ARITHMETIC FOR ALL - II

UNIT I

Pipes and Cisterns

Formulas:

1) If a pipe can fill a tank in x hours then part filled in 1 hour $= \frac{1}{x}$ of the tank

2) If a pipe can empty a full tank in y hours then part emptied in 1 hour $= \frac{1}{y}$ of the tank.

3) If a pipe can fill a tank in x hours and another pipe can empty the full tank in y hours where $(y > x)$ then on opening both the pipes, the net part filled in 1 hour $= \frac{1}{x} - \frac{1}{y}$

4) If a pipe can fill a tank in x hours and another pipe can empty the full tank in y hours where $(x > y)$, then on opening both the pipes, the net part emptied in 1 hour $= \frac{1}{y} - \frac{1}{x}$

1) Two pipes A and B can fill a tank in 20 and 30 minutes respectively. If both the pipes are used together, then how long will it take to fill the tank?

Pipe A alone can fill the tank in 20 minutes

Pipe B alone can fill the tank in 30 minutes

In one minute A fills $\frac{1}{20}$ part of the tank.

In one minute B fills $\frac{1}{30}$ part of the tank

In one minute A & B together fills $\frac{1}{20} + \frac{1}{30}$ part of the tank

$$= \frac{3+2}{60} = \frac{5}{60}$$

$$= \frac{1}{12} \text{ part of the tank}$$

Therefore A and B together fill the tank in 12 mins

Notes

2. A Cistern can be filled by a tap in 4 hours while it can be emptied by another tap in 9 hours. If both the taps are opened simultaneously, then after how much time will the Cistern get filled?

In 1 hour the Cistern filled by tap A is $\frac{1}{4}$ part

In 1 hour the Cistern emptied by tap B is $\frac{1}{9}$ part

Here $(x > y)$

∴ In 1 hour if both tap A and B are open the net part filled $= \frac{1}{4} - \frac{1}{9}$

$= \frac{9-4}{36} = \frac{5}{36}$

$= \frac{1}{(36/5)} = \frac{1}{7.2}$

Therefore if both taps A&B are open then will be filled in 7.2 hours

(OR)

In 60 mins net part filled $= \frac{5}{36}$

In 1 minute net part filled $= \frac{5}{36 \times 60} = \frac{1}{36 \times 12} = \frac{1}{432}$

The tank will be filled in 432 mins

(or) 7 hours 12 minutes //

Notes

JANUARY 2008
Wk | S M T W T F S
1 | 1 2 3 4 5
2 | 6 7 8 9 10 11 12
3 | 13 14 15 16 17 18 19
4 | 20 21 22 23 24 25 26
5 | 27 28 29 30 31

3) pipe A can fill the tank in 5 hours, pipe B in 10 hours and pipe C in 30 hours. If all the pipes are open, in how many hours will the tank be filled?

Part of tank filled by A in 1 hour $= \frac{1}{5}$

Part of tank filled by B in 1 hour $= \frac{1}{10}$

Part of tank filled by C in 1 hour $= \frac{1}{30}$

Part of tank filled by A, B, C in 1 hour $= \frac{1}{5} + \frac{1}{10} + \frac{1}{30}$

$= \frac{6+3+1}{30} = \frac{10}{30}$

$= \frac{1}{3}$

If A, B, C are open tank will be filled in 3 hours.

4) Pipes A and B can fill a tank in 5 and 6 hours respectively. Pipe C can empty it in 12 hours. If all the three pipes are opened together, then the tank will be filled in ___ hours.

Part filled in 1 hour if A and B are open $= \frac{1}{5} + \frac{1}{6}$

$= \frac{6+5}{30}$

$= \frac{11}{30}$

Part emptied by C in 1 hour $= \frac{1}{12}$

Net part filled in 1 hour when A & B & C are open $= \frac{11}{30} - \frac{1}{12}$

$= \frac{22-5}{60}$

$= \frac{17}{60}$

Therefore the tank will be filled in $\frac{60}{17}$ hours if A, B, C are open.

Notes

$17 \times 3 = 51$

$\frac{60}{17}$ hours $= 3\frac{9}{17}$ hours

JANUARY 2008

Wk	S	M	T	W	T	F	S
1			1	2	3	4	5
2	6	7	8	9	10	11	12
3	13	14	15	16	17	18	19
4	20	21	22	23	24	25	26
5	27	28	29	30	31		

Sol. Let the reservoir be filled by first pipe in x hours.

Then, second pipe will fill it in $(x + 10)$ hours.

$\therefore \quad \frac{1}{x} + \frac{1}{(x+10)} = \frac{1}{12} \quad \Leftrightarrow \quad \frac{x+10+x}{x(x+10)} = \frac{1}{12}$

$\Leftrightarrow \quad x^2 - 14x - 120 = 0 \quad \Leftrightarrow \quad (x - 20)(x + 6) = 0$

$\Leftrightarrow \quad x = 20.$ [*neglecting the – ve value of x*]

So, the second pipe will take (20 + 10) hrs *i.e.*, 30 hrs to fill the reservoir.

Ex. 4. ***A cistern has two taps which fill it in 12 minutes and 15 minutes respectively. There is also a waste pipe in the cistern. When all the three are opened, the empty cistern is full in 20 minutes. How long will the waste pipe take to empty the full cistern?***

Sol. Work done by the waste pipe in 1 minute

$$= \frac{1}{20} - \left(\frac{1}{12} + \frac{1}{15}\right) = -\frac{1}{10}$$ [*– ve sign means emptying*]

$\therefore$ Waste pipe will empty the full cistern in 10 minutes.

Ex. 5. ***An electric pump can fill a tank in 3 hours. Because of a leak in the tank it took $3\frac{1}{2}$ hours to fill the tank. If the tank is full, how much time will the leak take to empty it?***

Sol. Work done by the leak in 1 hour $= \left[\frac{1}{3} - \frac{1}{\left(\frac{7}{2}\right)}\right] = \left(\frac{1}{3} - \frac{2}{7}\right) = \frac{1}{21}.$

$\therefore$ The leak will empty the tank in 21 hours.

Ex. 6. ***Two pipes can fill a cistern in 14 hours and 16 hours respectively. The pipes are opened simultaneously and it is found that due to leakage in the bottom it took 32 minutes more to fill the cistern. When the cistern is full, in what time will the leak empty it?***

Sol. Work done by the two pipes in 1 hour $= \left(\frac{1}{14} + \frac{1}{16}\right) = \frac{15}{112}.$

$\therefore$ Time taken by these pipes to fill the tank $= \frac{112}{15}$ hrs $= 7$ hrs 28 min.

Due to leakage, time taken = 7 hrs 28 min + 32 min = 8 hrs

$\therefore$ Work done by (two pipes + leak) in 1 hour $= \frac{1}{8}.$

Work done by the leak in 1 hour $= \left(\frac{15}{112} - \frac{1}{8}\right) = \frac{1}{112}.$

$\therefore$ Leak will empty the full cistern in 112 hours.

Ex. 7. ***Two pipes A and B can fill a tank in 36 min. and 45 min. respectively. A waste pipe C can empty the tank in 30 min. First A and B are opened. After 7 minutes, C is also opened. In how much time, the tank is full?***

Sol. Part filled in 7 min. $= 7\left(\frac{1}{36} + \frac{1}{45}\right) = \frac{7}{20}.$

Remaining part $= \left(1 - \frac{7}{20}\right) = \frac{13}{20}.$

Net part filled in 1 min. when A, B and C are opened $= \left(\frac{1}{36} + \frac{1}{45} - \frac{1}{30}\right) = \frac{1}{60}$.

Now, $\frac{1}{60}$ part is filled in 1 min.

$\frac{13}{20}$ part is filled in $\left(60 \times \frac{13}{20}\right) = 39$ min.

Total time taken to fill the tank = (39 + 7) min. = 46 min.

Ex. 8. ***Two pipes A and B can fill a tank in 24 min. and 32 min. respectively. If both the pipes are opened simultaneously, after how much time B should be closed so that the tank is full in 18 minutes ?***

Sol. Let B be closed after x minutes. Then,

part filled by (A + B) in x min. + part filled by A in $(18 - x)$ min. = 1

$\therefore \quad x\left(\frac{1}{24} + \frac{1}{32}\right) + (18 - x) \times \frac{1}{24} = 1 \quad \Leftrightarrow \quad \frac{7x}{96} + \frac{18 - x}{24} = 1$

$\Rightarrow \quad 7x + 4(18 - x) = 96 \quad \Leftrightarrow \quad x = 8.$

Hence, B must be closed after 8 minutes.

EXERCISE 16A

(OBJECTIVE TYPE QUESTIONS)

Directions : *Mark (✓) against the correct answer :*

1. Two pipes A and B can fill a tank in 20 and 30 minutes respectively. If both the pipes are used together, then how long will it take to fill the tank ? **(M.A.T. 2003)**

(a) 12 min (b) 15 min (c) 25 min (d) 50 min

2. A cistern can be filled by a tap in 4 hours while it can be emptied by another tap in 9 hours. If both the taps are opened simultaneously, then after how much time will the cistern get filled ? **(Hotel Management, 1997)**

(a) 4.5 hrs (b) 5 hrs (c) 6.5 hrs (d) 7.2 hrs

3. A tap can fill a tank in 6 hours. After half the tank is filled, three more similar taps are opened. What is the total time taken to fill the tank completely ?

(a) 3 hrs 15 min (b) 3 hrs 45 min (c) 4 hrs (d) 4 hrs 15 min **(S.S.C. 2003)**

4. A water tank is two-fifth full. Pipe A can fill a tank in 10 minutes and pipe B can empty it in 6 minutes. If both the pipes are open, how long will it take to empty or fill the tank completely ? **(Bank P.O. 1999)**

(a) 6 min. to empty (b) 6 min. to fill (c) 9 min. to empty

(d) 9 min. to fill (e) None of these

5. Pipe A can fill a tank in 5 hours, pipe B in 10 hours and pipe C in 30 hours. If all the pipes are open, in how many hours will the tank be filled ? **(C.B.I. 1997)**

(a) 2 (b) 2.5 (c) 3 (d) 3.5

6. Pipes A and B can fill a tank in 5 and 6 hours respectively. Pipe C can empty it in 12 hours. If all the three pipes are opened together, then the tank will be filled in :

(a) $1\frac{13}{17}$ hours (b) $2\frac{8}{11}$ hours (c) $3\frac{9}{17}$ hours (d) $4\frac{1}{2}$ hours **(Bank P.O. 2002)**

Time and distance

Formulae:

1. Speed $= \frac{\text{distance}}{\text{time}}$

2. Conversion:

$1\ km = 1000\ m$

$1\ hour = 60\ mins = 3600\ secs$

$\Rightarrow 1\ m = \frac{1}{1000}\ km$

$\Rightarrow 1\ sec = \frac{1}{3600}\ hour$

$\therefore x\ \frac{km}{hr} = x \times \frac{1000}{3600}\ \frac{m}{sec}$

$= x \times \frac{5}{18}\ m/sec$

(or) $x\ m/sec = x \times \frac{3600}{1000}\ km/hr$

$= x \times \frac{18}{5}\ km/hr$

3. If the ratio of the speeds of A and B is $a:b$, then ratio of time taken to cover the same distance is $\frac{1}{a} : \frac{1}{b}$.

4. Suppose a man covers a certain distance at x km/hr and an equal distance at y km/hr, then the average speed during the whole journey is $\frac{2xy}{x+y}\ km/hr$.

Problems:

1. How many minutes does Aditya take to cover a distance of 400 m if he runs at a speed of 20 km/hr?

Ans: Speed $= \frac{\text{distance}}{\text{time}}$

Here 20 = 4000 / time Aditya Speed = 20 km/hr

time = 4000 / 20

distance = 400 m

∴ Change speed to m/sec

$20\ km/hr = 20 \times \frac{1000}{3600}\ m/sec$

$= \frac{200}{36}\ m/sec$

$= \frac{50}{9}\ m/sec$

$Speed = \frac{distance}{time}$

$\frac{50}{9} = \frac{400}{time}$

$time = \frac{400 \times 9}{50}\ sec$

$= 72\ secs$ //

1 min = 60 sec

$1 sec = \frac{1}{60}\ min$

$72\ secs = 72 \times \frac{1}{60}\ min$

$= 1.2\ minutes$ //

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2. A Cyclist covers a distance of 750 m in 2 min 30 sec. What is the speed in km/hr of the cyclist?

Ans: distance = 750 m

time = 2min30sec

$= (2 \times 60) + 30\ sec$

$= 150\ secs$

$Speed = \frac{dist}{time} = \frac{750}{150}\ m/sec$

$= 5\ m/sec$

$= 5 \times \frac{3600}{1000}\ km/hr$

$= 18\ km/hr.$

3. A dog takes 4 leaps for every 5 leaps of a hare but 3 leaps of a dog are equal to 4 leaps of the hare. Compare their speeds.

Soln:

Distance Covered in 1 leap of dog = x m

Distance Covered in 1 leap of hare = y m

$\therefore 3x = 4y \Rightarrow x = \frac{4}{3}y$

Speed ratio dog : hare $= 4x : 5y$

$= 4 \times \frac{4}{3}y : 5y$

$= \frac{16}{3}y : 5y$

$= \frac{16}{3} : 5$

$= 16 : 15$

4. While Covering a distance of 24 km, a man noticed that after walking for 1 hour 40 minutes, the distance covered by

him was $\frac{5}{7}$ of the remaining distance. What was his speed in metres per second?

Solution:

To find Speed formula is

$$\text{Speed} = \frac{\text{dist}}{\text{time}}$$

Time taken = 1 hr 40 mins

$= \left(1 + \frac{40}{60}\right)$ hr

$= 1\frac{2}{3}$ hrs. (or) $\frac{5}{3}$ hrs

Total distance to be Covered = 24 km

Remaining distance = x

distance Covered by him = $\frac{5}{7}x$

$\frac{5}{7}x + x = 24$

$\frac{5x + 7x}{7} = 24$

$12x = 24 \times 7$

$x = 2 \times 7 = 14$ km

distance Covered by him $= \frac{5}{7} \times 14 = 10$ k

$\therefore$ distance covered by him = 10 km

$\therefore$ Speed $= \frac{10}{5/3}$ km/hr

$= \frac{30}{5}$ km/hr

$= 6$ km/hr //

$= 6 \times \frac{1000}{3600}$ m/sec

$= \frac{10}{6}$ m/sec $= 1\frac{2}{3}$ m/sec //

5) Peter can cover a certain distance in 1 hr 24 min by covering two-third of the distance at 4 km/hr and the rest at 5 km/hr. Find the total distance.

Solution: Total distance = x km

Speed for $\frac{2}{3}x$ = 4 km/hr

Speed for $\frac{1}{3}x$ = 5 km/hr

~~Avg speed for~~ ~~x distance~~ } ~~2 avg~~ / ~~x avg~~

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time taken to cover 4 km = 1 hr

time taken to cover $\frac{2}{3}x$ km $= \frac{1}{4} \times \frac{2x}{3}$ hrs

$= \frac{x}{6}$ hrs

time taken to cover 5 km = 1 hr

time taken to cover $\frac{1}{3}x$ km $= \frac{1}{5} \times \frac{x}{3}$ hrs

$= \frac{x}{15}$ hrs

$\left(\frac{x}{6} + \frac{x}{15}\right)$ hrs = 1 hr 24 mins

$= \left(1 + \frac{24}{60}\right)$ hrs

$\frac{5x + 2x}{30} = \frac{84}{60}$ hrs

$\frac{7x}{30} = \frac{84}{60}$

$x = \frac{84}{60} \times \frac{30}{7} = 6$

$\therefore$ $x = 6$ kms //

6) A man travelled from the village to the post-office at the rate of 25 kmph and walked back at the rate of 4 kmph. If the whole journey took 5 hours 48 minutes, find the distance of the post-office from the village.

Soln:

$$\text{Avg. speed} = \frac{2xy}{x+y} \text{ km/h}$$

$$= \frac{2 \times 25 \times 4}{25+4} \text{ km/h.}$$

$$= \frac{200}{29} \text{ km/h.}$$

Time taken (Village to PO & PO to village) = 5 hrs 48 mins

$$= \left(5 + \frac{48}{60}\right) \text{ hrs}$$

$$= \left(5 + \frac{4}{5}\right) \text{ hrs}$$

$$= \frac{29}{5} \text{ hrs}$$

$$\text{Speed} = \frac{\text{dist}}{\text{time}}$$

$$\frac{200}{29} = \frac{\text{dist}}{29/5} \frac{\text{km}}{\text{hrs}}.$$

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$$\text{dist} = \frac{200}{29} \times \frac{29}{5} \text{ km}$$

$$= 40 \text{ km}$$

Total distance Covered = 40 km

Distance between PO & village = 20 km //

7. An aeroplane flies along the four sides of a square at the speeds of 200, 400, 600 and 800 km/hr. Find the average speed of the plane around the field.

Soln: If the length of a side of a square = x

If x hr the dist covered = 200 km

If speed is 200 km/hr

In 1 hr dist. Covered = 200 km

(or)

200 km dist covered in 1 hr

x km dist. Covered in $\frac{1}{200} \times x$ hrs

1st Side x kms Covered in $\frac{x}{200}$ hrs

2nd Side x kms Covered in $\frac{x}{400}$ hrs

3rd Side x kms Covered in $\frac{x}{600}$ hrs

4th Side x kms Covered in $\frac{x}{800}$ hrs

(4x) kms Covered in $\left(\frac{x}{200} + \frac{x}{400} + \frac{x}{600} + \frac{x}{800}\right)$ hrs

4x kms Covered in $\left(\frac{12x + 6x + 4x + 3x}{2400}\right)$ hrs

$= \left(\frac{25x}{2400}\right)$ hrs

Speed = $\frac{\text{dist}}{\text{time}}$

$= \frac{4x}{\left(\frac{25x}{2400}\right)}$ km/hr

$= \left(4x \times \frac{2400}{25x}\right)$ km/hr

Speed = 384 km/hr

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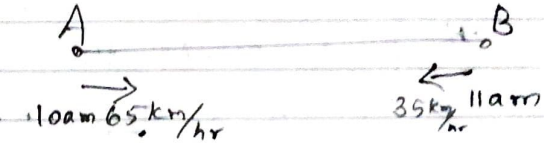
8) A and B are two stations 390 km apart. A train starts from A at 10 a.m and travels towards B at 65 km/hr. Another train starts from B at 11 a.m and travels towards A at 35 km/hr. At what time do they meet?

Soln:



If they meet at x hrs

In 1 hr (AB) travels 65 km

In x hr AB travels (65 x x) km

In 1 hr BA travels 35 km

In (x-1) hrs BA travels (35 x (x-1)) km

Total distance = 390 km

$65x + 35(x-1) = 390$

$65x + 35x - 35 = 390$

$100x - 35 = 390$

$100x = 390 + 35$

$100x = 425$

$x = \frac{425}{100}$

$x = 4.25$ hrs

If train AB starts at 10 am then it meets the train BA at

2.25 pm

2 hrs $(.25 \times 60)$ hrs

$(2 + (.25 \times 60))$ hrs

2.15 pm //

Problems on Trains

Formulae:

1. x km/hr $= x \times \frac{1000}{3600}$ m/s

2. x m/s $= x \times \frac{3600}{1000}$ km/hr

3. Time taken by a train of length l metres to pass a pole or a standing man or a signal post is equal to the time taken to cover l metres.

4. Time taken by a train of length l metres to pass a stationary object of length b metres is the time taken by the train to cover $(l+b)$ meters.

5. Suppose two trains or two bodies are moving in the same direction at u m/s

and v m/s, where u > v, then their relative speed = (u−v) m/s.

6. Suppose two trains or two bodies are moving in the opposite directions at u m/s and v m/s, then their relative speed = (u+v) m/s

7. If two trains of length a metres and b metres are moving in opposite directions at u m/s and v m/s., then time taken by the trains to cross each other = $\frac{a+b}{u+v}$ sec

8. If two trains of length a metres and b metres are moving in the same direction at u m/s and v m/s, then the time taken by the faster train to cross the slower train = $\left(\frac{a+b}{u-v}\right)$ sec.

9. If two trains (or bodies) start at the same time from points A and B towards each other and after crossing they take a and b sec in reaching B

and A respectively, then ratio of

A's speed : B's speed = $\sqrt{b} : \sqrt{a}$.

Problems:

1. A train 100m long is running at the speed of 30 km/hr. Find the time taken by it to pass a man standing near the railway line.

Soln: length of the train = 100 m

$= \frac{100}{1000}$ km

$= 0.1$ km

Time taken to pass the standing man = Time taken to cover the 0.1 km

Speed = 30 km/hr

Time taken to cover 30 km = 1 hr

Time taken to cover 0.1 km = $\left(\frac{1}{30} \times 0.1\right)$ hr

$= \frac{1}{300}$ hr (or) $\frac{1}{300} \times 3600$ sec

$= \frac{1}{300}$ hrs (or) $\frac{1}{300} \times 3600$ Secs

$= 12$ Secs //

2) A train is moving at a speed of 132 km/hr. If the length of the train is 110 metres, how long will it take to cross a railway platform 165 metres long?

Soln:

Time taken to Cover the plat form } = Time taken to Cross (110+165) mtrs

In 1 hr
132 km covered in 1 hr
(275 mtrs) or $\left(\frac{275}{1000} \text{ km}\right)$ Covered in $\left(\frac{1}{132} \times \frac{275}{1000}\right)$ hrs

$= \frac{275}{132 \times 1000}$ hrs

$= \frac{11}{132 \times 40}$ hrs $= \frac{1}{480}$ hrs

$= \frac{1}{480} \times 3600$ Secs $= \frac{30}{4}$ Secs $= 7\frac{1}{2}$ Secs //

3) A man is standing on a railway bridge which is 180m long. He finds that a train crosses the bridge in 20 seconds but himself in 8 seconds. Find the length of the train and its speed.

Soln:

Let the length of the train = x metres

Time taken to Cover (x+180) m = 20 Secs

Time taken to Cover x m = 8 Secs

Speed of the train = $\frac{\text{dist}}{\text{time}}$

$\frac{x+180}{20} = \frac{x}{8}$

$8(x+180) = 20x$

$1440 = 20x - 8x$

$1440 = 12x$

$x = \frac{1440}{12} = 120$ m

∴ Length of the train = 120 metres

Speed of the train = $\frac{120}{8}$ m/s

= 15 m/sec

(or) $15 \times \frac{3600}{1000}$ km/hr = 54 km/hr

4) A train 150m long is running with a speed of 68 kmph. In what time will it pass a man who is running at 8 kmph in the same direction in which the train is going?

Soln:

Speed of the train relative to man = (68 − 8) kmph

= 60 km/hr

$$\text{Speed} = \frac{\text{dist}}{\text{time}}$$

$$60 = \frac{\left(\frac{150}{1000}\right) \text{km}}{\text{time in hrs}}$$

$$\text{time} = \frac{150}{1000} \times \frac{1}{60} = \frac{1}{400} \text{ hrs}$$

$$= \frac{1}{400} \times 3600 \text{ secs} = 9 \text{ secs}$$

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5) A train 220m long is running with a speed of 59 km/hr. In what time will it pass a man who is running at 7 kmph in the direction opposite to that in which the train is going?

Soln:

Speed of the train relative to man = (59 + 7) km/hr

= 66 km/hr

$$1 \text{ m} = \frac{1}{1000} \text{ km}$$

$$220 \text{ m} = \frac{220}{1000} \text{ km}$$

$$\text{Speed} = \frac{\text{dist}}{\text{time}}$$

$$66 = \frac{\left(\frac{220}{1000}\right) \text{km}}{\text{time}} \text{ km/hr}$$

$$\text{time} = \frac{220}{1000} \times \frac{1}{66} \text{ hr}$$

$$= \frac{1}{300} \text{ hr}$$

$$= \frac{3600}{300} \text{ secs}$$

= 12 secs

6) A train 100 metres long takes 6 secs to cross a man walking at 5 kmph in a direction opposite to that of the train. Find the speed of the train.

Soln:

Let the speed of the train = x m/s

Speed of the man = 5 km/h

$= 5 \times \frac{1000}{3600}$ m/s

$= \frac{50}{36}$ m/s

Speed of the train relative to man = $\left(x + \frac{50}{36}\right)$ m/s

$= \left(\frac{36x + 50}{36}\right)$ m/s

$\text{Speed} = \frac{\text{dist}}{\text{time}}$

$\frac{36x + 50}{36} = \frac{100}{6}$

$36x + 50 = \frac{100}{6} \times 36$

$36x + 50 = 600$

$36x = 550$

$x = \frac{550}{36}$ m/s

$x = \frac{550}{36}$ m/s

(or) $= \frac{550}{36} \times \frac{3600}{1000}$ km/h.

Speed of train = 55 km/h //