UNIT-II

COLLISIONS OF ELASTIC BODIES

Definition-1:

The internal force which acts when a body tends to recover its original shape after a deformation or compression is called the force of restitution. (Eg: rubber band)

Definition-2:

The property which causes the solid body to recover it's shape is called elasticity. The property of force of restitution which causes the body to recover it's shape is also called elasti

Definition-3:

If the body doesn't tend to recover it's shape, it will cause no force of restitution a such a body is said to be inelastic. ($e=0$)

Definition-4:

If $v=u$, the velocity with which the ball leaves the floor is the same as that with which it strikes it. In this case, the ball is said to be perfectly elastic. When $e=1$, the bod perfectly elastic

It is said to be inelastic.

When a body completely retains it's shape after a collision, it is said to be perfectly elastic.

When a body does not retains it's original shape after a collision, it is said to be perfectly inelastic.

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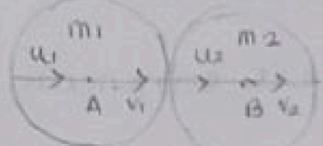
Definition-5: Impinge directly:

Two bodies are said to Impinge directly when the direction of motion of each before impact is along the common normal curve at the pt where they touch.

AB common normal



Definition-6:

Impinge Obliquely:

Two bodies are said to be Impinge obliquely, if the direction of motion of either bodies or both is not along the common normal at the point where they touch.

Definition-7: Line of impact:

The common normal at the point of contact is called line of impact.

In the case of two spheres, the line of

Fundamental laws of impact:

Law. 1 Newton's Experimental Law (NEL):

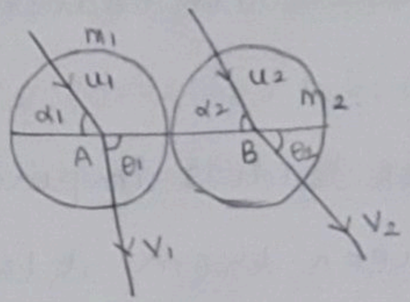
*. When two bodies impinge directly, their relative velocity after impact bears a constant ratio to their relative velocity before impact and is in the opposite direction.

If u_1, u_2 are the components of velocities of two impinging bodies along their common normal before impact and v_1, v_2 are the components of velocities of two impinging bodies along their common normal after impact.

e is the coefficient of restitution.

Then. $\dfrac{v_2 - v_1}{u_2 - u_1} = -e$

* If the two bodies impinge obliquely, their relative velocity resolved all along their common normal after impact bears a constant ratio to their relative velocity before impact, resolved in the same direction, and is of opposite sign.



$$\frac{v_2 \cos\theta_2 - v_1 \cos\theta_1}{u_2 \cos\alpha_2 - u_1 \cos\alpha_1} = -e.$$

(or resilience)

Defn: coefficient of elasticity (or restitution):

(modulus of elasticity).

The constant ratio depends on the material of which the bodies are made and is it is independent of their masses. It is denoted by. e, and it is called the coefficient of elasticity.

Law-2: Principle of conservation of momentum:

The algebraic sum of momenta of the impinging body after impact is equal to the algebraic sum of momenta before impact, all momenta being measured along the common normal.

Direct impact:

$$m_1 v_1 + m_2 v_2 = m_1 u_1 + m_2 u_2.$$

Oblique impact:

$$m_1 v_1 \cos\theta_1 + m_2 v_2 \cos\theta_2 = m_1 u_1 \cos\alpha_1 + m_2 u_2 \cos\alpha_2.$$

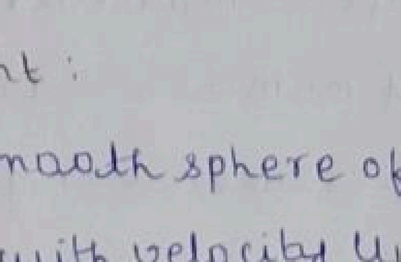
Law-2: Motion of two smooth bodies I'er to the line of impact:-

When two smooth bodies impinge, the only force between between them at the time of impact is the mutual reaction which acts along the common normal. There is no force along all the common tangent and hence there is no change of velocity in that direction. Hence the velocity of either body resolved in a direction I'r to line of impact is not altered by impact.

Book work: 1:- Direct impact of two smooth spheres:

Statement:

A smooth sphere of mass M_1 impinges directly with velocity u_1 on another smooth sphere of mass m_2, moving in the same direction with velocity u_2. if the coefficient of restitution is e, to find their velocities after impact.



Proof:

AB is the line of impact.

due to the impact. there is no tangential force and hence, for either sphere the velocity along the tangent is not altered by impact.

But before impact, the spheres had been moving along the line AB.

Hence tangential velocity after impact = tangential velocity before impact = 0.

Let v_1 and v_2 be the velocities after impact.

By NEL, $\frac{v_2 - v_1}{u_2 - u_1} = -e \Rightarrow v_2 - v_1 = -e(u_2 - u_1) \longrightarrow (1)$

By principle of conservation of momentum,

$$m_1 v_1 + m_2 v_2 = m_1 u_1 + m_2 u_2 \longrightarrow (2)$$

$(1) \times m_2$, $m_2 v_2 - m_2 v_1 = -e m_2 u_2 + m_2 e u_1 \longrightarrow (3)$

(2)-(3),

$\therefore m_1 v_1 + m_2 v_1 = m_1 u_1 + m_2 u_2 + e m_2 u_2 - m_2 e u_1$

$\Rightarrow v_1(m_1 + m_2) = u_1(m_1 - e m_2) + m_2 u_2 (1+e)$

$$\Rightarrow \boxed{v_1 = \frac{u_1(m_1 - e m_2) + m_2 u_2 (1+e)}{m_1 + m_2}} \longrightarrow (4)$$

Now, $(1) \times m_1$, $m_1 v_2 - m_1 v_1 = -e m_1 u_2 + m_1 e u_1 \longrightarrow (5)$

(2)+(5),

$\therefore m_2 v_2 + m_1 v_2 = m_1 u_1 + m_2 u_2 - e m_1 u_2 + m_1 e u_1$

$\Rightarrow v_2(m_1 + m_2) = m_1 u_1 (1+e) + u_2(m_2 - e m_1)$

$$\Rightarrow \boxed{v_2 = \frac{m_1 u_1 (1+e) + u_2 (m_2 - e m_1)}{m_1 + m_2}} \longrightarrow (6)$$

Eqn (4) & (6) gives velocities of sphere after impact.

Corollary-1 :

2m
If two equal perfectly elastic spheres impinge directly, they interchange their velocities.

Proof :

If the spheres are perfectly elastic,

$\Rightarrow e = 1$

Given: $m_1 = m_2$.

substitute in (4) & (6), we get,

$$v_1 = \frac{u_1 (0) + m_1 u_2 (1+1)}{m_1 + m_1}$$

$$= \frac{2 m_1 u_2}{2 m_1}$$

$\therefore v_1 = u_2$

Also, $v_2 = \dfrac{m_1 u_1 (1+1) + u_2 (0)}{2 m_1}$

$$v_2 = \frac{2 m_1 u_1}{2 m_1}$$

$$v_2 = u_1$$

$\therefore v_2 = u_1$.

equal mass they interchange their velocities.

Corollary - 2:

The impulsive blow on the sphere A of mass m_1 with equal to the impulsive blow on the sphere B of mass m_2 and opposite to that.

Proof:

Impulsive force = change of momentum.

The impulse of the blow on the sphere A of mass m_1,

$= $ change of momentum of A

$= m_1 v_1 - m_1 u_1$

$= m_1 (v_1 - u_1)$

$$= m_1 \left[\frac{u_1 (m_1 - e m_2) + m_2 u_2 (1+e)}{(m_1 + m_2)} - u_1 \right]$$

$$= m_1 \left[\frac{u_1 \cancel{m_1} - u_1 e m_2 + m_2 u_2 + e m_2 u_2 - m_1 u_1 - u_1 m_2}{m_1 + m_2} \right]$$

$$= m_1 \left[\frac{m_2 u_2 (1+e) - u_1 m_2 (1+e)}{m_1 + m_2} \right]$$

$$= m_1 \left[\frac{(1+e)(m_2 u_2 - u_1 m_2)}{m_1 + m_2} \right]$$

$$= \frac{m_1 m_2 (1+e)(u_2 - u_1)}{m_1 + m_2}.$$

Hence the impulsive blow on m_2 will be equal and opposite to impulsive blow on m_1.

2020. Bookwork - 2 : Loss of kinetic energy due to direct impact of two smooth spheres :

Proof :

Let m_1 and m_2 be the masses of the spheres. Let u_1 and u_2 be the velocities before impact. Let v_1 and v_2 be the velocities after impact and e be the coefficient of restitution.

By NEL, $v_2 - v_1 = -e(u_2 - u_1)$ ---> (1)

By the principle of conservation of momentum,

$$m_1v_1 + m_2v_2 = m_1u_1 + m_2u_2 \quad \text{---> (2)}$$

Total K.E before impact $= \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2$

Total K.E after impact $= \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$

change in K.E = Initial K.E - final K.E

$$= \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 - \frac{1}{2}m_1v_1^2 - \frac{1}{2}m_2v_2^2$$

$$= \frac{1}{2}m_1(u_1^2 - v_1^2) + \frac{1}{2}m_2(u_2^2 - v_2^2)$$

$$= \frac{1}{2}m_1(u_1 - v_1)(u_1 + v_1) + \frac{1}{2}m_2(u_2 - v_2)(u_2 + v_2)$$

change in K.E $= \frac{1}{2}m_1(u_1 - v_1)(u_1 + v_1) + \frac{1}{2}m_1(v_1 - u_1)(u_2 + v_2)$

($\because$ from (2), $m_2(u_2 - v_2) = m_1(u_1 - v_1)$)

$$= \frac{1}{2}m_1(u_1 - v_1)[u_1 + v_1 - (u_2 + v_2)]$$

$$= \frac{1}{2}m_1(u_1 - v_1)[(u_1 - u_2) - (v_2 - v_1)]$$

$$= \frac{1}{2}m_1(u_1 - v_1)[(u_1 - u_2) + e(u_2 - u_1)] \text{ from (1)}$$

$$= \frac{1}{2}m_1(u_1 - v_1)(u_1 - u_2)[1 - e] \quad \text{---> (3)}$$

Now from (2),

$m_1(u_1 - v_1) = m_2(v_2 - u_2)$

$$\Rightarrow \frac{u_1 - v_1}{m_2} = \frac{v_2 - u_2}{m_1} \text{ and each } = \frac{u_1 - v_1 + v_2 - u_2}{m_1 + m_2}$$

$$= \frac{(u_1 - u_2) + (v_2 - v_1)}{m_1 + m_2}$$

$$= \frac{(u_1 - u_2) - e(u_2 - u_1)}{m_1 + m_2}$$

[from (1)]

$$= \frac{(u_1 - u_2)(1 + e)}{m_1 + m_2}$$

$$\therefore \frac{u_1 - v_1}{m_2} = \frac{(u_1 - u_2)(1 + e)}{m_1 + m_2} \quad \text{---} \rightarrow (4)$$

sub (4) in (3), we get,

$$\text{change in k.E} = \frac{1}{2} m_1 m_2 \frac{(u_1 - u_2)(1 + e)}{m_1 + m_2} (u_1 - u_2)(1 - e)$$

$$= \frac{1}{2} \frac{m_1 m_2 (u_1 - u_2)^2 (1 - e^2)}{m_1 + m_2}$$

$$\therefore \text{change in k.E} = \frac{1}{2} \frac{m_1 m_2 (u_1 - u_2)^2 (1 - e^2)}{m_1 + m_2} \quad \text{--} \rightarrow (5)$$

Remark:

Perfectly elastic (e=1), then there is no change in k.E

- As $e < 1$, the eqn (5) is always positive. so there is a loss of total kinetic energy by collision.

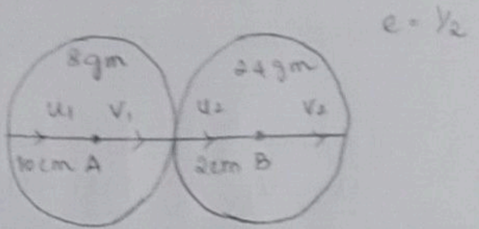
- When $e = 1$, (i.e), the bodies are perfectly elastic, the eqn (5) becomes zero and hence t total kinetic energy is unchanged by impact

Example :

A ball of mass 8 gm moving with a velocity of $10 cms^{-1}$ impinges directly on other of mass 24 gm moving at $2 cms^{-1}$ in the same direction. If $e = 1/2$, find the velocities after impact. Also find the loss in kinetic energy.

Soln:-

Given:



$m_1 = 8gm ; m_2 = 24 gm$

$u_1 = 10 cms^{-1} ; u_2 = 2 cms^{-1}$

To find v_1 and v_2 :

By NEL, $v_2 - v_1 = -e(u_2 - u_1)$

Substituting.

$$v_2 - v_1 = \frac{-1}{2}(2-10)$$

$$\Rightarrow v_2 - v_1 = 4 \ cms^{-1} \quad \dashrightarrow (1)$$

By principle of conservation of momentum,

$$m_1 v_1 + m_2 v_2 = m_1 u_1 + m_2 u_2$$

Substituting,

$$8v_1 + 24 v_2 = 8(10) + 24(2)$$

$$8v_1 + 24 v_2 = 80 + 48$$

$$\Rightarrow v_1 + 3v_2 = 10 + 6$$

$$\Rightarrow v_1 + 3v_2 = 16 \quad \dashrightarrow (2)$$

Solving (1) & (2),

$$4v_2 = 20$$

$$v_2 = 5$$

Sub in (1)

$$5 - v_1 = 4$$

$$v_1 = +1$$

$\therefore v_1 = 1\,cms^{-1}$; $v_2 = 5\,cms^{-1}$

Total k.E before impact, $= \frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2$

$$= \frac{1}{\cancel{2}} (\overset{4}{\cancel{8}})(100) + \frac{1}{\cancel{2}}(24)(\overset{2}{4})$$

$$= 400 + 48$$

$$= 448 \text{ dynes}$$

Total k.E after impact $= \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$

$$= \frac{1}{\cancel{2}} (\overset{4}{\cancel{8}})(1) + \frac{1}{\cancel{2}}(\overset{12}{\cancel{24}})(25)$$

$$= 4 + 300$$

$$= 304 \text{ dynes}$$

12(25)
60
24
300

change in k.E $= 448 - 304 = 144$ dynes

$\therefore$ Loss in k.E $= 144$ dynes.

2. A ball of mass 8gm moving with a velocity of $10\,cms^{-1}$ impinges directly on another of mass 24 gm moving at $2\,cms^{-1}$ in the opposite direction. If $e = \frac{1}{2}$, find the velocities after impact. Also

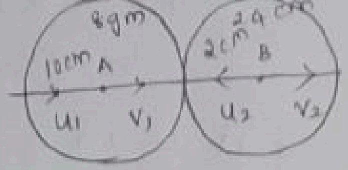
Soln:-

Given:

$m_1 = 8\,gm$; $m_2 = 24\,gm$

$u_1 = 10\,cms^{-1}$; $u_2 = -2\,cms^{-1}$

$e = 1/2$



To find v_1 and v_2:

By NEL, $v_2 - v_1 = -e(u_2 - u_1)$

substituting,

$$v_2 - v_1 = \frac{-1}{2}(-2-10)$$

$$v_2 - v_1 = 6 \rightarrow (1)$$

By principle of conservation of momentum,

$$m_1 v_1 + m_2 v_2 = m_1 u_1 + m_2 u_2$$

sub,

$$8v_1 + 24v_2 = 8(10) + 24(-2)$$

$$8v_1 + 24v_2 = 80 - 48$$

$$\Rightarrow v_1 + 3v_2 = 10 - 6$$

$$\Rightarrow v_1 + 3v_2 = 4 \rightarrow (2)$$

solving (1) & (2),

$$4v_2 = 10$$

$$v_2 = \frac{5}{2}$$

sub,

$$\frac{5}{2} - v_1 = 6$$

$$\frac{5}{2} - 6 = v_1$$

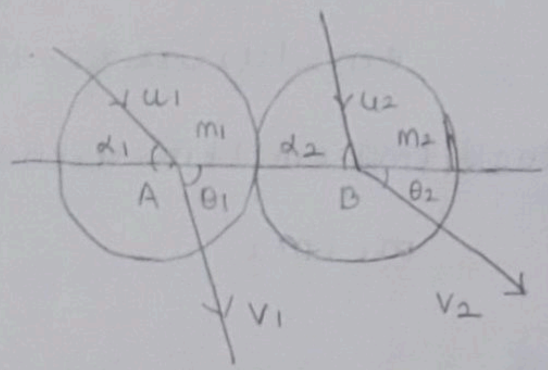
$$\Rightarrow v_1 = \frac{5-12}{2} = \frac{-7}{2}$$

$\therefore v_1 = \frac{-7}{2}$; $v_2 = \frac{5}{2}$ // (they move away from each other).

10 Bookwork - 3 : Oblique impact of two smooth spheres :

Statement :

A smooth sphere of mass m_1 impinges obliquely with velocity u_1 on another smooth sphere of mass m_2 moving with velocity u_2. If the directions of motion before impact make angles α_1 and α_2 respectively with the line joining the centres of the spheres and if the coefficient of restitution be e, to find the velocities and directions of motion after impact.



Proof :

Let the velocities of the spheres after impact be v_1 and v_2 in directions inclined at angles θ_1 and θ_2 respectively to the line of centres. Since the spheres are smooth, there is no force perpendicular to the line of centres and therefore, for each sphere the velocities.

impact.

No need to study for exam

$\therefore V_1 \sin\theta_1 = u_1 \sin\alpha_1 \rightarrow (1)$ and

$V_2 \sin\theta_2 = u_2 \sin\alpha_2 \rightarrow (2)$

By Newton's law concerning velocities along the common normal AB,

$V_2\cos\theta_2 - V_1\cos\theta_1 = -e(u_2\cos\alpha_2 - u_1\cos\alpha_1) \rightarrow (3)$

By the principle of conservation of momentum along AB,

$m_2 V_2 \cos\theta_2 + m_1 V_1 \cos\theta_1 = m_2 u_2 \cos\alpha_2 + m_1 u_1 \cos\alpha_1 \rightarrow (4)$

(4) - (3) $\times m_2$ gives.

$V_1 \cos\theta_1 (m_1 + m_2) = m_2 u_2 \cos\alpha_2 + m_1 u_1 \cos\alpha_1 + e m_2 (u_2\cos\alpha_2 - u_1\cos\alpha_1)$

(ie), $$V_1\cos\theta_1 = \frac{u_1\cos\alpha_1(m_1 - e m_2) + m_2 u_2 \cos\alpha_2(1+e)}{m_1+m_2} \rightarrow (5)$$

(4) + (3) $\times m_1$ gives,

$$V_2\cos\theta_2 = \frac{u_2\cos\alpha_2(m_2 - e m_1) + m_1 u_1 \cos\alpha_1(1+e)}{m_1+m_2} \rightarrow (6)$$

From (1) and (5), by squaring and adding, we obtain V_1^2 and by division, we have $\tan\theta_1$. Similarly from (2) and (6) we get V_2^2 and $\tan\theta_2$. Hence the motion after impact is completely

Notes to study for exam

Corollary 1:

If the two spheres are perfectly elastic an[d] of equal mass, then $e=1$ and $m_1=m_2$

Then from eqn (5) and (6) we have,

$$v_1 \cos\theta_1 = \frac{0 + m_1 u_2 \cos\alpha_2 . 2}{2m_1} = u_2 \cos\alpha_2$$

and $$v_2 \cos\theta_2 = \frac{0 + m_1 u_1 \cos\alpha_1 . 2}{2m_1} = u_1 \cos\alpha_1$$

Hence if two equal perfectly elastic spheres impinge, they interchange their velocities in the direction of the line of centres

Corollary 2: Usually, in most problems on oblique impact, one of the spheres is at rest. Suppose m_2 is at rest (ie). $u_2=0$

From eqn (2), $v_2 \sin\theta_2 = 0$ (ie), $\theta_2 = 0$. Hence m_2 moves along AB after impact. This is seen independently, since the only force on m_2 during impact is along the line of centres.

Corollary 3:

The impulse of the blow on the sphere A of mass m_1

= change of momentum of A along the common normal.

$$= m_1(v_1\cos\theta_1 - u_1\cos\alpha_1)$$

$$= m_1\left[\frac{u_1\cos\alpha_1(m_1 - em_2) + m_2u_2\cos\alpha_2(1+e)}{m_1+m_2} - u_1\cos\alpha_1\right]$$

$$= \frac{m_1[m_1u_1\cos\alpha_1 - em_2u_1\cos\alpha_1 + m_2u_2\cos\alpha_2 + em_2u_2\cos\alpha_2 - m_1u_1\cos\alpha_1 - m_2u_1\cos\alpha_1]}{m_1+m_2}$$

$$= \frac{m_1[m_2u_2\cos\alpha_2(1+e) - m_2u_1\cos\alpha_1(1+e)]}{m_1+m_2}$$

$$= \frac{m_1m_2(1+e)}{m_1+m_2}(u_2\cos\alpha_2 - u_1\cos\alpha_1)$$

The impulsive blow on m_2 will be equal and opposite to the impulsive blow on m_1.

Bookwork-4 : Loss of kinetic energy due to oblique impact of two smooth spheres :

statement :

Two spheres of masses m_1 and m_2, moving with velocities u_1 and u_2 at angles α_1 and α_2 with their line of centres, come into collision. To find an expression for the loss of kinetic energy

Let v_1 and v_2 be the velocities of the spheres after impact, in directions inclined at angles θ_1 and θ_2 respectively to the line of centres. The tangential velocity of each sphere is not altered b impact.

$\therefore v_1 \sin\theta_1 = u_1 \sin\alpha_1 \dashrightarrow (1)$

and $v_2 \sin\theta_2 = u_2 \sin\alpha_2 \dashrightarrow (2)$

By Newton's of rule,

$v_2\cos\theta_2 - v_1\cos\theta_1 = -e(u_2\cos\alpha_2 - u_1\cos\alpha_1) \dashrightarrow (3)$

By conservation of momenta,

$m_2 v_2 \cos\theta_2 + m_1 v_1 \cos\theta_1 = m_2 u_2 \cos\alpha_2 + m_1 u_1 \cos\alpha_1$

$\Rightarrow m_2 v_2 \cos\theta_2 - m_2 u_2 \cos\alpha_2 = m_1 u_1 \cos\alpha_1 - m_1 v_1 \cos\theta_1$

(ie)., $m_1(u_1\cos\alpha_1 - v_1\cos\theta_1) = m_2(v_2\cos\theta_2 - u_2\cos\alpha_2)$

$\Rightarrow m_2(v_2\cos\theta_2 - u_2\cos\alpha_2) = m_1(u_1\cos\alpha_1 - v_1\cos\theta_1) \dashrightarrow (4)$

change in K.E, $\Rightarrow m_1(u_1\cos\alpha_1 - v_1\cos\theta_1) = m_2(v_2\cos\theta_2 - u_2\cos\alpha_2)$

$= \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 - \frac{1}{2}m_1v_1^2 - \frac{1}{2}m_2v_2^2$

$= \frac{1}{2}m_1u_1^2(\cos^2\alpha_1 + \sin^2\alpha_1) + \frac{1}{2}m_2u_2^2(\cos^2\alpha_2 + \sin^2\alpha_2)$

$= \frac{1}{2}m_1u_1^2\cos^2\alpha_1 + \frac{1}{2}m_1u_1^2\sin^2\alpha_1 + \frac{1}{2}m_2u_2^2\cos^2\alpha_2 + \frac{1}{2}m_2u_2^2\sin^2\alpha_2 - \frac{1}{2}m_1v_1^2\cos^2\theta_1 - \frac{1}{2}m_1v_1^2\sin^2\theta_1$

$- \frac{1}{2}m_1v_1^2(\cos^2\theta_1 + \sin^2\theta_1) - \frac{1}{2}m_2v_2^2(\cos^2\theta + \sin^2\theta)$

$- \frac{1}{2}m_2v_2^2\cos^2\theta - \frac{1}{2}m_2v_2^2\sin^2\theta$

$= \frac{1}{2}m_1u_1^2\cos^2\alpha_1 + \frac{1}{2}m_2u_2^2\cos^2\alpha_2 - \frac{1}{2}m_1v_1^2\cos^2\theta_1 -$

$= \frac{1}{2}m_1(u_1^2\cos^2\alpha_1) + \frac{1}{2}m_1(u_1^2\sin^2\theta_1 - v_1^2\sin^2\theta_1) + \frac{1}{2}m_2u_2^2\cos^2\alpha + \frac{1}{2}$

$\frac{1}{2}m_2v_2^2\cos^2\theta_2$ (using (1) & (2))

$= \frac{1}{2}m_1(u_1^2\cos^2\alpha_1 - v_1^2\cos^2\theta_1) + \frac{1}{2}m_2(u_2^2\cos^2\alpha_2 - v_2^2\cos^2\theta_2)$

$\frac{1}{2}m_2(u_2\cos\alpha_2 - v_2\cos\theta_2)(u_2\cos\alpha_2 + v_2\cos\theta_2)$

$= \frac{1}{2}m_1(u_1\cos\alpha_1 - v_1\cos\theta_1)(u_1\cos\alpha_1 + v_1\cos\alpha_1)$

$- \frac{1}{2}(u_2\cos\alpha_2 + v_2\cos\theta_2).m_1.(u_1\cos\alpha_1 - v_1\cos\theta_1)$

using (4)

$= \frac{1}{2}m_1(u_1\cos\alpha_1 - v_1\cos\theta_1)(u\cos\alpha_1 + v_1\cos\theta_1 - u_2\cos\alpha_2 - v_2\cos\theta_2)$

$= \frac{1}{2}m_1(u_1\cos\alpha_1 - v_1\cos\theta_1)[u_1\cos\alpha_1 - u_2\cos\alpha_2 \; e(u_2\cos\alpha_2 - u_1\cos\alpha_1)]$

using (3)

$$= \frac{1}{2} m_1 (u_1 \cos\alpha_1 - v_1 \cos\theta_1)(u_1 \cos\alpha_1 - u_2 \cos\alpha_2)(1-e) \quad \text{---} \rightarrow (5)$$

Now from (4),

$$\frac{u_1 \cos\alpha_1 - v_1 \cos\theta_1}{m_2} = \frac{v_2 \cos\theta_2 - u_2 \cos\alpha_2}{m_1}$$

and each $= \dfrac{u_1 \cos\alpha_1 - v_1 \cos\theta_1 + v_2 \cos\theta_2 - u_2 \cos\alpha_2}{m_1 + m_2}$

$$= \frac{(u_1 \cos\alpha_1 - u_2 \cos\alpha_2) + (v_2 \cos\theta_2 - v_1 \cos\theta_1)}{m_1 + m_2}$$

$$= \frac{u_1 \cos\alpha_1 - u_2 \cos\alpha_2 - e\,(u_2 \cos\alpha_2 - u_1 \cos\alpha_1)}{m_1 + m_2}$$

[using (3)]

$$= \frac{(u_1 \cos\alpha_1 - u_2 \cos\alpha_2)(1+e)}{m_1 + m_2}$$

$$\therefore u_1 \cos\alpha_1 - v_1 \cos\theta_1 = \frac{m_2 (1+e)}{m_1 + m_2} (u_1 \cos\alpha_1 - u_2 \cos\alpha_2)$$

substituting in (5),

$$\text{change in KE} = \frac{1}{2} \frac{m_1 m_2 (1+e)}{m_1 + m_2} (u_1 \cos\alpha_1 - u_2 \cos\alpha_2) \times (u_1 \cos\alpha_1 - u_2 \cos\alpha_2)(1-e)$$

$$= \frac{1}{2} \frac{m_1 m_2}{m_1 + m_2} (1-e^2)(u_1 \cos\alpha_1 - u_2 \cos\alpha_2)^2$$

If the spheres are perfectly elastic, $e = 1$.

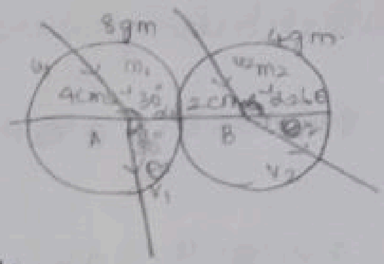
and the loss of kinetic energy is zero.

Eg:

A ball of mass 8gm moving with the velocity $4cms^{-1}$, impinges on a ball of mass 4gms moving with velocity $2cmsec^{-1}$. If their velocities before impact, been inclined at an angle $30°$ and $60°$ to the line joining their centres at the moment of impact. Find their velocities after impact when $e = \frac{1}{2}$

Soln:-

Given:



$m_1 = 8gm$; $m_2 = 4gm$.

$u_1 = 4cms^{-1}$; $u_2 = 2cms^{-1}$.

$\alpha_1 = 30°$; $\alpha_2 = 60°$; $e = 1/2$.

Let V_1 and V_2 be the velocities after impact in the directions making θ_1 and θ_2 angles respectively with A and B

The tangential velocity of each sphere is not affected by impact.

$V_1 \sin\theta_1 = u_1 \sin\alpha_1 \rightarrow (1)$

$V_2 \sin\theta_2 = u_1 \sin\alpha_2 \rightarrow (2)$

Substituting,

$V_1 \sin\theta_1 = (4) \sin 30° = (4)\left(\frac{1}{2}\right) = 2$

$V_2 \sin\theta_2 = (2) \sin 60° = (2)\frac{\sqrt{3}}{2} = \sqrt{3}$.

$\therefore V_1 \sin\theta_1 = 2 \rightarrow (1)$

$V_2 \sin\theta_2 = \sqrt{3} \rightarrow (2)$

By NEL, we have,

$V_2 \cos\theta_2 - V_1 \cos\theta_1 = -e(u_2 \cos\alpha_2 - u_1 \cos\alpha_1)$

$V_2 \cos\theta_2 - V_1 \cos\theta_1 = \frac{-1}{2}\left((2)\cos 60° - (4)\cos 30°\right)$

$$= \frac{-1}{2}\left((2)\frac{1}{2} - 4\cdot\frac{\sqrt{3}}{2}\right)$$

$$= \frac{1}{2}\left(2\sqrt{3} - 1\right)$$

$$\Rightarrow \therefore V_2\cos\theta_2 - V_1\cos\theta_1 = \frac{1}{2}(2\sqrt{3}-1) \dashrightarrow (3)$$

By principles of conservation of momentum,

$$m_1V_1\cos\theta_1 + m_2V_2\cos\theta_2 = m_1u_1\cos\theta_1 + m_2u_2\cos\theta_2$$

substituting,

$$8V_1\cos\theta_1 + 4V_2\cos\theta_2 = (8)(4)\cos 30^\circ + (4)(2)\cos 60$$

$$8V_1\cos\theta_1 + 4V_2\cos\theta_2 = \overset{16}{\cancel{32}}\cdot\frac{\sqrt{3}}{\cancel{2}} + 8\cdot\frac{1}{2}$$

$$\Rightarrow \therefore 8V_1\cos\theta_1 + 4V_2\cos\theta_2 = 16\sqrt{3} + 4 \dashrightarrow (4).$$

Solving (3) and (4) and adding by $\times 8$

$$(3)\times 8 \Rightarrow -8V_1\cos\theta_1 + 8V_2\cos\theta_2 = 8\sqrt{3} - 4$$

$$(4)\times 1 \Rightarrow \quad 8V_1\cos\theta_1 + 4V_2\cos\theta_2 = 16\sqrt{3} + 4$$

$$12V_2\cos\theta_2 = 24\sqrt{3}$$

$$\therefore V_2\cos\theta_2 = 2\sqrt{3} \longrightarrow (5)$$

Sub, in (3)

$$12\sqrt{3} - V_1\cos\theta_1 = \frac{1}{2}(2\sqrt{3}-1)$$

$$V_1\cos\theta_1 = 12\sqrt{3} - 2\sqrt{3} + \frac{1}{2}$$

$$= 10\sqrt{3} + \frac{1}{2} + \sqrt{3}.$$

$\therefore v_1 \cos\theta_1 = \frac{1}{2} + \sqrt{3} \rightarrow (6)$

Squaring (1) and (6), and adding.

$$v_1^2 \sin^2\theta_1 + v_1^2 \cos^2\theta_1 = 4 + \frac{1}{4} + 3 + \sqrt{3}$$

$$v_1^2 (1) = \frac{16+1+12+4\sqrt{3}}{4} = \frac{29+4\sqrt{3}}{4}$$

$$\therefore v_1 = \frac{\sqrt{29+4\sqrt{3}}}{2}$$

2 $2+$ $4+\frac{1}{1}$ $\frac{1}{4}+3+\sqrt{3}$. $\sqrt{3}$

Squaring (2) and (5), and adding,

$$v_2^2 \sin^2\theta_2 + v_2^2 \cos^2\theta_2 = 3 + 4(3)$$

$$v_2^2 (1) = 15$$

$$\therefore v_2 = \sqrt{15}$$

To find the directions:-

(1) ÷ (6), we get,

$$\frac{v_1 \sin\theta_1}{v_1 \cos\theta_1} = \frac{2}{\frac{1}{2}+\sqrt{3}}$$

$$\tan\theta_1 = \frac{4}{1+2\sqrt{3}}$$

$$\therefore \theta_1 = \tan^{-1}\left(\frac{4}{1+2\sqrt{3}}\right)$$

then (2) ÷ (5) we get

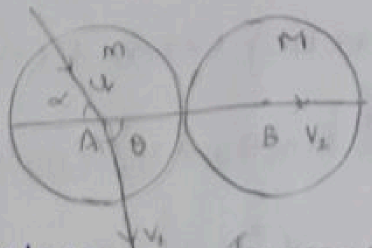
$$\frac{v_2 \sin\theta_2}{v_1 \cos\theta_2} = \frac{\sqrt{3}}{2\sqrt{3}}$$

$$\therefore \tan\theta_2 = \frac{1}{2}$$

$$\theta_2 = \tan^{-1}\left(\frac{1}{2}\right)$$

A smooth sphere of mass m impinges oblique on a smooth sphere of mass M which is at rest. Show that if $m = eM$, the directions of motion after impact are at right angles [e = coefficient of restitution).

Soln:-



Considering the sphere M,

$m = eM$.

To prove: $\theta = 90°$

its tangential velocity before impact is zero.

$\alpha_1 = \alpha$ $m_1 = m$
$\alpha_2 = 0$ $m_2 = M$
$\alpha_2 = 0$ $u_1 = u$; $\theta_1 = \theta$

Hence, after impact also its tangential velocity is zero.

Hence after impact m will move along AB.

Let its velocity be v_2 and let the velocity of m be v_1 at an angle θ_2 to AB after impact.

By NEL,

$$V_2\cos\theta_2 - V_1\cos\theta_1 = -e(u_2\cos\alpha_2 - u_1\cos\alpha_1)$$

(by diagram, as there is no θ_2 omit it).

$\Rightarrow$ $V_2\cos\theta_2 - V_1 u$

$\Rightarrow$ $V_2 - V_1\cos\theta = -e(0 - u\cos\alpha)$

$\Rightarrow$ $V_2 - V_1\cos\theta = eu\cos\alpha \rightarrow (1)$

By principle of conservation of momentum, along AB.

$$m_2 v_2 \cos\theta_2 + m_1 v_1 \cos\theta_1 = m_2 u_2 \cos\alpha_2 + m_1 u_1 \cos\alpha_1$$

$$\Rightarrow M v_2 + m v_1 \cos\theta = M.0 + m.u\cos\alpha$$

$$\Rightarrow M v_2 + m v_1 \cos\theta = m u \cos\alpha \rightarrow (2)$$

$$(1) \times M \Rightarrow M v_2 - M v_1 \cos\theta = Me\, u\cos\alpha \rightarrow (3)$$

Now (2-3)

$$(m+M) v_1 \cos\theta = (m - Me) u \cos\alpha$$

$$\Rightarrow v_1 \cos\theta = \frac{m - Me}{m+M} u\cos\alpha$$

$\Rightarrow$ sub $Me = m$, we get

$$\Rightarrow v_1 \cos\theta = 0$$

$$\Rightarrow \cos\theta = 0 \qquad (\because v_1 \neq 0)$$

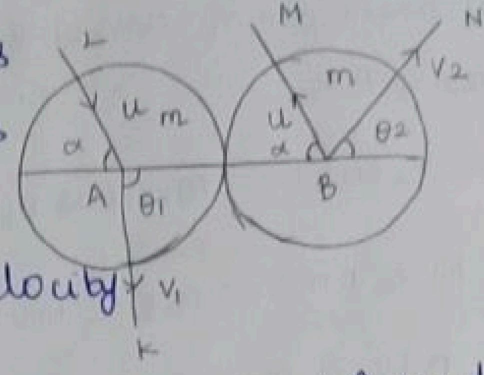
$$\Rightarrow \theta = \cos^{-1}(0)$$

$$\Rightarrow \theta = 90^\circ$$

8. Two equal elastic balls moving in opposite parallel direction with equal speeds impinge on one another. If their inclination of their direction of motion to the line of centres be $\tan^{-1}(\sqrt{e})$ where e is the coefficient of restitution. Show that their direction of motion will be turned through a right angle

Soln:-

Let m be the mass of either sphere AB is the line of impact.



Let 'u' be their velocity Before impact,

the direction of motions are, LA and BM making the same angle 'α' with AB.

After impact,

let the sphere A proceed in the direction AK with the velocity V_1 at angle θ_1 to AB and the sphere B proceed in the direction BN with velocity V_2 at an angle θ_2 to AB.

The tangential velocity of either sphere is not affected by impact.

$\therefore V_1 \sin\theta_1 = u \sin\alpha \rightarrow (1)$

$V_2 \sin\theta_2 = u \sin\alpha \rightarrow (2)$

By NEL,

$V_2 \cos\theta_2 - V_1 \cos\theta_1 = -e(u_2 \cos\alpha_2 - \cancel{u_1} \cos\alpha_2)$

$\Rightarrow V_2 \cos\theta_2 - V_1 \cos\theta_1 = -e(-u\cos\alpha - u\cos\alpha)$

second sphere's direction changes (ie) $-u$

Sub $e=1$,

$V_2 \cos\theta_2 - V_1 \cos\theta_1 = -e(-2u\cos\alpha)$

($\because$ two balls are elastic $e=1$)

$\rightarrow V_2 \cos\theta_2 - V_1 \cos\theta_1 = 2u\cos\alpha \rightarrow (3)$

By principle of conservation of momentum.

$$m_2 v_2 \cos\theta_2 + m_1 v_1 \cos\theta_1 = m_2 u_2 \cos\alpha_2 + m_1 u_1 \cos\alpha_1$$

$$\Rightarrow m v_2 \cos\theta_2 + m v_1 \cos\theta_1 = -mu\cos\alpha + mu\cos\alpha$$

$$\Rightarrow m v_2 \cos\theta_2 + m v_1 \cos\theta_1 = 0 \rightarrow (4)$$

$$(3) \times m \Rightarrow m v_2 \cos\theta_2 - m v_1 \cos\theta_1 = 2mue\cos\alpha \rightarrow (5)$$

Adding (4)+(5), we get,

$$\Rightarrow 2m v_2 \cos\theta_2 = 2mue\cos\alpha$$

$$\Rightarrow v_2 \cos\theta_2 = ue\cos\alpha \rightarrow (6)$$

Sub in (3),

$$\Rightarrow ue\cos\alpha - v_1\cos\theta_1 = 2ue\cos\alpha$$

$$\Rightarrow v_1\cos\theta_1 = -ue\cos\alpha \rightarrow (7)$$

Divide (1) by (7), we get,

$$\Rightarrow \frac{v_1 \sin\theta_1}{v_1\cos\theta} = \frac{u\sin\alpha}{-ue\cos\alpha}$$

$$\Rightarrow \tan\theta_1 = \frac{-1}{e}\tan\alpha$$

$$\Rightarrow \tan\theta_1 = \frac{-1}{e}\sqrt{e} = \frac{-1}{\sqrt{e}} \quad [\because \alpha = \tan^{-1}\sqrt{e}]$$

$$\Rightarrow \tan\theta_1 = \frac{-1}{\tan\alpha} = -\cot\alpha$$

$$\Rightarrow \tan\theta_1 = \tan(90^\circ + \alpha)$$

$$\Rightarrow \theta_1 = 90^\circ + \alpha$$

Dividing (2) by (6), we get,

$$\frac{v_2 \sin\theta_2}{v_2 \cos\theta_2} = \frac{u \sin\alpha}{ue \cos\alpha}$$

$$\Rightarrow \tan\theta_2 = \frac{1}{e}\tan\alpha$$

$$= \frac{1}{e}\sqrt{e}$$

$$= \frac{1}{\sqrt{e}}$$

$$= \frac{1}{\tan\alpha}$$

$$= \cot\alpha$$

$$= \tan(90^\circ - \alpha)$$

$$\theta_2 = 90^\circ - \alpha$$

Hence their direction motions are turned through their right angle.

1/2020

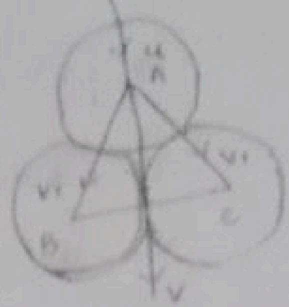
Q Two equal billiard green balls are in contact in smooth surface and a third ball moving along their tangent strikes them simultaneously. Prove that $\frac{3}{5}(1-e^2)$ of its kinetic energy is lost by the impact, e being the coefficient of restitution for each pair of balls.

Soln:

Let A be the centre of the impinging balls and B, C be the centres of the other two balls ...

Since the balls are equal.

ΔABC is equilateral.



Let 'u' be the velocity of A before impact and 'V' be the velocity of A after impact.

Let 'V_1' be the common velocity of 'B' and 'C'.

'V' will be in the same line as 'u' while 'B' and 'C' move along AB and AC respectively.

Let 'm' be the mass of each ball.

By the principle of conservation of momentum for all the three balls along the direction of motion of A,

$$mv + mv_1 \cos 30^\circ + m v_1 \cos 30^\circ = mu$$

$$mv + mv_1 \frac{\sqrt{3}}{2} + mv_1 \frac{\sqrt{3}}{2} = mu$$

$$\div m, \quad v + v_1 \frac{\sqrt{3}}{2} + v_1 \frac{\sqrt{3}}{2} = u$$

$$\Rightarrow \quad v + 2v_1 \frac{\sqrt{3}}{2} = u$$

$$\Rightarrow \quad v + v_1\sqrt{3} = u \quad \longrightarrow (1)$$

Applying NEL, along AB,

$$v_1 - v \cos 30^\circ = -e(0 - u\cos 30^\circ)$$

$$\Rightarrow v_1 - v_1 \frac{\sqrt{3}}{2} = eu\frac{\sqrt{3}}{2}$$

$$\Rightarrow \frac{2v_1 - v_1\sqrt{3}}{2} = \frac{eu\sqrt{3}}{2}$$

$$\Rightarrow 2v_1 - v\sqrt{3} = eu\sqrt{3} \longrightarrow (2).$$

① × $\sqrt{3}$ + ②

$\Rightarrow \sqrt{3}v + 3v_1 = u\sqrt{3}$

$\Rightarrow \sqrt{3}v + 3v_1 = u\sqrt{3}$

$-\cancel{v}\sqrt{3} + 2v_1 = eu\sqrt{3}$

$5v_1 = (e+1)u\sqrt{3}$

$\therefore v_1 = \frac{u\sqrt{3}}{5}(e+1) \dashrightarrow (3)$

Sub in (1), we get

$\Rightarrow v + \frac{u\sqrt{3}}{5}(e+1)^{\sqrt{3}} = u$

$\Rightarrow 5v + u\sqrt{3}(e+1)^{\sqrt{3}} = 5u$

$\Rightarrow 5v = 5u - u\sqrt{3}(e+1)\sqrt{3}$ $u(5-\sqrt{3}e-\sqrt{3})$

$\Rightarrow 5v = u(5-\sqrt{3}(e+1))$

$\Rightarrow v = \frac{u}{5}(5-\overset{3}{\sqrt{3}}(e+1))$

$\Rightarrow v = \frac{u}{5}(5-3e-3)$

$\Rightarrow v = \frac{u}{5}(2-3e)$

The loss of K.E = Initial K.E − Final K.E

$= \frac{1}{2}mu^2 - (\frac{1}{2}mv^2 + \frac{1}{2}mv_1^2 + \frac{1}{2}mv_1^2)$

$= \frac{1}{2}mu^2 - \frac{1}{2}mv^2 - \frac{2mv_1^2}{2}$

$= \frac{1}{2}mu^2 - \frac{1}{2}mv^2 - mv_1^2$

$$= \frac{1}{2}mu^2 - \frac{1}{2}m\frac{u^2}{25}(2-3e)^2 - m\frac{u^2(3)}{25}(e+1)$$

$$= \frac{1}{2}mu^2 \left(1 - \frac{1}{25}(4+9e^2-12e) - \frac{6}{25}(e^2+1+2e) \right]$$

$$= \frac{1}{50}mu^2 (25 - 4 - 9e^2 + 12e - 6e^2 - 6 - 12e]$$

$$= \frac{1}{50}mu^2 [-15e^2 + 15]$$

$$= \frac{15}{50}mu^2 [1 - e^2]$$

$$= \frac{3}{10}mu^2 (1-e^2)$$

$$= \frac{1}{2}mu^2 \cdot \frac{3}{5}(1-e^2)$$

$$= \frac{3}{5}(1-e^2) \times \text{the original K.E}$$

$\therefore$ Loss of K.E $= \frac{3}{5}(1-e^2)$ of its kinetic energy.

Hence proved.

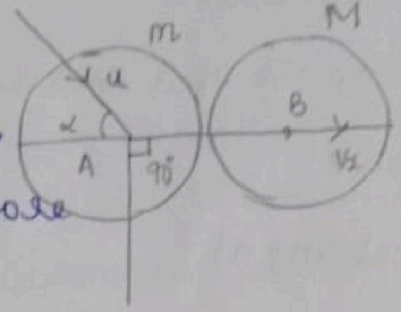
(5). A smooth sphere impinges on another one at rest. After collision, their directions of motion are at right angles. Show that if they are assumed perfectly elastic, their masses must be equal.

(6). A sphere of mass m moving on a horizontal plane with velocity v impinges obliquely on a sphere of mass m' at rest on the same plane. If e = 1 and m = m', prove that their direction of motions after impact are at right angles.

V.Q

(Q-98)

5. Soln:



Consider the sphere m, its tangential velocity before impact is zero.

Hence after impact also its tangential velocity is zero.

Hence after impact M will move along AB. Let its velocity be V_2 and let the velocity of m be V_1 at an angle $\theta(=90^\circ)$ to AB after impact.

By NEL,

$$V_2\cos\theta_2 - V_1\cos\theta_1 = -e(u_2\cos\alpha_2 - u_1\cos\alpha_1)$$

$$V_2 - V_1\cos 90^\circ = -e(-u\cos\alpha)$$

$$\Rightarrow V_2 = eu\cos\alpha \rightarrow (1)$$

By principle of conservation of momentum along AB,

$$m_2V_2\cos\theta_2 + m_1V_1\cos\theta_1 = m_2u_2\cos\alpha_2 + m_1u_1\cos\alpha_1$$

$$\Rightarrow MV_2 + mV_1\cos 90^\circ = mu\cos\alpha \rightarrow (2)$$

$$\Rightarrow MV_2 + mV_1(0) = mu\cos\alpha$$

$$\Rightarrow MV_2 = mu\cos\alpha \rightarrow (3)$$

By (1), if $e=1$, then, $V_2 = u\cos\alpha \rightarrow (4)$

Sub (4) in (3), we get,

$$\Rightarrow M(u\cos\alpha) = m(u\cos\alpha)$$

$$\Rightarrow M = m$$

Hence their masses must be equal.

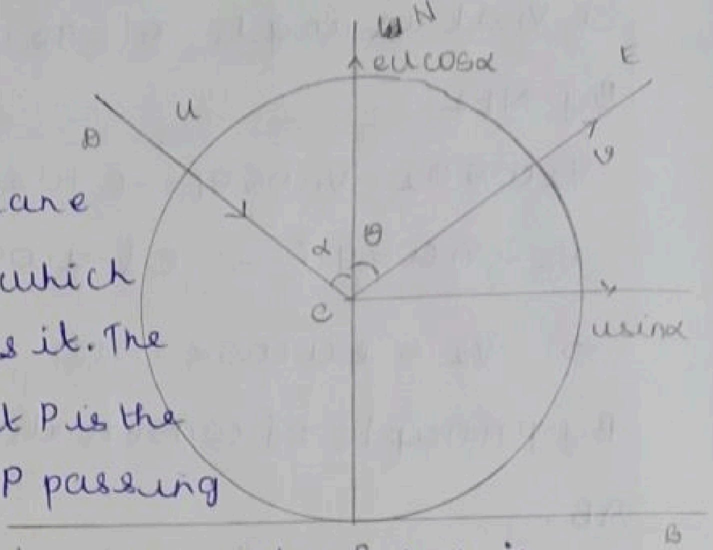
20. Bookwork -

Impact of a smooth sphere on a fixed smooth plane:

Statement:

A smooth sphere, or particle whose mass is m and whose coefficient of restitution is e, impinges obliquely on a smooth fixed plane; to find its velocity and direction of motion after impact.

Proof:



Let AB be the plane and P the point at which the sphere strikes it. The common normal at P is the vertical line at P passing through the centre of the sphere. Let it be PC. This is the line of impact. Let the velocity of the sphere before impact be u at an angle α with CP and v its velocity after impact at an angle θ with CN.

Since the plane and the sphere are smooth, the only force acting during impact is the impulsive reaction and this is along the common normal. There is no force parallel to the plane during impact. Hence the velocity of the sphere, resolved in a direction parallel to the plane is unaltered by the impact.

Hence $v \sin\theta = u \sin\alpha$ ---> (1)

By NEL, the relative velocity of the sphere along the common normal after impact is (-e) time its relative velocity along the common normal before impact. Hence,

$v\cos\theta - 0 = -e(-u\cos\alpha - 0)$

(ie), $v\cos\theta = eu\cos\alpha$ ---> (2)

Squaring (1) and (2), and adding, we have,

$v^2 = u^2(\sin^2\alpha + e^2\cos^2\alpha)$ (ie), $v = u\sqrt{\sin^2\alpha + e^2\cos^2\alpha}$ ---> (3)

Dividing (2) by (1), we have $\cot\theta = e\cot\alpha$ ---> (4)

Hence (3) and (4) give the velocity and direction of motion after impact.

corollary-1:

If $e=1$, we find that form (3). $v=u$ and from (4), $\theta=\alpha$.

Hence if a perfectly elastic sphere impinges on a fixed smooth plane, its velocity is not altered by impact and the angle of reflection is equal to the angle of incidence

corollary-2:

If $e=0$, then from (2), $v\cos\theta = 0$ and $v = u\sin\alpha$

Hence, $\cos\theta = 0$ (ie)., $\theta = 90^\circ$

Hence the inelastic sphere slides along the plane with velocity $u\sin\alpha$.

Corallary-3:

If the impact is direct we have $\alpha = 0$, then $\theta = 0$ and from (3), $v = eu$. Hence if an elastic sphere strikes a plane normally with velocity u, it will rebound in the same direction with velocity eu

Corallary-4:

The impulse of the pressure on the plane is equal and opposite to the impulse of the pressure on the sphere. The impulse I on the sphere is measured by the change in momentum of the sphere along the common normal.

$$I = mv\cos\theta - (-mu\cos\alpha) = m(v\cos\theta + u\cos\alpha)$$

$$= m(eu\cos\alpha + u\cos\alpha)$$

$$= mu\cos\alpha(1+e)$$

Corallary-5:

Loss of kinetic energy due to impact.

$$= \frac{1}{2}mu^2 - \frac{1}{2}mv^2$$

$$= \frac{1}{2}mu^2 - \frac{1}{2}mu^2(\sin^2\alpha + e^2\cos^2\alpha)$$

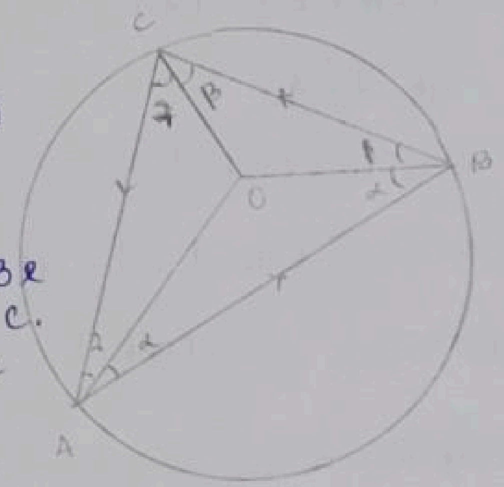
$$= \frac{1}{2}mu^2(1 - \sin^2\alpha - e^2\cos^2\alpha)$$

$$= \frac{1}{2}(1-e^2)mu^2\cos^2\alpha$$

If the sphere is perfectly elastic, $e = 1$ an the loss of kinetic energy is zero.

A smooth circular table is surrounded by smooth rim whose interior surface is vertical. Show that a ball projected along the table from a point A on the rim in a direction making an angle 'α' with the radius through 'a' will return through the point of projection after two impact. If $\tan\alpha = \frac{e^{(3/2)}}{\sqrt{1+e+e^2}}$. Prove also that when the ball returns to the point of projection, its velocity is to its original velocity as $e^{3/2} : 1$

soln:



Let the ball starting from A return to it, after two impact at B & C.

Velocity of projection of A is 'u'.

Angle between radius and the direction of projection is 'α'.

Let 'v' and 'w' be the velocities after first and second impact and β and γ be the angle of reflection after the impact which is shown in the figure.

Let $\angle OBC = \beta$

and $\angle OCA = \gamma$

then $\angle OCB = \beta$

and $\angle OAC = \gamma$

Considering the impact at B,

$\tan\beta = \frac{1}{e}\cdot\tan\alpha \dashrightarrow (1)$

We know that, from B.4.(5)

$\cot\theta = e\cot\alpha$

$\frac{1}{\tan\theta} = e\cdot\frac{1}{\tan\alpha}$

$\frac{1}{e}\cdot\tan\alpha = \tan\theta$

Impact at C,

$\tan\gamma = \frac{1}{e}\cdot\tan\beta \dashrightarrow (2)$

$\Rightarrow \tan\gamma = \frac{1}{e}\cdot\frac{1}{e}\cdot\tan\alpha$

$\Rightarrow \tan\gamma = \frac{1}{e^2}\cdot\tan\alpha \dashrightarrow (3)$

1120. In ΔABC, $\angle A + \angle B + \angle C = 180^\circ$

$\Rightarrow 2\alpha + 2\beta + 2\gamma = 180^\circ$

$\Rightarrow \alpha + \beta + \gamma = 90^\circ$

$\Rightarrow \alpha = 90^\circ - (\beta+\gamma)$

$\Rightarrow \tan\alpha = \tan(90^\circ - (\beta+\gamma))$

$\Rightarrow \tan\alpha = \cot(\beta+\gamma)$

$\Rightarrow \tan\alpha = \frac{1}{\tan(\beta+\gamma)}$

$\Rightarrow \tan\alpha = \frac{1}{\frac{\tan\beta+\tan\gamma}{1-\tan\beta\tan\gamma}}$

$\Rightarrow \tan\alpha = \frac{1-\tan\beta\tan\gamma}{\tan\beta+\tan\gamma}$

$\Rightarrow \tan\alpha(\tan\beta+\tan\gamma) = 1-\tan\beta.\tan\gamma$

$\Rightarrow \tan\alpha\left(\frac{1}{e}\tan\alpha + \frac{1}{e^2}\tan\alpha\right) = 1 - \frac{1}{e}\tan\alpha.\frac{1}{e^2}\tan\alpha$

(By (1) & (3))

$\Rightarrow \frac{1}{e}\tan^2\alpha + \frac{1}{e^2}\tan^2\alpha = 1 - \frac{1}{e^3}\tan^2\alpha$

$\Rightarrow \frac{1}{e}\tan^2\alpha + \frac{1}{e^2}\tan^2\alpha + \frac{1}{e^3}\tan^2\alpha = 1$

$\Rightarrow \tan^2\alpha\left(\frac{1}{e} + \frac{1}{e^2} + \frac{1}{e^3}\right) = 1$

$\Rightarrow \tan^2\alpha\left(\frac{e^2+e+1}{e^3}\right) = 1$

$\Rightarrow \tan^2\alpha = \frac{e^3}{e^2+e+1} \quad \dashrightarrow (4)$

$\Rightarrow \tan\alpha = \frac{e^{3/2}}{\sqrt{1+e+e^2}}$

Hence proved

(ii). W.k.T, $V^2 = u^2(\sin^2\alpha + e^2\cos^2\alpha) \dashrightarrow (5)$

III'ly $W^2 = V^2(\sin^2\beta + e^2\cos^2\beta) \dashrightarrow (6)$

Sub (5) in (6),

$W^2 = u^2(\sin^2\alpha + e^2\cos^2\alpha).(\sin^2\beta + e^2\cos^2\beta)$

$= u^2\cos^2\alpha\left(\frac{\sin^2\alpha}{\cos^2\alpha} + e^2\right).\cos^2\beta\left(\frac{\sin^2\beta}{\cos^2\beta} + e^2\right)$

$= u^2\cos^2\alpha\left(\tan^2\alpha + e^2\right).\cos^2\beta\left(\tan^2\beta + e^2\right)$

$= u^2.\frac{1}{\sec^2\alpha}(\tan^2\alpha + e^2).\frac{1}{\sec^2\beta}(\tan^2\beta + e^2)$

$= \frac{u^2(\tan^2\alpha + e^2)(\tan^2\beta + e^2)}{(1+\tan^2\alpha)(1+\tan^2\beta)}$

$= \frac{u^2(\tan^2\alpha + e^2)\left(\frac{1}{e^2}.\tan^2\alpha + e^2\right)}{(1+\tan^2\alpha)\left(1+\frac{1}{e^2}\tan^2\alpha\right)}$ using (1)

(2)

$$= \frac{u^2 \cancel{(\tan^2\alpha + e^2)} \left(\frac{\tan^2\alpha + e^4}{\cancel{e^2}}\right)}{(1+\tan^2\alpha) \; \frac{\cancel{(e^2 + \tan^2\alpha)}}{\cancel{e^2}}}$$

$$= \frac{u^2(\tan^2\alpha + e^4)}{(1+\tan^2\alpha)}$$

$$\Rightarrow w^2 = \frac{u^2 \left(\frac{e^3}{1+e+e^2} + e^4\right)}{\left(1 + \frac{e^3}{1+e+e^2}\right)}$$

$$= \frac{u^2 \left(\frac{e^3+e^4+e^5+e^6}{\cancel{1+e+e^2}}\right)}{\frac{1+e+e^2+e^3}{\cancel{(1+e+e^2)}}}$$

$$= u^2 \frac{e^3+e^4+e^5+e^6}{1+e+e^2+e^3}$$

$$= u^2 \frac{e^3 \cancel{(1+e+e^2+e^3)}}{\cancel{1+e+e^2+e^3}}$$

$$w^2 = u^2 e^3$$

$$\Rightarrow w = u e^{3/2}$$

$$\Rightarrow \frac{w}{u} = \frac{e^{3/2}}{1}$$

$$\Rightarrow w : u = e^{3/2} : 1$$ Hence proved.

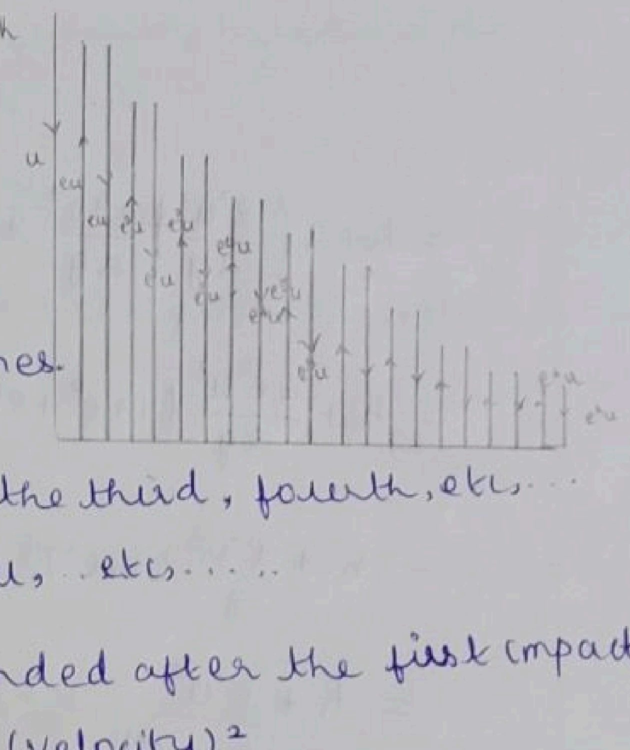
(2). A particle falls from a height 'h' upon a fixed horizontal plane. If 'e' be the coefficient of restitution, show that the whole distance described before the particle has finished rebounding is $h\left(\frac{1+e^2}{1-e^2}\right)$. Show also the whole time taken is $\frac{1+e}{1-e}\left(\sqrt{\frac{2h}{g}}\right)$.

soln:

Let 'u' be the velocity of the particle on first hitting the plane, then $u^2 = 2gh$ --→(1)

After the first impact, the particle rebounce with a velocity 'eu' and ascends a certain height, and makes a second impact with a plane with velocity. eu

After the second impact, it rebounce with a velocity "e^2u" and the process is repeated number of times.



The velocities after the third, fourth, etc,... impacts are e^3u, e^4u, ..etc.....

The height ascended after the first impact with velocity, $eu = \frac{(\text{velocity})^2}{2g}$

$$\text{Height first impact} = \frac{e^2u^2}{2g}$$

The height ascended after the second impact with velocity, $e^2u = \frac{(\text{velocity})^2}{2g}$

$$= \frac{e^4u^2}{2g}$$

The height after the third impact with velocity, $e^3u = \frac{e^6u^2}{2g}$

and so on

Total distance travelled before the particle stops rebounding $\Bigg\} = h + \left(\frac{e^2u^2}{2g} + \frac{e^2u^2}{2g}\right) + \left(\frac{e^4u^2}{2g} + \frac{e^4u^2}{2g}\right) + \left(\frac{e^6u^2}{2g} + \frac{e^6u^2}{2g}\right) + \ldots$

$$= h + 2\left(\frac{e^2u^2}{2g} + \frac{e^4u^2}{2g} + \frac{e^6u^2}{2g} + \ldots\right)$$

$$= h + \frac{2e^2u^2}{2g}(1 + e^2 + e^4 + \ldots)$$

$$= h + \frac{e^2u^2}{g}(1 - e^2)^{-1}$$

$$= h + \frac{e^2u^2}{g(1-e^2)}$$

Total distance $= h + \frac{e^2u^2}{g(1-e^2)}$

$$= h + \frac{e^2 \cdot 2\cancel{g}h}{\cancel{g}(1-e^2)} \quad [\text{from (1)}]$$

$$= h\left[1+\frac{2e^2}{1-e^2}\right]$$

$$= h\left[\frac{1-e^2+2e^2}{1-e^2}\right]$$

$$= h\left[\frac{1+e^2}{1-e^2}\right]$$

$\therefore$ Total distance $= h\left[\frac{1+e^2}{1-e^2}\right]$ //.

considering the motion before the first impact, we have the initial velocity, $v_1 = 0$, acceleration $= g$, final velocity $= u_2$ and so if 't' is time taken, $u = 0 + gt$,

$$\therefore t = \frac{u}{g} = \frac{\text{velocity}}{g}$$

Time interval between first and second impact is,

= 2 × time taken for gravity to reduce the velocity eu to 0.

$$= 2 \times \frac{\text{velocity}}{g} = \frac{2eu}{g}$$

Similarily time interval between the second and third impacts,

$$= \frac{2e^2u}{g} \text{ and so on.}$$

So total time taken,

$$= \frac{u}{g} + 2\left(\frac{eu}{g} + \frac{e^2u}{g} + \frac{e^3u}{g} + \dots \infty\right)$$

$$= \frac{u}{g} + \frac{2eu}{g}\left(1+e^2+e^3+\dots\infty\right)$$

$$= \frac{u}{g} + \frac{2eu}{g} \cdot \frac{1}{1-e} = \frac{u}{g}\left[1+\frac{2e}{1-e}\right]$$

$$= \frac{u}{g}\left(\frac{1+e}{1-e}\right)$$

$$= \frac{\sqrt{2gh}}{g}\left(\frac{1+e}{1-e}\right)$$

$$= \left(\frac{1+e}{1-e}\right)\sqrt{\frac{2h}{g}}$$

Hence proved.

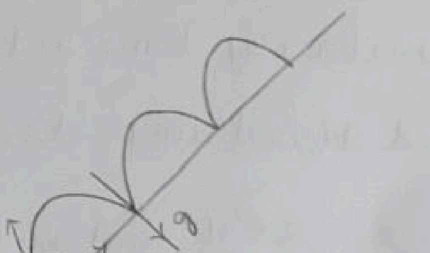
19/11/2020 Q. A particle is projected from a point on an inclined plane and at the r^{th} impact, it strikes the plane perpendicularily and at the nth impact, is at the point of projection. Show that

$e^n - 2e^r + 1 = 0$.

Soln:

Let 'α' be the inclination of the plane to the horizontal and 'u' be the velocity of projection at an angle θ to the inclined plane.

This velocity can be ressolved into two components:



(i). $u\cos\theta$ along the upward inclined plane.

(ii) $u\sin\theta$ $\perp$'r to the inclined plane.

The acceleration 'g' can be ressolved into two components:

(i). $g\sin\alpha$ along the downward inclined plane.

(ii). $g\cos\alpha$ ⊥'r to the inclined plane

Consider the motion ⊥'r to the plane. Let 't_1' be the time up to first impact.

Distance travelled ⊥'r to the plane in time t_1 is 0.

$$\therefore 0 = u\sin\theta . t_1 - \frac{1}{2} g\cos\alpha . t_1^2$$

$$0 = (u\sin\theta - \frac{1}{2} g\cos\alpha t_1) . t_1$$

$$\Rightarrow u\sin\theta - \frac{1}{2} g\cos\alpha t_1 = 0$$

$$\Rightarrow \frac{1}{2} g\cos\alpha t_1 = u\sin\theta$$

$$\Rightarrow t_1 = \frac{2u\sin\theta}{g\cos\alpha} \dashrightarrow (1)$$

The particle strikes the plane first time with the velocity $u\sin\theta$ ⊥'r to it.

After this impact, this component is reversed as $eu\sin\theta$.

∴ Time interval between the first & second impact is $= \frac{2eu\sin\theta}{g\cos\alpha}$

The particle strikes the plane a second time with a velocity $eu\sin\theta$ ⊥'r to it.

After this II^{nd} impact, this component is reversed as $e^2u\sin\theta$.

∴ Time interval between the second & third impact $= \frac{2e^2u\sin\theta}{g\cos\alpha}$

$\therefore$ Time till the r^{th} impact,

$$= \frac{2u\sin\theta}{g\cos\alpha} + \frac{2eu\sin\theta}{g\cos\alpha} + \frac{2e^2u\sin\theta}{g\cos\alpha} + \dots + r\text{ ter}$$

$$= \frac{2u\sin\theta}{g\cos\alpha}(1+e+e^2+\dots r\text{ terms})$$

$$= \frac{2u\sin\theta}{g\cos\alpha}\left(\frac{1-e^r}{1-e}\right) \dashrightarrow (2)$$

At the end of this time, the particle strikes the plane $\perp$'r. So the velocity parallel to the plane at that instant $= 0$. $(v = u + at)$

$$\therefore u\cos\theta - g\sin\alpha \frac{2u\sin\theta}{g\cos\alpha}\left(\frac{1-e^r}{1-e}\right) = 0$$

$$\Rightarrow u\cos\theta = \frac{2u\sin\theta\sin\alpha}{\cos\alpha}\left(\frac{1-e^r}{1-e}\right)$$

$$\Rightarrow \cos\theta\cos\alpha(1-e) = 2\sin\theta\sin\alpha(1-e^r) \dashrightarrow (3)$$

Put $r = n$, in (2),

Time till the n^{th} Impact $= \frac{2u\sin\theta}{g\cos\alpha}\left(\frac{1-e^n}{1-e}\right)$

Now it is at the point of projection.

Hence distance travelled parallel to the plane upto this time $= 0$

$s = ut + \frac{1}{2}at^2$

$$u\cos\theta \cdot \frac{2u\sin\theta}{g\cos\alpha}\left(\frac{1-e^n}{1-e}\right) - \frac{1}{2}g\sin\alpha\left(\frac{2u\sin\theta}{g\cos\alpha}\left(\frac{1-e^n}{1-e}\right)\right.$$

$$\Rightarrow \left[u\cos\theta - \frac{1}{2}g\,n\frac{2u\sin\theta}{g\cos\alpha}\left(\frac{1-e^n}{1-e}\right)\right]\left[\frac{2u\sin\theta}{g\cos\alpha}\left(\frac{1-e^n}{1-e}\right)\right]$$

term

(either must be zero, so)

$$\Rightarrow u\cos\theta - \frac{u\sin\theta \overset{\sin\alpha}{}}{\cos\alpha}\left(\frac{1-e^n}{1-e}\right) = 0$$

$$\Rightarrow u\cos\theta = \frac{u\sin\theta(1-e^n)\cdot\sin\alpha}{\cos\alpha(1-e)}$$

$$\Rightarrow \cos\theta\cos\alpha(1-e) = \sin\theta\sin\alpha(1-e^n) \rightarrow (4)$$

Divide (3) by (4),

at

$$\frac{\cos\theta\cos\alpha(1-e)}{\cos\theta\cos\alpha(1-e)} = \frac{2\sin\theta\sin\alpha(1-e^r)}{\sin\theta\sin\alpha(1-e^n)}$$

$$\Rightarrow 1 = \frac{2(1-e^r)}{1-e^n}$$

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$v^2 = u^2 + 2as$

$$\Rightarrow 1-e^n = 2-2e^r$$

$$\Rightarrow 2e^r - 2 + 1 - e^n = 0$$

$$\Rightarrow 2e^r - e^n - 1 = 0$$

$$\Rightarrow e^n - 2e^r + 1 = 0.$$

Hence proved.

(4). A ball is thrown from the point on a smooth horizontal ground with a speed V at an angle α to the horizon. If 'e' be the coefficien of restitution, show that the total time for whi the ball rebounce on the ground is $\frac{2v\sin\alpha}{g(1-e)}$ and the horizontal distance travelled by it is

$$\frac{V^2\sin 2\alpha}{g(1-e)}.$$

20. Proof:

The initial horizontal and vertical components of velocity are,

$v\cos\alpha$ and $v\sin\alpha$

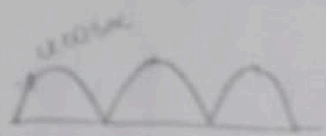
The particle describes the parabola and strikes the horizontal plane with velocity $u\sin\alpha$.

Due to impact, the horizontal velocity is not affected and the vertical component is reversed as $eV\sin\alpha$.

III'ly the vertical components of the velocity after 2 second, third, etc... impacts are $e^2V\sin\alpha$, $e^3V\sin\alpha$...

Let $t_1, t_2, t_3, \ldots$ be the times for the successive trajectories.

$$t_1 = \frac{2V\sin\alpha}{g}$$



$$t_2 = \frac{2eV\sin\alpha}{g}$$

$$t_3 = \frac{2e^2V\sin\alpha}{g}$$

and so on

$\therefore$ Total time that elapses before the particle stops rebounding $\Big\} = t_1 + t_2 + t_3 + \ldots$

$$= \frac{2V\sin\alpha}{g} + \frac{2eV\sin\alpha}{g} + \frac{2e^2V\sin\alpha}{g}$$

$$= \frac{2V\sin\alpha}{g}(1 + e + e^2 + \ldots)$$

$$= \frac{2V\sin\alpha}{g}(1-e)^{-1}$$

$$= \frac{2V\sin\alpha}{g(1-e)}$$

Throughout this time,

the horizontal component $V\cos\alpha$ is not affecte

$\therefore$ Horizontal distance described during this time

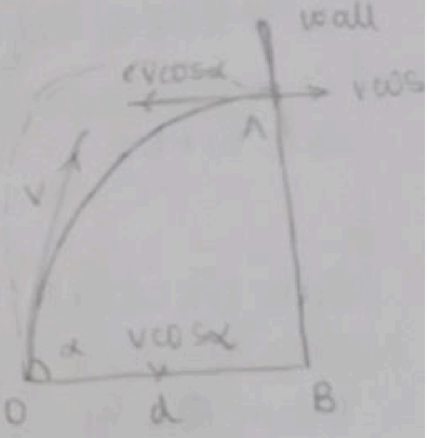
$$= V\cos\alpha \cdot \frac{2V\sin\alpha}{g(1-e)}$$

$$= \frac{V^2\, 2\sin\alpha\cos\alpha}{g(1-e)}$$

$$= \frac{V^2\sin 2\alpha}{g(1-e)}$$

Hence proved.

(5). An elastic sphere is projected from the given point 'O' with a given velocity 'V' at an inclinatio 'α' to the horizontal. After hitting the smooth vertical wall, at a distance 'd' from O and return to 'O'. Prove that $d = \frac{V^2\sin 2\alpha}{g} \frac{e}{1+e}$ where e is the coefficient of restitution.



Let the particle strikes the wall at A. From O to A, the particle describes the parabola with constant horizontal velocity, $V\cos\alpha$.

Let t_1 be the time for this.

$$\therefore V\cos\alpha \cdot t_1 = d$$

At the impact at 'A', there is no force parallel to the wall.

The component $v\cos\alpha$ being ⊥'r to the wall is reversed as $ev\cos\alpha$. The particle will describe another parabola with constant horizontal velocity $ev\cos\alpha$ and return to 'O'. Let t_2 be the time for this.

$$ev\cos\alpha \cdot t_2 = d \dashrightarrow (2)$$

The vertical motion is not affected by impact and throughout the interval t_1+t_2, it is subject to retardation, g, only.

As the particle returns to 'O', the vertical distance described in time t_1+t_2 is zero.

$$0 = v\sin\alpha\,(t_1+t_2) - \frac{1}{2}g\,(t_1+t_2)^2 \quad s = ut + \frac{1}{2}at^2$$

$$\Rightarrow v\sin\alpha(t_1+t_2) = \frac{1}{2}g\,(t_1+t_2)^2$$

$$\Rightarrow v\sin\alpha = \frac{1}{2}g\,(t_1+t_2)$$

$$\Rightarrow t_1+t_2 = \frac{2v\sin\alpha}{g} \dashrightarrow (3)$$

Sub (1), (2) in (3), we get,

From ①, $t_1 = \frac{d}{v\cos\alpha}$, from ②, $t_2 = \frac{d}{ev\cos\alpha}$

∴ (3) becomes,

$$\frac{d}{v\cos\alpha} + \frac{d}{ev\cos\alpha} = \frac{2v\sin\alpha}{g}$$

$$\Rightarrow \frac{d}{v\cos\alpha}\left(\frac{1+e}{e}\right) = \frac{2v\sin\alpha}{g}$$

$$\Rightarrow d = \frac{2v\sin\alpha \cdot v\cdot\cos\alpha \cdot e}{g(1+e)}$$

$$= \frac{2v^2 e\sin\alpha\cos\alpha}{g(1+e)}$$

$$= \frac{v^2 e\sin 2\alpha}{g(1+e)}$$

$$\Rightarrow d = \frac{v^2\sin 2\alpha}{g}\cdot\frac{e}{1+e}$$

Hence proved

(6). A heavy ball drops from the ceiling of a room and after rebounding twice from the floor reaches a height $= \frac{1}{2}$ of that of ceiling. Show that the coefficient of restitution is $\left(\frac{1}{2}\right)^{1/4}$.

(7) A ball falls from a height of 64 cm on a smooth horizontal plane. If the coefficient of restitution be $\frac{1}{2}$. Find the height to which it rises after rebounding 4 times. ($8/9$)

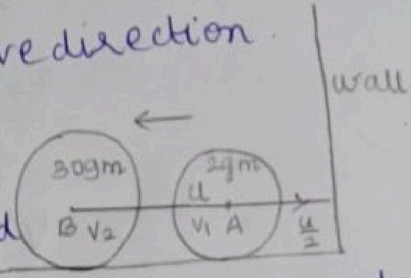
Examples to direct impact :

(5).(8). Two equal spheres A and B of masses 2 g and 3.0 g respectively lie on a smooth floor, so that their line of centres is ⊥'r to the fixed vertical wall A being nearer to the wall. A is projected towards B show that if their coefficient of restitution between the two spheres and that between the first sphere and the wall is 3/5, then A will be reduced to rest after it's second impact with B.

soln:

Consider the impact between A and B.

Taking AB as positive direction.



Let the velocity of A before impact be 'u' and B is at rest. After the impact, let the velocities of A and B be V_1 and V_2 respectively in the same direction.

Given, $e = \frac{3}{5}$

By NEL,

$V_2 - V_1 = -e(0-u)$

$\Rightarrow V_2 - V_1 = \frac{3}{5}u \dashrightarrow (1)$

By principle of conservation of momentum alon AB,

$30V_2 + 2V_1 = 30(0) + 2(u)$

$\Rightarrow 30V_2 + 2V_1 = 2u$

$\Rightarrow 15V_2 + V_1 = u \rightarrow (2)$

solving (1) and (2), sub adding (1) and (2),

$16V_2 = \left(\frac{3}{5}+1\right)u$

$\Rightarrow 16V_2 = \frac{8}{5}u$

$\Rightarrow 2V_2 = \frac{1}{5}u$

$\Rightarrow V_2 = \frac{u}{10}$

Sub v_2 in (1),

$\Rightarrow \frac{u}{10} - v_1 = \frac{3}{5}u$

$\Rightarrow v_1 = -\frac{3u}{5} + \frac{u}{10}$

$\Rightarrow v_1 = \frac{-2u}{10} = \frac{-u}{2}$

Since v_1 is negative, the velocity of A after impact towards the wall and is $= \frac{u}{2}$

and the velocity of $B = \frac{u}{10}$ away from the wall.

Now A strikes the wall with a velocity $\frac{u}{2}$ after this impact, its velocity will be reversed as,

$e v_1 = e \cdot \frac{u}{2} = \frac{3}{5} \cdot \frac{u}{2} = \frac{3u}{10}$

With this velocity A moves in the direction away from the wall and strikes B a second time.

Let the velocities of A and B be v_3 and v_4 after this impact.

	A(2)	B(30)
Before impact	$\frac{3u}{10}$	$\frac{u}{10}$
After impact	v_3	v_4

By NEL,

$(v_4 - v_3) = -e(u_4 - u_3)$

$\Rightarrow v_4 - v_3 = -\frac{3}{5}\left(\frac{u}{10} - \frac{3u}{10}\right)$

$\Rightarrow v_4 - v_3 = \frac{-3}{5}\left(\frac{-2u}{10}\right)$

$\Rightarrow v_4 - v_3 = \frac{3u}{25} \quad \text{---}\rightarrow (3)$

By principle of conservation of momentum along AB,

$\Rightarrow 30v_4 + 2v_3 = 30.\frac{u}{2} + 2\,\frac{3u}{10}$

$\Rightarrow (15v_4 + v_3) = \frac{15u+3u}{10}$

$\Rightarrow 15v_4 + v_3 = \frac{18u}{10}$

$\Rightarrow 15v_4 + v_3 = \frac{9u}{5}$ —→(4)

Solving (3) and (4),

$\Rightarrow 16v_4 = \frac{3u}{25} + \frac{45u}{25}$

$\Rightarrow 16v_4 = \frac{48u}{25}$

$\Rightarrow \therefore v_4 = \frac{3u}{25}$

Sub v_4 in (3),

$\Rightarrow \frac{3u}{25} - v_3 = \frac{3u}{25}$

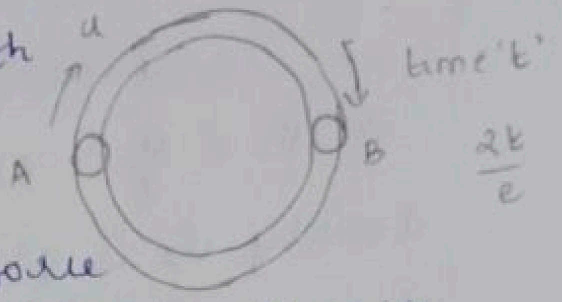
$\therefore v_3 = 0$

$\therefore v_3 = 0 \Rightarrow$ A is reduced to rest after its rest its second impact with "B".

Q. Two equal marble balls A, B lie in the horizontal circular grove at the opposite ends of the diameter. A is projected along the grove and after time 't' impinges on B. Show that a second impact takes place after the further interval $\frac{2t}{e}$.

Soln:

Let the ball A move with velocity 'u'.



As there is no tangential force acting on A at any point of its path, then its speed remains the same throughout.

Hence it impinges on 'B' with a velocity 'u'.

Since the time from 'A' to 'B' is = 't'. We get $ut = \pi r$

$$\Rightarrow u = \frac{\pi r}{t} \dashrightarrow (1)$$

Let v_1 and v_2 be the velocities of A and B respectively after impact.

By Newton's Exprimental law.

$$\Rightarrow v_2 - v_1 = -e(u_2 - u_1)$$

$$\Rightarrow v_2 - v_1 = -e(0 - u)$$

$$\Rightarrow v_2 - v_1 = eu \dashrightarrow (2)$$

By principle of conservation of momentum,

$$\Rightarrow mv_2 + mv_1 = mu_2 + mu_1 \quad (= mu)$$

where 'm' being the mass of each ball.

$$\Rightarrow v_2 + v_1 = u \dashrightarrow (3)$$

Solving (2) and (3),

$$\Rightarrow v_2 = \frac{u(1+e)}{2}$$

Sub,

$$\frac{u(e+1)}{2} - v_1 = eu$$

$$u(e+1) - eu$$

$$\Rightarrow v_1 = \frac{eu + u - 2eu}{2}$$

$$= u - eu/2$$

$$\Rightarrow v_1 = \frac{u}{2}(1-e)$$

clearly v_2 is greater than v_1.

Hence B more in advance of A.

let it strike A again t_1 sec, after the imp which is first.

The velocity of 'B' relative to 'A' after the first impact $\left.\right\}$ $= v_2 - v_1$

$= eu$ (from (2))

Before striking again B should cover the distance in equal length to the circumference relative to 'A'.

$$\therefore (v_2 - v_1).t_1 = 2\pi r \quad \because \text{(from (2))}$$

$$\Rightarrow eu.t_1 = 2\pi r$$

$$\Rightarrow t_1 = \frac{2\pi r}{eu}$$

$$\Rightarrow t_1 = \frac{2\pi r}{\frac{e\pi r}{t}} \quad \text{(from (1))}$$

$$\Rightarrow t_1 = \frac{2t}{e}$$

$\therefore$ Second impact occurs $\frac{2t}{e}$ sec after the first impact.