UNIT - V

Moment Of Inertia

If m is the mass of a particle and r is its $\perp^r$ distance from a given line, the quantity mr^2 is called the moment of inertia of the particle about the line.

If a series of particles of masses $m_1, m_2, \ldots$ etc are arranged at points where $\perp^r$ distance from a given straight line are $r_1; r_2, \ldots$, then the quantity $m_1 r_1^2 + m_2 r_2^2 + \ldots$ is defined to be the moment of inertia of the system of particles about the line and written as $\Sigma m r^2$.

If we write moment of inertia of a body of mass M about an axis as Mk^2, then k is called the radius of gyration of the body about the line.

The theorem

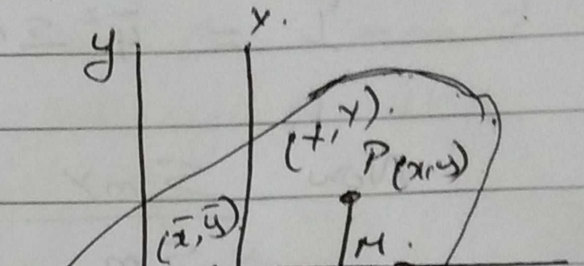
The theorem of Parallel Axes.

If I is the M.I. of a body about any axis and I_G is its M.I. about a parallel axis through G, the C.G. of body then $I = I_G + Mh^2$, where M is the mass of the body and h is the distance between the two parallel axes.

Proof:-

Let us consider the case of a plane lamina.

Let its M.I. about a line Ox in its plane be I.



Oy is $\perp^r$ to Ox in the same plane.

Let G be the C.G. of the lamina with coordinates $(\bar{x}, \bar{y})$ referred to Ox, Oy.

Take an elementary mass m at the point P whose coordinates are (x, y) referred to Ox, Oy.

Let P be (X, Y) referred to GX and GY, a set of $\parallel^{el}$ axes to Ox and Oy.

$\therefore x = ON = OK + KN = \bar{x} + GM = \bar{x} + X$

$$y = NP = NM + MP = KG + MP = \bar{y} + Y.$$

Now I = M.I. of the lamina about Ox

$$= \Sigma m\, PN^2$$

$$= \Sigma m y^2 = \Sigma m (\bar{y} + Y)^2$$

$$= \Sigma m (\bar{y}^2 + 2\bar{y}Y + Y^2).$$

$$I = \bar{y}^2 \Sigma m + 2\bar{y}\, \Sigma mY + \Sigma m Y^2 \quad \text{——} (1).$$

Now $\dfrac{\Sigma mY}{\Sigma m}$ will give the y-coordinate of the C.G. of the lamina with respect to axes GX and GY.

But it must be equal to zero as G itself is the origin in this system.

Hence the 2nd term in (1) is zero.

Also Σm = total mass M.

$\Sigma m Y^2 = \Sigma m\, PM^2$ = M.I. about $GX = I_G$.

$\bar{y} = GK$ = distance b/w the ||el axes Ox and GX

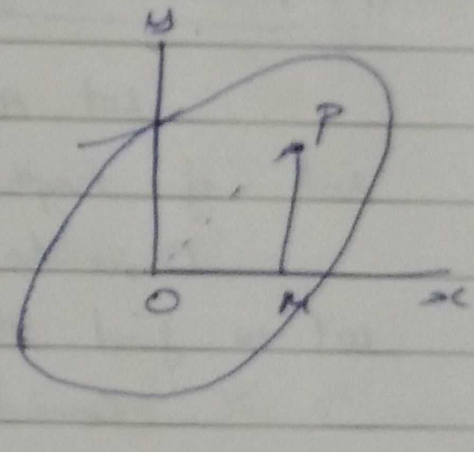
$= h.$

$\therefore \; ① \Rightarrow I = h^2 M + I_G$

$= I_G + M h^2$ //

The Theorem Of Perpendicular Axes:

If I_x and I_y denote the moment of inertia of a plane lamina about two rectangular axes Ox and Oy in the plane, the $I_x + I_y$ will be its moment of inertia about an axis through O $\perp^r$ to its plane (ie) to the plane xOy.



Let Ox and Oy be two rectangular axes in the plane of the lamina

Let OZ be the axis through O $\perp^r$ to the plane xOy

Consider an elementary mass m, at P. Draw PM $\perp^r$ to Ox.

I_x = M.I. of the lamina about Ox

$= \Sigma m\, PM^2$.

I_y = M.I. about Oy $= \Sigma m\, OM^2$

M.I. of the lamina about $OZ = \Sigma m\, OP^2$ ($\because$ OP is the $\perp^r$ distance of P from OZ)

$$= \sum m (PM^2 + OM^2)$$

$$= \sum m PM^2 + \sum m OM^2$$

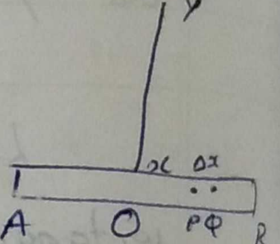
$$= I_x + I_y$$

Note :-

The theorem is applicable only to a plane lamina while the $\parallel^{el}$ axes theorem is true for any rigid body.

Moments of Inertia in some particular Cases.

(1) A thin uniform rod.



Let AB be a thin uniform rod of length 2a.

O is the midpoint and OY is $\perp^r$ to AB.

Let us find the M.I. of the rod about OY.

Consider an elementry section PQ of the rod of length Δx at a distance x from O.

If ρ is the mass per unit length of the rod,

mass of the section PQ $= \rho . \Delta x$.

The M.I of this section about OY $= \rho . \Delta x . x^2$.

Hence M.I of the rod AB about OY $\Big\} = \sum_{x=-a}^{a} \rho \cdot \Delta x \cdot x^2$

$$= \int_{-a}^{a} \rho x^2 dx = \rho \left[\frac{x^3}{3}\right]_{-a}^{a}$$

$$= \rho \left[\frac{a^3}{3} - \left(-\frac{a^3}{3}\right)\right]$$

$$= \rho \left[\frac{2a^3}{3}\right].$$

$$= \frac{M}{2a} \cdot \frac{2a^3}{3} \quad \left(\because \rho = \frac{M}{2a}\right)$$

$$= \frac{Ma^2}{3}.$$

$2a - M$

$1 - (\rho)$

$\rho = \frac{M}{2a}$

Note:

(i) M.I of the rod AB about the line through A $\perp^r$ to AB

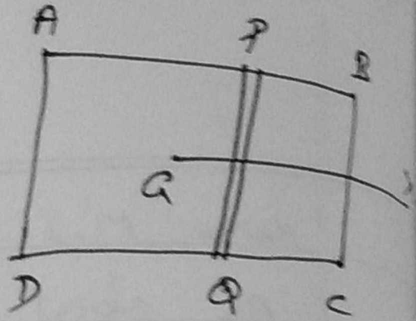
$= M.I$ about $OY + M.(AO)^2$ using the $\parallel^{el}$ axes theorem.

$$= \frac{Ma^2}{3} + Ma^2 = \frac{4Ma^2}{3}.$$

(ii) M.I of the rod AB about itself $= 0$.

(2) Rectangular lamina:

Let ABCD be a uniform rectangular lamina $\ni$ $AB = 2a$, $BC = 2b$ and



G its centre of inertia.

To obtain its M.I about Gx, $\parallel$el to AB, we divide the rectangle into elementary strips such as PQ $\perp$r to Gx.

Each strip is of length 2b and can be considered as a thin uniform rod.

Hence M.I. of the strip about $Gx =$ its mass $\times \frac{b^2}{3}$.

Summing up the moment of intertia of such strips, we get the M.I. of the whole rectangle about Gx to be $\frac{Mb^2}{3}$ where M is the mass of the lamina.

$\parallel$ly M.I. about a line through G $\parallel$el to the side 2b is $\frac{Ma^2}{3}$.

By the theorem of $\perp$r axes, The M.I. of the ~~rod~~ lamina about an axis through G $\perp$r to its plane $= \frac{Ma^2}{3} + \frac{Mb^2}{3} = \frac{M}{3}(a^2+b^2)$.

Also by the theorem of $\parallel$el axes

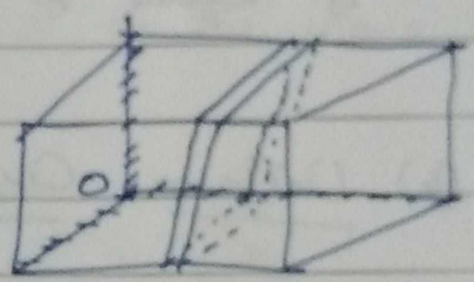
$$\text{M.I about AB} = \text{M.I about } Gx + Mb^2$$
$$= \frac{Mb^2}{3} + Mb^2 = \frac{M4b^2}{3}.$$

|||ly M.I. about AD = $M.\frac{4a^2}{3}$.

M.I. about an axis through A ⊥r to its plane = $M.\frac{4a^2}{3} + M.\frac{4b^2}{3} = \frac{4M}{3}(a^2+b^2)$.

(3) Uniform rectangular parallelopiped of edge 2a, 2b, 2c

Let us find the M.I. about an axis through the centre O ||el to the side 2a.



Divide the solid into a very large number of thin ||el rectangular slices all ⊥r to the above axis

Each ~~rectangle is~~ slice is a rectangle with sides 2b and 2c and its

M.I. about the axis = mass$\cdot\left(\frac{b^2+c^2}{3}\right)$.

Hence M.I. of the whole solid about the axis $\Big\}$ = $M.\left(\frac{b^2+c^2}{3}\right)$

where M is the whole mass.

|||ly the M.I. about the two axes through

the centre ||el to the edges of length 2b and 2c are

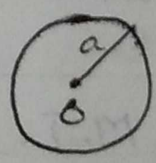
$$\frac{M}{3}(a^2+c^2) \text{ and } \frac{M}{3}(a^2+b^2).$$

If the rectangular parallelopiped is a cube of edge 2a, then in the above result $b=c=a$ and hence M.I. about any of the three axes of symmetry through the centre is equal to $\frac{M}{3}(2a^2)$.

(4) <u>Uniform</u> <u>Circular Ring</u>.

(a) About an axis through the centre $\perp^r$ to its plane

Let a be the radius of the ring. Then each particle of the ring is at the same distance 'a' from the above axis.



Hence M.I of the element = mass $\times a^2$.

So M.I. of the ring $= \sum \text{mass} \times a^2 = Ma^2$

where M is the mass of the ring.

(b) ~~Abot~~ <u>About</u> a <u>diameter</u>.

Let I be the M.I. of the circular ring about a diameter.

By symmetry, I is also the M.I. about any other diameter.

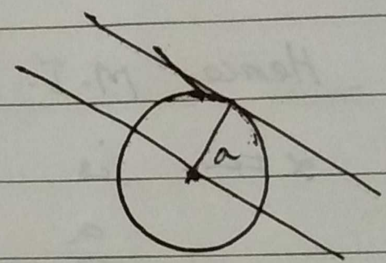
Take two diameters at right angles.

By the theorem of $\perp^r$ axes

$I + I$ = M.I. of the ring about an axis through the centre $\perp^r$ to the plane

$= Ma^2$.

$\Rightarrow I = \frac{Ma^2}{2}$.



(c) <u>About a tangent line</u>

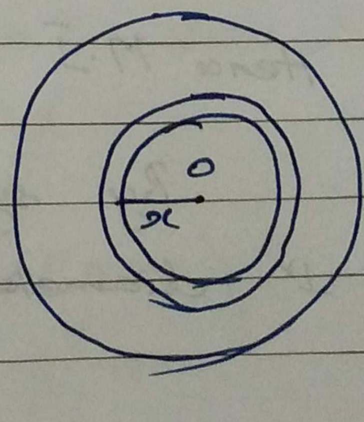
By theorem of $\parallel^{el}$ axes

M.I. of the circular ring about a tangent line = M.I. about a $\parallel^{el}$ diameter $+ Ma^2$.

$= \frac{Ma^2}{2} + Ma^2$.

$= M\frac{3a^2}{2}$.

5). <u>Uniform Circular Disc</u>



Let O be the centre of the disc, a its radius and ρ be the mass per unit ~~ar~~ area.

Divide the disc into concentric rings of breath Δx.

The area contained between two consecutive rings of radii x and $x+\Delta x$ is $2\pi x \cdot \Delta x$.

The mass of this element is $2\pi \rho x \cdot \Delta x$.

M.I. of this ring about a diameter $= \text{mass}\,\frac{x^2}{2} = 2\pi \rho x \cdot \frac{x^2}{2}\Delta x$

$$= \pi \rho x^3 \Delta x.$$

Hence M.I. of the whole disc about a diameter $x = a$ is

$$\sum_{x=0}^{a} \pi \rho x^3 \Delta x = \pi\rho \int_0^a x^3 dx. = \pi\rho \left(\frac{x^4}{4}\right)_0^a.$$

$$= \pi \rho \frac{a^4}{4}.$$

The area of the whole disc $= \pi a^2$.

and so $\pi a^2 \rho = M$, its total mass.

Hence M.I $= \pi a^2 \rho \cdot \frac{a^2}{4.} = \frac{Ma^2}{4}$.

By symmetry M.I. of the disc about a $\perp^r$ diameter is also $\frac{Ma^2}{4}$.

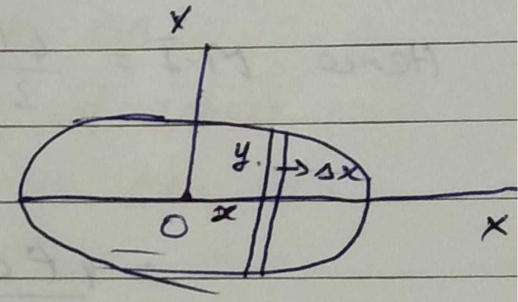
Hence by the theorem of $\perp^r$ axes, M.I about an axis through the centre $\perp^r$ to the disc $= \frac{Ma^2}{4} + \frac{Ma^2}{4} = \frac{Ma^2}{2}$.

Note:

By the theorem of $\parallel^{el}$ axes.
M.I. of the circular disc about a tangent line = M.I. about a $\parallel^{el}$ diameter + Ma^2.

$$= \frac{Ma^2}{4} + Ma^2 = M.\frac{5a^2}{2}.$$

6). Uniform elliptic lamina (axes 2a, 2b) about these axes.



To find the M.I. about the major axis, let (x, y) be any point on the ellipse.

Divide the area into elementary rectangular strips $\perp^r$ to the major axis.

Area of one typical section of length 2y and breadth $\Delta x = 2y\Delta x$.

If ρ is the mass per unit area, the mass of this section $= 2y\rho\Delta x$.

M.I. about the major axis $= 2y\rho\Delta x \cdot \frac{y^2}{3}$

$$= 2\rho \frac{y^3}{3} \Delta x.$$

$$\int_0^{\pi/2} \sin^n\theta \, d\theta = \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdots \frac{\pi}{2} \quad \text{" " " odd}$$

$$= \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdots \frac{3}{2}$$

Hence M.I. of the whole ellipse $= 2 \times \sum_{x=0}^{a} 2\rho \frac{y^3}{3} \Delta x$

$$= \frac{4\rho}{3} \int_0^a y^3 \, dx.$$

We know that in the ellipse $x = a\cos\theta$ and $y = b\sin\theta$ where θ is the eccentric angle of the point

$x = a\cos\theta \Rightarrow dx = -a\sin\theta \, d\theta$.

$x = 0 \Rightarrow a\cos\theta = 0 \Rightarrow \theta = \pi/2$.

$x = a \Rightarrow a\cos\theta = a \Rightarrow \theta = 0$.

Hence $M.I = \frac{4\rho}{3} \int_{\pi/2}^{0} b^3 \sin^3\theta \, (-a\sin\theta \, d\theta)$.

$$= \frac{4\rho a b^3}{3} \int_0^{\pi/2} \sin^4\theta \, d\theta.$$

$$= \frac{4\rho a b^3}{3} \cdot \frac{3}{4} \cdot \frac{1}{2} \cdot \frac{\pi}{2} = \frac{\pi \rho a b^3}{4}.$$

The whole area being πab, we have $\pi ab \rho = M$, the total mass.

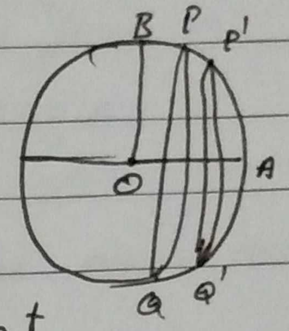
So $M.I = \pi a b \rho \cdot \frac{b^2}{4} = \frac{Mb^2}{4}$.

$|||^{ly}$ M.I. of the ellipse about minor axis is $\frac{Ma^2}{4}$

Hence by the theorem of $\perp^r$ axes,
M.I. of the ellipse about an axis through the centre $\perp^r$ to its plane $= M\left(\frac{a^2+b^2}{4}\right)$.

(7) A solid sphere about its diameter

Taking the $\perp^r$ radii OA, OB of a circle of radius a as axes of x & y, its equation is $x^2+y^2=a^2$.



By revolving this circle about OA, we get a sphere.

Divide the sphere into a number of circular discs by planes $\perp^r$ to OA.

PQQ'P' is one such section of sphere cut off at distance x and $x+\Delta x$ from O.

Elementary volume b/w these two sections $= \pi y^2 \Delta x$.

elementary mass $= \rho . \pi y^2 \Delta x$

where ρ is the mass per unit ~~length~~ volume.

This section is a circular plate of radius y and hence

M.I about OA $= \rho \pi y^2 \Delta x \cdot \frac{y^2}{2}$.

Hence M.I. of the whole sphere $= \cancel{2}\int_0^a \frac{\rho\pi y^4}{\cancel{2}}\, dx$

$$\cancel{= \pi\rho \left(\frac{y^3}{3} \right)}$$

$$= \pi\rho \int_0^a (a^2 - x^2)^2 \, dx \quad \text{as } x^2 + y^2 = a^2.$$

$$= \pi\rho \int_0^a (a^4 + x^4 - 2a^2x^2) \, dx$$

$$= \pi\rho \left[a^4 x + \frac{x^5}{5} - \frac{2a^2x^3}{3} \right]_0^a$$

$$= \pi\rho \left(a^5 + \frac{a^5}{5} - \frac{2a^5}{3} \right)$$

$$= \pi\rho \left(\frac{15a^5 + 3a^5 - 10a^5}{15} \right).$$

$$= \pi\rho \cdot \left(\frac{8a^5}{15} \right).$$

The volume of the whole sphere $= \frac{4}{3}\pi a^3$

$$\therefore \rho = \frac{M}{\frac{4}{3}\pi a^3} = \frac{3M}{4\pi a^3}$$

$$\left.\begin{array}{r}\text{Hence M.I. of the sphere}\\ \text{about OA}\end{array}\right\} = \frac{\overset{2}{\cancel{8}}\cancel{\pi}a^{\cancel{5}}}{\underset{5}{\cancel{15}}} \cdot \frac{\cancel{3}M}{\cancel{4}\cancel{\pi}\cancel{a^3}} = \frac{2Ma^2}{5}$$

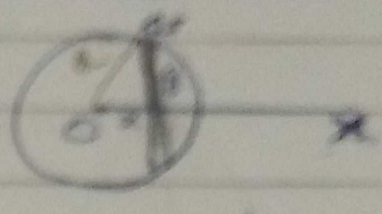
<u>Note:</u>

M.I. of the sphere about a tgt line = M.I. about a parallel diameter $+ Ma^2$.

$$= \frac{2Ma^2}{5} + Ma^2 = \frac{7Ma^2}{5}$$

(8) A hollow sphere about its diameter

Divide the hollow sphere into thin circular rings ⊥ to OX.



Consider a ring of radius y and arcual width Δs, at a distance x from O.

Surface area of this ring $= 2\pi y \Delta s$

$\therefore$ mass $= \rho \cdot 2\pi y \Delta s$, where ρ is mass per unit area of the surface

$$\left.\begin{array}{l}\text{M.I of the circular ring}\\ \text{about } OX\end{array}\right\} = \rho \cdot 2\pi y \Delta s \cdot y^2 = 2\pi \rho y^2 \Delta s$$

$$\therefore \left.\begin{array}{l}\text{M.I of the hollow}\\ \text{sphere about } OX\end{array}\right\} = 2\int_0^a 2\pi \rho y^3 \, ds$$

But $x^2+y^2=a^2$

$\therefore 2x + 2y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -x/y$.

$$\frac{ds}{dx} = \sqrt{1+\left(\frac{dy}{dx}\right)^2} = \sqrt{1+\frac{x^2}{y^2}} = \sqrt{\frac{y^2+x^2}{y^2}} = \sqrt{\frac{a^2}{y^2}} =$$

$$\Rightarrow ds = \frac{a}{y}\,dx.$$

Hence $M.I = \int_0^a 4\pi\rho \cdot y^{\cancel{3}} \cdot \frac{a}{\cancel{y}}\, dx$

$$= 4\pi\rho a \int_0^a y^2\, dx$$

$$= 4\pi\rho a \int_0^a (a^2 - x^2)\, dx$$

$$= 4\pi\rho a \left[a^2 x - \frac{x^3}{3}\right]_0^a$$

$$= 4\pi\rho a \left(a^3 - \frac{a^3}{3}\right)$$

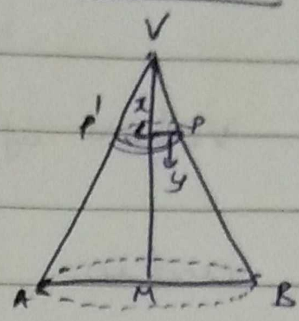
$$= \frac{8\pi\rho a^4}{3}$$

Total surface area of the hollow sphere $= 4\pi a^2$

$\therefore \rho = \frac{M}{4\pi a^2}$

Hence $M.I = \frac{\overset{2}{\cancel{8}}\cancel{\pi} a^{\cancel{4}}}{3} \cdot \frac{M}{\cancel{4}\cancel{\pi}\cancel{a^2}} = \frac{2Ma^2}{3}$

(9). A solid right circular cone about its axis:



Let V be the vertex,
VM - the axis
r - the base radius
h - height of the cone.

Let PP' be a circular section of the cone, by a distant x from V

Let y be the radius LP. of this section and Δx its width.

volume of this section $= \pi y^2 \Delta x$.

$\therefore$ mass " " $= \rho \pi y^2 \Delta x$ where ρ is the mass per unit volume.

This is in the form of a circular disc and its M.I. about VM $= \rho \pi y^2 \Delta x \cdot \frac{y^2}{2}$.

$\therefore$ M.I. of the cone about VM $= \rho \frac{\pi}{2} \int_0^h y^4 dx$.

Triangles VLP and VMB are similar.

Hence $\frac{LP}{MB} = \frac{VL}{VM}$ (ie) $\frac{y}{r} = \frac{x}{h}$.

$\Rightarrow y = \frac{xr}{h}$.

$\therefore$ required M.I $= \rho \frac{\pi}{2} \int_0^h \frac{x^4 r^4}{h^4} dx = \frac{\rho \pi r^4}{2h^4} \left(\frac{x^5}{5} \right)_0^h$.

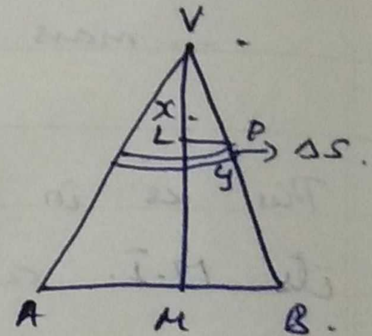
$$= \frac{\rho \pi r^4 h}{10}.$$

volume of the cone is $\frac{1}{3}\pi r^2 h$.

and so $\rho = \dfrac{M}{\frac{1}{3}\pi r^2 h} = \dfrac{3M}{\pi r^2 h}$.

So required $M.I = \dfrac{3M}{\cancel{\pi} r^2 h} \cdot \dfrac{\cancel{\pi} r^4 h}{10} = \dfrac{3r^2 M}{10}$.

(c). <u>A hollow cone</u> <u>about its axis</u>.



Consider a belt-like section of the surface area, at a distance x from the vertex V and having actual width $= \Delta s$.

Radius of this section $= y$.

Let r, h be the radius and height of the cone.

The surface area of this section $= 2\pi y \cdot \Delta s$.

So mass $= \rho \cdot 2\pi y \, \Delta s$ where ρ is the surface density

M.I of the ring about VM = $\rho \cdot 2\pi y \Delta s \cdot y^2$

$= 2\rho\pi y^2 \Delta s$

So the ~~whole~~ M.I. of the whole cone

$\therefore = 2\pi\rho \int_0^h y^2 ds$

As $\triangle$ VLP and VMB are similar

$$\frac{y}{r} = \frac{x}{h} \Rightarrow y = \frac{r}{h}x$$

$$\therefore \frac{dy}{dx} = \frac{r}{h}$$

$$\frac{ds}{dx} = \sqrt{1+\left(\frac{dy}{dx}\right)^2} = \sqrt{1+\frac{r^2}{h^2}} = \sqrt{\frac{h^2+r^2}{h^2}} = \frac{l}{h}$$

where l is the slant height

$$\therefore M.I = 2\pi\rho \int_0^h \frac{r^2x^2}{h^2}\cdot\frac{l}{h}\,dx$$

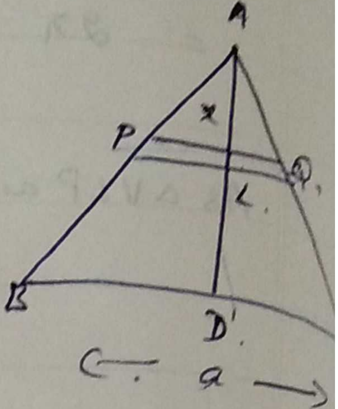
$$= \frac{2\pi\rho r^2 l}{h^4}\left[\frac{x^4}{4}\right]_0^h = \frac{\pi\rho r^2 l}{2}$$

Surface area of cone $= \pi r l$

Hence $\rho = \dfrac{M}{\pi r l}$

$$\therefore M.I = \frac{\cancel{\pi} r^3 \cancel{l}}{2}\cdot\frac{M}{\cancel{\pi} r \cancel{l}} = \frac{Mr^2}{2}$$

1). Show that the M.I. of a triangular lamina of mass M about a side is $\frac{Mh^2}{6}$ where h is the altitude from the opposite vertex.



Divide the lamina into strips $\parallel^{el}$ to BC.
Let PQ be one such strip at a distance x from A and its width be Δx.

Area of the strip $= PQ \cdot \Delta x$.

As APQ & ABC are similar Δ^{les}.

$$\frac{PQ}{BC} = \frac{AL}{AD} = \frac{x}{h}.$$

Hence $PQ = \frac{ax}{h}$ where $BC = a$.

If ρ is the areal density, mass of the ~~strip~~ strip PQ $= \frac{\rho a x}{h} \Delta x$.

M.I. of PQ about BC = M.I. of PQ about a line through the midpoint of PQ, $\parallel^{el}$ to BC + (mass of PQ) $(LD)^2$
(by parallel axis theorem)

$= $ M.I. of PQ itself + (mass of PQ) $(LD)^2$

$$= 0 + \rho \frac{ax}{h} \Delta x \, (h-x)^2.$$

Hence M.I of the whole ΔABC about BC $\Bigg\} = \int_0^h \frac{\rho a x}{h}(h^2+x^2-2hx)\,dx$.

$$= \frac{\rho a}{h}\int_0^h (h^2x + x^3 - 2hx^2)\,dx$$

$$= \frac{\rho a}{h}\left[\frac{h^2x^2}{2} + \frac{x^4}{4} - \frac{2hx^3}{3}\right]_0^h$$

$$= \frac{\rho a}{h}\left[\frac{h^4}{2} + \frac{h^4}{4} - \frac{2h^4}{3}\right]$$

$$= \frac{\rho a}{h}\left[\frac{6h^4+3h^4-8h^4}{12}\right] = \frac{\rho a}{\cancel{h}}\left[\frac{h^4}{12}\right]$$

$$= \frac{\rho a h^3}{12}.$$

Area of the $\Delta ABC = \frac{1}{2} BC \cdot AD = \frac{1}{2}ah$.

$$\therefore \rho = \frac{M}{\frac{1}{2}ah} = \frac{2M}{ah}.$$

Hence $M.I = \frac{ah^3}{\cancel{12}_{6}} \cdot \frac{\cancel{2}M}{ah} = \frac{Mh^2}{6}$.

2). Show that the M.I. of a hollow sphere whose external and internal radii are a and b about a diameter is $\frac{2M}{5}\left(\frac{a^5-b^5}{a^3-b^3}\right)$. Deduce the M.I. of a hollow sphere of radius a.

The given hollow sphere can be got by removing from a solid sphere of radius a, an inner solid sphere of radius b.

Let M_1 and M_2 be the masses of these spheres and ρ be the mass per unit volume.

$$M_1 = \frac{4}{3}\pi a^3 \rho \quad \text{and} \quad M_2 = \frac{4}{3}\pi b^3 \rho.$$

$$\therefore \frac{M_1}{M_2} = \frac{a^3}{b^3} \quad \text{(ie)} \quad \frac{M_1}{a^3} = \frac{M_2}{b^3} = \frac{M_1 - M_2}{a^2 - b^3} = \frac{M}{a^3 - b^3}$$

$$\therefore M_1 = \frac{Ma^3}{a^3 - b^2} \quad \text{and} \quad M_2 = \frac{Mb^3}{a^3 - b^3}.$$

reqd. M.I = M.I. of outer sphere − M.I. of inner sphere.

$$= M_1\left(\frac{2a^2}{5}\right) - M_2\left(\frac{2b^2}{5}\right)$$

$$= \frac{2a^2}{5} \cdot \frac{Ma^3}{a^3 - b^3} - \frac{2b^2}{5} \cdot \frac{Mb^3}{a^2 - b^3}.$$

$$= \frac{2M}{5}\left(\frac{a^5 - b^5}{a^3 - b^3}\right).$$

Now as $b \to a$, the ~~bod~~ above body becomes a thin hollow sphere.

$\therefore$ M.I of a hollow sphere of radius a

$$= \lim_{b \to a} \frac{2M}{5} \left(\frac{a^5 - b^5}{a^3 - b^3} \right).$$

$$= \lim_{b \to a} \frac{2M}{5} \frac{\cancel{(a-b)}(a^4 + a^3 b + a^2 b^2 + a b^3 + b^4)}{\cancel{(a-b)}(a^2 + ab + b^2)}$$

$$= \frac{2M}{5} \left(\frac{a^4 + a^4 + a^4 + a^4 + a^4}{a^2 + a^2 + a^2} \right)$$

$$= \frac{2M}{\cancel{5}} \cdot \frac{\cancel{5} a^4}{3 \cancel{a^2}} = \frac{2Ma^2}{3} //.$$

Dr. Routh's Rule:

A body has three axes of symmetry. Its moment of ~~inter~~ inertia about an axis of symmetry

$$= \frac{\text{Mass} \times (\text{sum of the squares of the } \perp^r \text{ semi axis})}{D}.$$

where D is 3 or 4 or 5 according as the body is rectangular, elliptical (including circle) or ellipsoidal (including spherical).