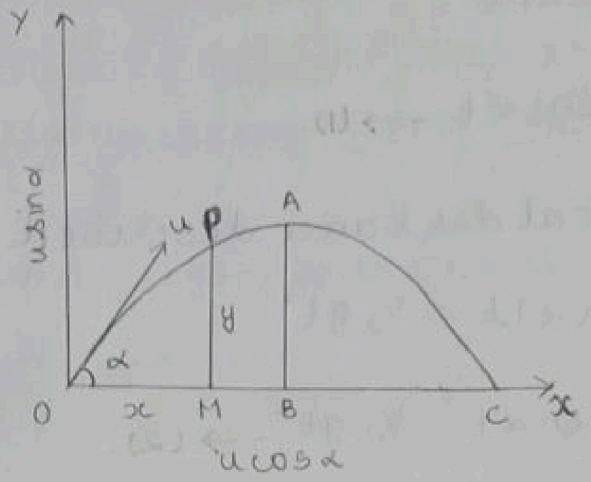
* The weight of the particle acting vertically downwards will have its full effect on the vertical motion of the particle.

The weight mg acting vertically downwards on a particle of mass m will produce an acceleration g vertically downwards. Hence, the vertical component of the velocity will be subject to a retardation g.

Book Work - 1 :

To show that the path of a projectile is a parabola.

Proof :



Let a particle be projected from O, with a velocity 'u' at an angle α to the horizon.

Take O as the origin.

The horizontal and upward vertical through O as x-axis and y-axis respectively.

The initial velocity 'u' can be split into two components which are u cos α and and u sin α which are in horizontal and vertical direction respectively.

The horizontal component u cos α is constant throughout the motion as there is no horizontal acceleration.

The vertical component u sin α is subject to an acceleration g downwards

Let $P(x, y)$ be the position of the particle at time 't' seconds after projection. Then,

x = horizontal distance described in 't' secs

$= (u\cos\alpha)t$

$\Rightarrow x = u\cos\alpha\, t \dashrightarrow (1).$

$s = ut + \frac{1}{2}at^2$

$y = (u\sin\alpha)t + \frac{1}{2}(-g)t^2$

y = vertical distance described in 't' secs

$= u\sin\alpha\, t - \frac{1}{2}gt^2$

$= (u\sin\alpha)t - \frac{1}{2}gt^2$

$\Rightarrow y = u\sin\alpha\, t - \frac{1}{2}gt^2 \dashrightarrow (2).$

From (1), $t = \dfrac{x}{u\cos\alpha} \dashrightarrow (3)$

Sub (3) in (2), we get,

$$y = \cancel{u}\sin\alpha\,\frac{x}{\cancel{u}\cdot\cos\alpha} - \frac{1}{2}\,g\cdot\frac{x^2}{u^2\cos^2\alpha}$$

$$y = x\tan\alpha - \frac{gx^2}{2u^2}\,\frac{1}{\cos^2\alpha}$$

$$\Rightarrow y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha} \dashrightarrow (4).$$ [formula of path]

$$\Rightarrow y = \frac{x\tan\alpha\cdot 2u^2\cdot\cos^2\alpha - gx^2}{2u^2\cos^2\alpha}$$

$$\Rightarrow y(2u^2\cdot\cos^2\alpha) = 2u^2\cos^2\alpha\; x\tan\alpha - gx^2$$

$$\Rightarrow y(2u^2\cdot\cos^2\alpha) = 2u^2\cos\alpha\cdot x\sin\alpha - gx^2$$

(× by −1)

$$\Rightarrow gx^2 - 2xu^2\sin\alpha\cos\alpha = -2yu^2\cos^2\alpha \quad (\div g)$$

$$\Rightarrow x^2 - \frac{2u^2\sin\alpha\cos\alpha}{g}\,x = -\frac{2u^2\cos^2\alpha}{g}\,y$$

Add $\frac{u^4\sin^2\alpha\cos^2\alpha}{g^2}$ on both sides

$$\Rightarrow \left(x - \frac{u^2\sin\alpha\cos\alpha}{g}\right)^2 = \frac{u^4\sin^2\alpha\cos^2\alpha}{g^2} - \frac{2u^2\cos^2\alpha}{g}\,y$$

$$\Rightarrow \left[x - \frac{u^2\sin\alpha\cos\alpha}{g}\right]^2 = -\frac{2u^2\cos^2\alpha}{g}\left[y - \frac{u^2\sin^2\alpha}{2g}\right]$$

Transfer the origin to the point

$(x-h)^2 = 4a(y-k)$

x^2 $(h,k) \rightarrow$

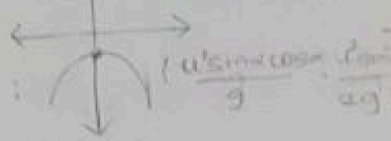
$$\left(\frac{u^2\sin\alpha\cos\alpha}{g}, \frac{u^2\sin^2\alpha}{2g}\right)$$

$x^2 = 4ay$

$x^2 = 4ay$ curve upwards

$x^2 = -4ay$ curve downwards

the above equation becomes,



$$x^2 = -\frac{2u^2\cos^2\alpha}{g}\,y \longrightarrow (5)$$

Eqn (5) is the eqn of parabola of latus rectum $\frac{2u^2\cos^2\alpha}{g}$ whose axis is vertical and downwards and whose vertex is the point $\left(\frac{u^2\sin\alpha\cos\alpha}{g}, \frac{u^2\sin^2\alpha}{2g}\right)$

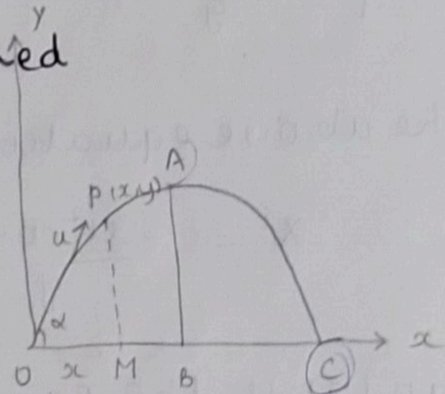
9. Characteristics of the motion of a projectile:

Let a particle be projected from O with velocity u at an angle α to the horizontal OX.

Let A be the highest point of the path and 'C' the point where it again meets the horizontal plane through 'O'.

Using the two-fundamental principles, we can derive the following results:

(i). **Greatest height attained by a projectile:**



At A, the highest point, the particle will be moving only horizontally, having lost all its vertical velocity.

Let AB = h

Initial upward vertical velocity = $u\sin\alpha$.

acceleration = $-g$

The final vertical velocity at A is = 0

Hence, $0 = (u\sin\alpha)^2 + 2(-g)(h)$ By eqn of moti[on] $v^2 = u^2 + 2as$

$0 = u^2\sin^2\alpha - 2gh$. Sub, we get.

$\Rightarrow 2gh = u^2\sin^2\alpha$

$$\Rightarrow h = \frac{u^2\sin^2\alpha}{2g}$$

(i). Time taken to reach the greatest height:

Let 'T' be the time from O to A.

Then, in time 'T', the initial vertical velocity is reduced to zero. (i.e)., $u \sin\alpha$ is reduced to '0')

and acceleration $= -g$.

Then, by eqn of motion is, $v = u + at$.

sub, $0 = u\sin\alpha - gT$

$$gT = u\sin\alpha$$

$$T = \frac{u\sin\alpha}{g}$$

(ii). Time of flight:

(i.e) is the time taken to return to the same horizontal level as 'O':-

When the particle arrives at 'O', the vertical distance it has described is zero.

If 't' is the time of flight, considering the vertical motion.

Then, by eqn of motion is, $s = ut + \frac{1}{2}at^2$.

sub, $0 = (u\sin\alpha)t - \frac{1}{2}gt^2$

$\Rightarrow t(u\sin\alpha - \frac{1}{2}gt) = 0$

$\Rightarrow t = 0$ or $u\sin\alpha - \frac{1}{2}gt = 0$

$\Rightarrow t = 0$ or $t = \frac{2u\sin\alpha}{g}$

t = 0 is the instant of projection when also the vertical distance travelled is zero.

$\therefore$ Time of flight $= \frac{2u\sin\alpha}{g}$

(iv). Range :

The range on the horizontal plane through the point of projection ~~is the~~ :

The time of flight in $t = \frac{2u\sin\alpha}{g}$

During this time, the horizontal velocity remains constant and it is equal to $u\cos\alpha$.

Hence OC = horizontal distance described in time 't'.

$$= u\cos\alpha . t$$

$$= u\cos\alpha . \frac{2u\sin\alpha}{g}$$

$$= \frac{2u^2\sin\alpha\cos\alpha}{g}$$

$$= \frac{u^2\sin2\alpha}{g}$$

$\therefore$ Horizontal range, $R = \frac{u^2\sin2\alpha}{g}$

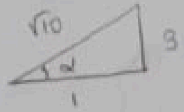
Eg-1: A body is projected with a velocity of 98 m's/sec in a direction making an angle $\tan^{-1}3$ with the horizon. Show that it rises to the vertical height of 441 m's and that it is time of flight is about 19 sec's. Find also the horizontal range through the point of projection. ($g = 9.8\ ms^{-2}$).

soln:

Given: $u = 98\ m/sec$.

$\alpha = \tan^{-1}3$.

$\Rightarrow \tan\alpha = 3/1$



$\sqrt{3^2+1^2} = \sqrt{10} = \sqrt{10}$.

$\Rightarrow \sin\alpha = \frac{3}{\sqrt{10}}$; $\cos\alpha = \frac{1}{\sqrt{10}}$

$g = 9.8\ m/sec^2$.

(i). Greatest height : $h = \frac{u^2\sin^2\alpha}{2g}$

sub, $h = \frac{(98)^2\left(\frac{3}{\sqrt{10}}\right)^2}{2(9.8)}$

$= \frac{98 \times 98 \times 3 \times 3}{10 \times 2 \times 9.8}$

$\therefore h = 441$ metres.

(ii). Time of flight, $t = \frac{2u\sin\alpha}{g}$

$\frac{6}{3.1622}$

sub, $t = \frac{2 \times 98 \times 3/\sqrt{10}}{9.8}$

$= 18.97$ sec

Range, $R = \frac{2u^2 \sin\alpha \cos\alpha}{g}$

$$= \frac{2 \times 98 \times 98 \times \frac{1}{\sqrt{10}} \times \frac{3}{\sqrt{10}}}{9.8}$$

$$= \frac{2 \times \cancel{98} 98 \times 98 \times 3}{\underset{5}{\cancel{10}} \times \underset{0.1}{\cancel{9.8}}}$$

$= 588\ m$

. A projectile is thrown with a velocity of $20 ms^{-1}$ at an elevation 30°. Find the greatest height at time and the horizontal range.

Soln:

Given, $u = 20 ms^{-1}$.

$\alpha = 30^\circ$

We know that,

greatest height, $h = \frac{u^2 \sin^2\alpha}{2g}$

$$= \frac{(\overset{100}{\cancel{400}})(\frac{1}{\cancel{4}})}{2 \times 9.8}$$

$$= \frac{100}{19.6}$$

$= 5.1020$

$h \simeq 5.1 m.$

Horizontal range, $R = \frac{2u^2 \sin\alpha \cos\alpha}{g}$

$$= \frac{2 \cdot \overset{100}{(400)} \cdot \frac{1}{2} \cdot \frac{\sqrt{3}}{2}}{9.8}$$

$$= \frac{200\sqrt{3}}{9.8}$$

$$= 35.346$$

$$\therefore R = 35.3\ m$$

2. If the greatest height obtained by the particle is a quater of its range on the horizontal plane through the point of projection. Find the angle of projection.

Soln:

Let 'u' be the initial velocity and

Let 'α' be the angle of projection.

We know that,

greatest height, $h = \frac{u^2 \sin^2\alpha}{2g}$

range, $R = \frac{2u^2 \sin\alpha \cos\alpha}{g}$

Given:

Greatest height $= \frac{1}{4} \times$ range

$$\frac{\cancel{u^2} \overset{\sin\alpha}{\cancel{\sin^2\alpha}}}{\cancel{2}\cancel{g}} = \frac{1}{\underset{2}{\cancel{4}}} \times \frac{2\cancel{u^2}\cancel{\sin\alpha}\cos\alpha}{\cancel{g}}$$

$$\sin\alpha = \cos\alpha$$

$$\Rightarrow \tan\alpha = 1$$

$\alpha = \tan^{-1} 1$

$\therefore \alpha = 45°$

$\therefore$ Angle of projection is $45°$.

4. A stone is thrown with a velocity $39.2\ ms^{-1}$ at $30°$ to the horizontal. Find at what times it will be at a height of $14.7m$ ($g = 9.8\ ms^{-2}$).

Soln: Given, $u = 39.2\ ms^{-1}$; $\alpha = 30°$.

Initial vertical velocity, $= u \sin\alpha$

$= (39.2) \sin 30$

$= 39.2 \times \frac{1}{2}$

$= 19.6\ ms^{-1}$.

$\therefore$ Initial vertical velocity $= 19.6\ ms^{-1}$

This is subject to acceleration '$-g$'.

Let the particle be height, $14.7\ m$ after time 't' sec's,

Applying the formula,

$\rightarrow u \sin\alpha$ (since thrown vertically).

"$s = ut + \frac{1}{2} at^2$"

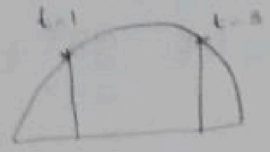
we get, ($u = 19.6$)

$14.7 = (19.6)t - \frac{1}{2}(9.8)t^2$

$14.7 = (19.6)t - \frac{9.8t^2}{2}$

$14.7 = 19.6t - 4.9t^2$. ($\div 4.9$)

$\Rightarrow t^2 - 4t + 3 = 0$



$\Rightarrow (t-1)(t-3) = 0$

$\rightarrow t = 1$ or $t = 3$.

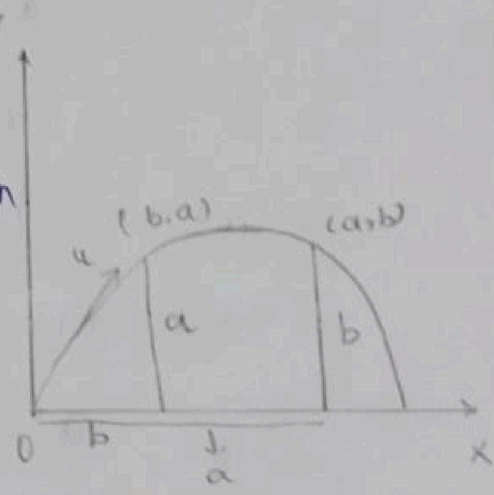
Hence at the end of 1 sec and again at the end of 3 sec it will be at the height of 14.7 m

10m

5. A particle is projected so as to graze the tops of two parallel walls, the first of height 'a' at a distance 'b', from the pt of projection and the second of height 'b' at a distance 'a' from the pt of projection. If the path of the particle lies in the plane ⊥'r to both the walls, find the range of on the horizontal plane and show that the angle of projection exceeds $\tan^{-1} 3$.

Soln:

Let 'u' be the initial velocity and 'α' be the angle of projection



The eqn to the path is,

$$y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha}.$$

put, $t = \tan\alpha$,

$$y = xt - \frac{gx^2}{2u^2}\sec^2\alpha.$$

$$y = xt - \frac{gx^2}{2u^2}(1+\tan^2\alpha)$$

$$\Rightarrow y = xt - \frac{gx^2}{2u^2}(1+t^2) \dashrightarrow (1).$$

The pt (b, a) lies in (1),

$$\Rightarrow a = bt - \frac{gb^2}{2u^2}(1+t^2) \rightarrow (2)$$

The pt (a, b) lies in (1),

$$\Rightarrow b = at - \frac{ga^2}{2u^2}(1+t^2) \rightarrow (3)$$

From (2), $a - bt = \frac{-gb^2}{2u^2}(1+t^2) \rightarrow (4)$

From (3), $b - at = \frac{-ga^2}{2u^2}(1+t^2) \rightarrow (5)$

$$\frac{(4)}{(5)} \Rightarrow \frac{a-bt}{b-at} = \frac{b^2}{a^2}$$

$$\Rightarrow a^2(a-bt) = b^2(b-at)$$

$$\Rightarrow a^3 - a^2bt = b^3 - b^2at$$

$$\Rightarrow a^3 - b^3 = a^2bt - b^2at$$

$$\Rightarrow a^3 - b^3 = abt(a-b)$$

$$\Rightarrow a^3 - b^3 = (a^2b - ab^2)t$$

$$\Rightarrow (\cancel{a-b})(a^2+ab+b^2) = (\cancel{a-b})(ab)t$$

$$\Rightarrow t = \frac{a^2+ab+b^2}{ab} \rightarrow (5^*)$$

$$\Rightarrow \tan\alpha = \frac{a^2+ab+b^2}{ab} \quad [\because \tan\alpha = t]$$

$$\Rightarrow \tan\alpha = \frac{a^2 - 2ab + 2ab + ab + b^2}{ab}$$

$$\Rightarrow \tan\alpha = \frac{a^2 - 2ab + b^2 + 3ab}{ab}$$

$$\Rightarrow \tan\alpha = \frac{(a-b)^2}{ab} + 3$$

$$\Rightarrow \tan\alpha = \frac{(a-b)^2}{ab} + 3 \dashrightarrow (6)$$

The term $\frac{(a-b)^2}{ab}$ is positive,

$$\therefore \tan\alpha > 3$$

$$\Rightarrow \alpha > \tan^{-1} 3$$

where $\sin 2\alpha = \frac{2\tan\alpha}{1+\tan^2\alpha}$

From (4), $\frac{a-bt}{-b^2} = \frac{g(1+t^2)}{2u^2}$

Alternate method

$$R = \frac{u^2 \sin 2\alpha}{g} = \frac{u^2}{g} \cdot \frac{2\tan\alpha}{(1+\tan^2\alpha)}$$

sub. we get final result.

$$\Rightarrow \frac{g(1+t^2)}{2u^2} = \frac{a-bt}{-b^2}$$

$$= \frac{bt-a}{b^2}$$

$$= \frac{\left[\cancel{b}\left(\frac{a^2+ab+b^2}{a\cancel{b}}\right) - a\right]}{b^2}$$

$$= \frac{\cancel{a^2}+ab+b^2-\cancel{a^2}}{ab^2}$$

$$= \frac{\cancel{b}(a+b)}{a\cancel{b^2}}$$

$$= \frac{(a+b)}{ab}$$

$$\Rightarrow \frac{g(1+t^2)}{2u^2} = \frac{a+b}{ab} \dashrightarrow (7)$$

Horizontal range, $R = \frac{u^2 \sin 2\alpha}{g}$

$$= \frac{2u^2 t}{g(1+t^2)}$$

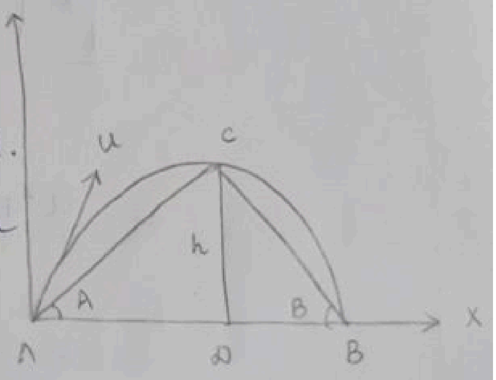
$$= t \cdot \frac{ab}{(a+b)} \quad (\text{from}(7))$$

$$= \frac{(a^2+ab+b^2)}{\cancel{ab}} \cdot \frac{\cancel{ab}}{(a+b)}$$

$$= \frac{a^2+ab+b^2}{a+b} \,//.$$

6. A particle is thrown over a triangle from one end of the horizontal base and grazing the vertex falls on the other end of the base. If A, B are the base angles and α the angle of projection, show that $\tan\alpha = \tan A + \tan B$.

soln:



Let 'u' be the velocity, 'α' the angle of projection. Let 't' secs be the time from A to C.

Draw $CD \perp AB$.

Let $CD = h$.

Considering vertical motion,

h = vertical distance described in time 't'.,

$$\Rightarrow h = u\sin\alpha\, t - \tfrac{1}{2} g t^2 \dashrightarrow (1).$$

AD = horizontal distance described in time 't'.

$AD = u\cos\alpha . t$ --> (2)

From ΔCAD,

$$\tan A = \frac{CD}{AD}$$

$$= \frac{h}{AD}$$

$$= \frac{u\sin\alpha . t - \frac{1}{2}gt^2}{u\cos\alpha . t}$$

$$= \frac{u\sin\alpha t}{u\cos\alpha t} - \frac{\frac{1}{2}gt^2}{u\cos\alpha . t}$$

$$= \frac{\sin\alpha}{\cos\alpha} - \frac{gt}{2u\cos\alpha}$$

$$\tan A = \frac{\sin\alpha}{\cos\alpha} - \frac{gt}{2u\cos\alpha}$$

$$\Rightarrow \tan A = \tan\alpha - \frac{gt}{2u\cos\alpha} \quad \text{--> (3)}$$

AB = horizontal range.

$$\Rightarrow AB = \frac{2u^2\sin\alpha\cos\alpha}{g} \quad \text{--> (4)}$$

$$\therefore DB = AB - AD$$

$$\Rightarrow DB = \frac{2u^2\sin\alpha\cos\alpha}{g} - u\cos\alpha . t \quad \text{--> (5)}$$

From ΔCDB,

$$\tan B = \frac{CD}{DB}$$

$$= \frac{h}{DB} \quad \text{(from (1) \& (5))}$$

$$= \frac{u\sin\alpha . t - \frac{1}{2}gt^2}{\frac{2u^2\sin\alpha\cos\alpha}{g} - u\cos\alpha . t}$$

$$\tan B = \frac{2u\sin\alpha.t - gt^2}{2\left(\frac{2u^2\sin\alpha\cos\alpha - ug\cos\alpha.t}{g}\right)}$$

$$\Rightarrow \tan B = \frac{g(2u\sin\alpha t - gt^2)}{2(2u^2\sin\alpha\cos\alpha - gu\cos\alpha t)}$$

$$\Rightarrow \tan B = \frac{gt(2u\sin\alpha - gt)}{2u\cos\alpha(2u\sin\alpha - gt)}$$

$$\Rightarrow \tan B = \frac{gt}{2u\cos\alpha} \dashrightarrow (6).$$

(3) + (6) gives,

$$\tan A + \tan B = \tan\alpha - \frac{gt}{2u\cos\alpha} + \frac{gt}{2u\cos\alpha}$$

$$\Rightarrow \tan\alpha = \tan A + \tan B.$$

Hence proved

10m

7. Show that the greatest height which a particle with a initial velocity v can reach on a vertical wall at a distance 'a' from the pt of projection is $\frac{v^2}{2g} - \frac{ga^2}{2v^2}$. Prove also that the greatest height above the pt of projection obtained by the particle in its flight is $\frac{v^6}{2g(v^4+g^2a^2)}$.

Soln:

The eqn to the path is,

$$y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha} \dashrightarrow (1)$$

Given : Initial velocity, $u = v$

and $x = a$

sub this in (1), we get,

$$y = a\tan\alpha - \frac{ga^2}{2v^2\cos^2\alpha}$$

$$\Rightarrow y = a\tan\alpha - \frac{ga^2}{2v^2}(\sec^2\alpha)$$

$$\Rightarrow y = a\tan\alpha - \frac{ga^2}{2v^2}(1+\tan^2\alpha)$$

put $t = \tan\alpha$, we get,

$$\Rightarrow y = at - \frac{ga^2}{2v^2}(1+t^2) \dashrightarrow (2)$$

$\Rightarrow y$ is a function of 't'.

y is maximum when $\frac{dy}{dt} = 0$ and $\frac{d^2y}{dt^2}$ is negative.

Diff (2) w. resp. to 't'.

$$\frac{dy}{dt} = a - \frac{ga^2}{2v^2}(2t)$$

$$\Rightarrow \frac{dy}{dt} = a - \frac{ga^2t}{v^2} \dashrightarrow (3)$$

Differ' (3) w. resp. 't' again,

$$\Rightarrow \frac{d^2y}{dt^2} = \frac{-ga^2}{v^2}$$

which is negative

(ie), $\frac{d^2y}{dt^2}$ = negative

So y is maximum, $\frac{dy}{dt} = 0$.

from (3),

$$\Rightarrow a - \frac{ga^2t}{v^2} = 0$$

$$\Rightarrow \frac{av^2 - ga^2t}{v^2} = 0$$

$$\Rightarrow av^2 - ga^2t = 0 \Rightarrow av^2 = ga^2t$$

$$\Rightarrow t = \frac{av^2}{ga^2}$$

$$\Rightarrow t = \frac{v^2}{ga} \dashrightarrow (4)$$

sub (4) in (2), we get,

$$y = a \cdot \frac{v^2}{ga} - \frac{ga^2}{2v^2}\left(1 + \frac{v^4}{g^2a^2}\right)$$

$$y = \frac{v^2}{g} - \frac{ga^2}{2v^2}\left[\frac{g^2a^2 + v^4}{g^2a^2}\right]$$

$$y = \frac{v^2}{g} - \frac{g^2a^2 - v^4}{2v^2g}$$

$$y = \frac{2v^4 - g^2a^2 - v^4}{2v^2g}$$

$$y = \frac{v^4 - g^2a^2}{2v^2g}$$

$$y = \frac{v^2}{2g} - \frac{a^2}{2v^2} \dashrightarrow (5).$$

Eqn (5) is the greatest height reached on the wall.

Greatest height attained during the flight:

$$\Rightarrow h = \frac{u^2 \sin^2\alpha}{2g}$$

here $u = v$, then

$$\Rightarrow h = \frac{v^2 \sin^2\alpha}{2g}$$

$$h = \frac{v^2}{2g} \cdot \frac{1}{\operatorname{cosec}^2\alpha}$$

$$h = \frac{v^2}{2g} \cdot \frac{1}{1+\cot^2\alpha}.$$

$$h = \frac{v^2}{2g} \cdot \frac{1}{\left(1+\frac{1}{\tan^2\alpha}\right)}$$

$$h = \frac{v^2}{2g} \cdot \frac{1}{\left(1+\frac{1}{t^2}\right)} \quad \rightarrow (6)$$

from (4), $t = \frac{v^2}{ga}$, we get,

$$h = \frac{v^2}{2g} \cdot \frac{1}{\left(1+\frac{1}{\frac{v^4}{g^2a^2}}\right)}$$

$$h = \frac{v^2}{2g} \cdot \frac{1}{\left(1+\frac{g^2a^2}{v^4}\right)}$$

$$h = \frac{v^2}{2g} \cdot \frac{v^4}{(v^4+g^2a^2)}$$

$$h = \frac{v^6}{2g(v^4+g^2a^2)}$$

Hence proved.

8. If T is time of flight, R the horizontal range and 'α' angle of projection, show that

$$gT^2 = 2R\tan\alpha$$

If $\alpha = 60°$. Find in terms of R, the height of the projectile when it has moved through a horizontal distance equal to $\frac{3R}{4}$.

Given:

(i) horizontal range $= 3R/4$

distance

We know that,

T is the time of flight, $T = \frac{2u\sin\alpha}{g}$ --> (1)

R the horizontal range $= \frac{u^2\sin 2\alpha}{g}$ --> (2)

$$\therefore \frac{T^2}{R} = \frac{\left(\frac{2u\sin\alpha}{g}\right)^2}{\frac{u^2\sin 2\alpha}{g}}$$

$$= \frac{\cancel{4}^{2}\cancel{u^2}\sin^{\cancel{2}}\alpha}{\cancel{g^2}_{g}} \times \frac{\cancel{g}}{\cancel{2}\cancel{u}\,\cancel{\sin\alpha}\cos\alpha}$$

$$= \frac{2\sin\alpha}{g\cos\alpha}$$

$$= \frac{2}{g}\cdot\tan\alpha$$

$$\Rightarrow gT^2 = 2\cdot R\cdot\tan\alpha$$

(ii) Given: $\alpha = 60°$; Horizontal distance $= \frac{3R}{4}$ → (3)

To find: Height of the projectile:

$$h = \frac{u^2 \sin^2\alpha}{2g} \rightarrow (4)$$

$$(3) \Rightarrow \frac{u^2 \sin 2\alpha}{g} = \frac{3R}{4}$$

$$\Rightarrow u^2 = \frac{3Rg}{4\sin 2\alpha} \rightarrow (5)$$

Sub (5) in (4),

$$h = \frac{3Rg}{4\sin 2\alpha} \cdot \frac{\sin^2\alpha}{2g}$$

$(2\sin\alpha\cos\alpha)$

$$= \frac{3Rg\sin\alpha}{16g\cos\alpha}$$

$$= \frac{3R}{16} \cdot \tan\alpha.$$

$$h = \frac{3R}{16} \cdot \tan 60°$$ (where $\alpha = 60°$).

$$\Rightarrow h = \frac{3R}{16} \cdot \sqrt{3}$$

If the time of flight of a shot is 'T' secs over a range of 'x' m, show that the elevation is, $\tan^{-1}\left(\frac{gT^2}{2x}\right) = \alpha$.

$$\tan^{-1}\left(\frac{gT^2}{2x}\right) =$$

9. If the time of flight of a shot is T sec over a range of 'x' meters, show that the elevation is $\alpha = \tan^{-1}\left(\frac{gT^2}{2x}\right)$.

Soln:

We know that,

T is the time of flight, $T = \frac{2u\sin\alpha}{g}$ --> (1)

R, the horizontal range $= \frac{u^2 \sin 2\alpha}{g}$ --> (2)

Here, 'x' denotes the range.

$$\Rightarrow \frac{u^2 \sin 2\alpha}{g} = x = R.$$

$$\Rightarrow \frac{T^2}{x} = \frac{\left(\frac{2u\sin\alpha}{g}\right)^2}{\frac{u^2 \sin 2\alpha}{g}}$$

$$\Rightarrow \frac{T^2}{x} = \frac{\overset{2}{\cancel{4}}\cancel{u^2}\sin^{\cancel{2}}\alpha}{g^{\cancel{2}}} \times \frac{\cancel{g}}{\cancel{2}\cancel{u^2}\cancel{\sin\alpha}\cos\alpha}.$$

$$\Rightarrow \frac{T^2}{x} = \frac{2\sin\alpha}{g\cos\alpha}$$

$$\Rightarrow \frac{T^2}{x} = \frac{2}{g}\tan\alpha$$

$$\Rightarrow \tan\alpha = \frac{gT^2}{2x}$$

$$\Rightarrow \alpha = \tan^{-1}\left(\frac{gT^2}{2x}\right)$$

Hence proved.

Determine when the horizontal range of a projectile is maximum, given the magnitude 'u' of the velocity of projection.

Proof:

If 'u' is the initial velocity & 'α' is the angle of projection, the horizontal range,

$$R = \frac{u^2 \sin 2\alpha}{g} \rightarrow (1)$$

for a given value of 'u', 'g' being the constant, the value of 'R' greatest when $\sin 2\alpha$ is greatest.

When $\sin 2\alpha = 1, \Rightarrow 2\alpha = 90^\circ \Rightarrow \alpha = 45^\circ$

$\therefore$ (1) becomes, $R = \frac{u^2(1)}{g} = \frac{u^2}{g}$

$$\boxed{\text{Maximum horizontal range } (R_{max}) = \frac{u^2}{g}}$$

Hence for the given velocity of projection, the horizontal range is maximum when the particle is projected at an angle of 45° to the horizontal.

Eg:

A bomb resting on level ground explodes sending fragments in all directions with a velocity of 98 ms^{-1}. What is the greatest distance from the bomb at which a fragment can fall?

Soln:

Given.

$u = 98\ ms^{-1}$.

$R_{max} = \frac{u^2}{g}$

Substituting:

$$R_{max} = \frac{(98)^2}{9.8}$$

$$= \frac{98 \times 98 \times 10}{\cancel{98}}$$

$$= 980\ ms^{-1}.$$

2. Find the radius of basin which could collect all the drops of water from a fountain at its centre assuming that th drop projects in all directions with the same velocity of $4.9\ ms^{-1}$.

Soln:

Given, $u = 4.9\ ms^{-1}$.

$R_{max} = \frac{u^2}{g}$

$$\frac{4.9 \times 49 \times 1}{9.8} \quad 2.45$$

Substituting,

$$R_{max} = \frac{(4.9)^2}{9.8}$$

$$= \frac{4.9 \times 4.9 \times 10}{9.8 \times 10}$$

$$= \frac{4.9 \times 49}{98}$$

$$= 2.45\ m$$

3. If h and h' be the greatest heights in the two paths of a projectile with a given velocity for a given range. Prove that $R = 4 \times \sqrt{hh'}$

soln:

Let α and α' be the two angles of projection, with a given velocity 'u'.

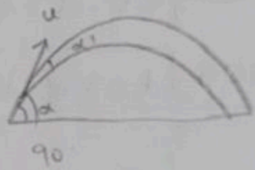
We know that,

$$\alpha + \alpha' = 90^\circ \dashrightarrow (1).$$

then. $$R = \frac{u^2 \sin 2\alpha}{g} = \frac{2u^2 \sin\alpha \cos\alpha}{g} \dashrightarrow (2)$$

$$h = \frac{u^2 \sin^2\alpha}{2g} \dashrightarrow (3)$$

$$h' = \frac{u^2 \sin^2\alpha'}{2g} \dashrightarrow (4).$$



$$hh' = \frac{u^2 \sin^2\alpha}{2g} \times \frac{u^2 \sin^2\alpha'}{2g}$$

$$= \frac{u^4 \sin^2\alpha \sin^2\alpha'}{4g^2}$$

$$\Rightarrow \sqrt{hh'} = \frac{u^2 \sin\alpha \sin\alpha'}{2g}$$

$$= \frac{u^2 \sin\alpha \sin(90^\circ - \alpha)}{2g} \qquad \begin{matrix} \alpha' = 90^\circ - \alpha \\ [\text{from } (1)] \end{matrix}$$

$$= \frac{u^2 \sin\alpha \cos\alpha}{2g} \qquad [\times, \div \text{ by } 2].$$

$$= \frac{2u^2 \sin\alpha \cos\alpha}{4g}$$

$$= \frac{u^2 \sin 2\alpha}{4g}$$

$\Rightarrow \sqrt{hh'} = \frac{1}{4} \cdot \frac{u^2 \sin 2\alpha}{g}$

$\Rightarrow 4\sqrt{hh'} = R.$

$\Rightarrow R = 4\sqrt{hh'}$

Hence proved.

4. The range of riffle bullet is 1000 m. When α is the angle of projection, show that if the bullet is fired with same elevation from a car wearing travelling 36 km/hour towards the target. the range will be increased by $\frac{1000\sqrt{\tan\alpha}}{7}$

Soln:

Let 'u' be the velocity of projection and 'α' the angle of projection. (given

The horizontal range, $R = \frac{2u^2 \sin\alpha \cos\alpha}{g} = 100$ → (1)

Given, also,

$R = \frac{2}{g}\{(u\cos\alpha)(u\sin\alpha)\}$

$= \frac{2}{g}$ (horizontal velocity) × (vertical velocity) → (2)

when the bullet is fired from a moving

car the horizontal velocity is increased and the increase

$= 36\ km/h$

$= \dfrac{36 \times 1000}{60 \times 60}$

$= 10\ ms^{-1}$

New horizontal velocity $= (u\cos\alpha + 10)$.

As there is no change in the vertical motion, new initial vertical velocity $= (u\sin\alpha)$

Hence in the second case, horizontal range,

$R' = \dfrac{2}{g}(u\cos\alpha + 10)(u\sin\alpha)$

$R' - R = \dfrac{2}{g}(u\cos\alpha + 10)(u\sin\alpha) - \dfrac{2u^2\sin\alpha\cos\alpha}{g}$

$= \dfrac{2}{g}[u^2\sin\alpha\cos\alpha + 10u\sin\alpha - u^2\sin\alpha\cos\alpha]$

$= \dfrac{2}{g}(10u\sin\alpha]$

$= \dfrac{20u\sin\alpha}{g} \rightarrow (3)$

From (1),

$\dfrac{2u^2\sin\alpha\cos\alpha}{g} = 1000$.

$u^2 = \dfrac{\overset{500}{\cancel{1000}}\, g}{\cancel{2}\sin\alpha\cos\alpha}$

$\Rightarrow u = \dfrac{\sqrt{500}\sqrt{g}}{\sqrt{\sin\alpha}\sqrt{\cos\alpha}}$

$$= \frac{\sqrt{500}\sqrt{g} \times \sqrt{g}}{\sqrt{g}\sqrt{\sin\alpha}\sqrt{\cos\alpha}}$$

$$= \frac{g\sqrt{500} \times \sqrt{\sin\alpha}}{\sqrt{g}\,\sin\alpha\sqrt{\cos\alpha}}$$

$$u = \frac{g\sqrt{500}\cdot\sqrt{\tan\alpha}}{\sqrt{g}\cdot\sin\alpha}$$

substituting in (3), we get,

$$R' - R = \frac{20}{g} \cdot \frac{\sqrt{500}\sqrt{\tan\alpha}}{\sqrt{g}\cdot\sin\alpha} \cdot g\sin\alpha$$

$$= \frac{20\sqrt{500}\sqrt{\tan\alpha}}{\sqrt{g}}$$

$$= \frac{20\sqrt{500}\sqrt{\tan\alpha} \times \sqrt{10}}{\sqrt{9.8} \times \sqrt{10}}$$

$$= \frac{20\sqrt{5000}\sqrt{\tan\alpha}}{\sqrt{49 \times 2}}$$

$$= \frac{200\sqrt{50}^{\sqrt{25}}\sqrt{\tan\alpha}}{7\sqrt{2}}$$

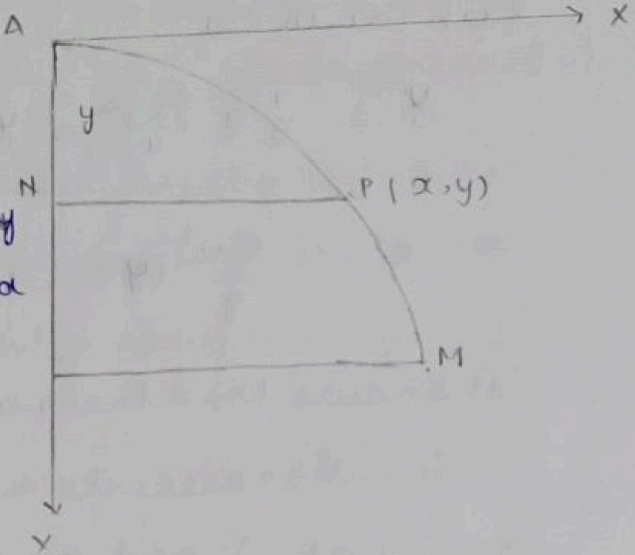
$$R' - R = \frac{1000\sqrt{\tan\alpha}}{7}$$

Hence proved.

Bookwork 2:

A particle is projected horizontally from a point at a certain height above the ground; to show that the path described by it is a parabola.

Proof:



Let a particle be projected with a velocity 'u' from a point 'A' at a height 'h' above the ground level.

Let it strike the ground at 'M'. Take 'A' as origin, the horizontal through 'A' as x-axis.

The downward vertical through A as y-axis.

Let $P(x,y)$ be the position of the particle at time 't'.

As there is no horizontal acceleration, the horizontal velocity remains constant throughout the motion.

So, x = horizontal distance described in the time 't'

$\therefore x = ut \dashrightarrow (1)$

Due to gravity, the vertical acceleration during the motion is 'g' downwards.

y = vertical distance described in time 't'

$$y = \frac{1}{2} g t^2 \dashrightarrow (2)$$

from (1),

$$t = \frac{x}{u} \dashrightarrow (3)$$

sub (3) in (2).

$$y = \frac{1}{2} g \left(\frac{x}{u}\right)^2 \Rightarrow y = \frac{1}{2} g \left(\frac{x^2}{u^2}\right).$$

$$\Rightarrow x^2 = \frac{2u^2}{g}(y) \dashrightarrow (4)$$

(4) shows that the quadratic equation of 'x'.

So, it represents a parabola with vertex at A and axis AN.

1. A bomb was released from an aeroplane when it was at a height of 1960 m above the point A on the ground and was moving horizontally with a speed of 100 ms^{-1}. Find the distance from A of the point where the bomb strikes the ground ($g = 9.8 ms^{-2}$).

Soln:-

Let us consider the motion of the bomb in the horizontal and vertical directions seperately.

Horizontal motion:

Initial velocity = 100 ms^{-1}.

Acceleration = 0.

Vertical motion:

Initial velocity = 0

Acceleration $= -9.8\ ms^{-2}$

distance $= -1960\ m$ (since it is downwards)

Let 't' be the time taken by the bomb to strike the point on the ground.

Consider the vertical motion and applying the formula,

"$s = ut + \frac{1}{2}gt^2$"

$\Rightarrow -1960 = (0).t + \frac{1}{2}(-9.8)t^2$

4.9

$+1960 = +\frac{1}{\cancel{2}}(\cancel{9.8})t^2$

$4.9t^2 = 1960$

$t^2 = \frac{1960}{4.9}$

$t^2 = 400$

$t = 20$ sec.

During this time, the horizontal velocity is constant.

$\therefore$ Horizontal distance described by the bomb in 20 sec $= 20 \times 100 = 2{,}000$ m