Unit III - Compound Interest (1)

Compound Interest: Sometimes it so happens that the borrower and the lender agree to fix up a certain ~~time~~ unit of time say half-yearly or yearly or Quarterly to settle the previous account.

In such cases, the amount after first unit of time becomes the principal for the second unit, the amount after second unit becomes principal for the third unit and so on.

After a specified period the difference between the amount and the money borrowed is called the Compound interest (C.I) for that period.

Formulae:

Let P - Principal

R% - Rate of Interest per annum

n - Time in Years.

(2)

1. When interest is compounded annually

$$\text{Amount} = P\left(1+\frac{R}{100}\right)^n$$

2. When interest is compounded half-yearly

$$\text{Amount} = P\left(1+\frac{R/2}{100}\right)^{2n}$$

3. When interest is compounded Quarterly

$$\text{Amount} = P\left(1+\frac{R/4}{100}\right)^{4n}$$

4. When interest is compounded annually but time is in fraction say $3\frac{2}{5}$ Years

$$\text{Amount} = P\left(1+\frac{R}{100}\right)^3\left(1+\frac{\frac{2}{5}R}{100}\right)$$

5. When the Rates are different for different years say $R_1\%$, $R_2\%$, $R_3\%$ for 1st, 2nd, 3rd years respectively.

Then $\text{Amount} = P\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right)\left(1+\frac{R_3}{100}\right)$

6. Present worth of x Rs. due n years is

$$\text{Present worth} = \frac{x}{\left(1+\frac{R}{100}\right)^n}$$

Problems

1. Find the Compound interest on Rs. 7500 at 4% per annum for 2 years, Compounded annually

Soln:

$$Amount = P\left(1+\frac{R}{100}\right)^n$$

$$= 7500\left(1+\frac{4}{100}\right)^2$$

$$= 7500 \times \frac{104}{100} \times \frac{104}{100}$$

$$= 8112$$

$\therefore$ Compound Interest $= 8112 - 7500$

$= Rs.\ 612$ //

2. Find the Compound interest on Rs. 8000 at 15% per annum for 2 years 4 months compounded annually.

Soln: $n = 2$ years 4 months

$= 2\frac{4}{12}$ Years

$= 2\frac{1}{3}$ years (In fraction)

$$Amount = P\left(1+\frac{R}{100}\right)^2\left(1+\frac{\frac{1}{3}R}{100}\right)$$

$$= 8000\left(1+\frac{15}{100}\right)^2\left(1+\frac{\frac{15}{3}}{100}\right)$$

$$= 8000 \times \left(1+\frac{3}{20}\right)^2\left(1+\frac{15}{300}\right)$$

$$= 8000 \times \left(\frac{23}{20} \times \frac{23}{20}\right) \times \left(1+\frac{5}{100}\right)$$

$$= 80 \times \frac{23}{2} \times \frac{23}{2} \times \left(1+\frac{1}{20}\right)$$

$$= 20 \times 23 \times 23 \times \frac{21}{20}$$

$$= 23 \times 23 \times 21$$

$$= 11109$$

Compound Interest $= 11109 - 8000$

$= 3109$ //

3. Find the Compound interest on Rs. 10,000 in 2 years at 4% per annum, the interest being compounded half-yearly.

Soln: Here interest Compounded half-yearly

$$\therefore Amount = P\left(1+\frac{R/2}{100}\right)^{2n}$$

$$= 10000\left(1+\frac{4/2}{100}\right)^{2\times 2}$$

$$= 10000\left(1+\frac{2}{100}\right)^4$$

$$= 10000 \times \frac{102}{100} \times \frac{102}{100} \times \frac{102}{100} \times \frac{102}{100}$$

$$= \frac{102 \times 102 \times 102 \times 102}{100 \times 100}$$

$$= 10824.3216$$

Compound Interest = ~~10824.32~~ (5)

$10824.32 - 10000$

$= 824.32$ //

4. Find the compound interest on Rs. 16,000 at 20% per annum for 9 months, compounded quarterly.

Soln: Here compounded quarterly

$\therefore$ Amount $= P\left(1+\frac{R/4}{100}\right)^{4n}$

here $n = 9$ months $= 9 \times \frac{1}{12}$ years

$= \frac{3}{4}$ years

$\therefore$ Amount $= 16000 \times \left(1+\frac{20/4}{100}\right)^{4\times\frac{3}{4}}$

$= 16000 \times \left(1+\frac{5}{100}\right)^3$

$= 16000 \times \left(1+\frac{1}{20}\right)^3$

$= 16000 \times \frac{21}{20} \times \frac{21}{20} \times \frac{21}{20}$

$= 2 \times 21 \times 21 \times 21$

$= 18522$

$C.I = 18522 - 16000$

$= 2522$ Rs. //

(6)

5. If the simple interest on a sum of money at 5% per annum for 3 years is Rs. 1200, find the compound interest on the same sum for the same period at the same rate.

Soln:

$S.I = 1200$

By formula $S.I = \frac{P \times R \times T}{100}$

$1200 = \frac{P \times 5 \times 3}{100}$

$P = \frac{1200 \times 100}{5 \times 3}$

$P = 8000$

To find C.I

Amount $= P\left(1+\frac{R}{100}\right)^n$

$= 8000 \times \left(1+\frac{5}{100}\right)^3$

$= 8000 \times \left(1+\frac{1}{20}\right)^3$

$= 8000 \times \frac{21}{20} \times \frac{21}{20} \times \frac{21}{20}$

$= 21 \times 21 \times 21$

$= 9261$

$C.I = 9261 - 8000$

$= 1261$ Rs. //

6. In what time will Rs. 1000 become Rs. 1331 at 10% per annum compounded annually?

Soln:

$$\text{Amount} = P\left(1+\frac{R}{100}\right)^n$$

$$1331 = 1000\left(1+\frac{10}{100}\right)^n$$

$$\frac{1331}{1000} = \left(1+\frac{1}{10}\right)^n$$

$$\frac{11\times11\times11}{1000} = \left(\frac{11}{10}\right)^n$$

$$\left(\frac{11}{10}\right)^3 = \left(\frac{11}{10}\right)^n$$

$\therefore$ n = 3 Years //

7. If Rs. 500 amounts to Rs. 583.2 in two years compounded annually, find the rate of interest per annum.

Soln:

$$\text{Amount} = P\left(1+\frac{R}{100}\right)^n$$

$$583.2 = 500\left(1+\frac{R}{100}\right)^2$$

$$\frac{583.2}{500} = \left(1+\frac{R}{100}\right)^2$$

$$\left(\frac{583.2}{500}\right)^{1/2} = 1+\frac{R}{100}$$

$$1.08 = 1+\frac{R}{100}$$

$$\frac{R}{100} = 1.08 - 1$$

$$\frac{R}{100} = 0.08$$

$$R = 0.08 \times 100$$

$$R = 8\%$$ //Ans

8. If the compound interest on a certain sum at $16\frac{2}{3}\%$ for 3 years is Rs. 1270, find the simple interest on the same sum at the same rate and for the same period.

Soln.

$$P+1270 = P\left(1+\frac{R}{100}\right)^3$$

here $R = 16\frac{2}{3} = \frac{50}{3}$

$$P+1270 = P\left(1+\frac{50/3}{100}\right)^3$$

$$= P\left(1+\frac{50}{3\times100}\right)^3$$

$$= P\left(1+\frac{1}{6}\right)^3$$

$$P+1270 = P\left(\frac{7}{6}\times\frac{7}{6}\times\frac{7}{6}\right)$$

$$P + 1270 = P \times \frac{343}{216}$$

$$\frac{343}{216}P - P = 1270$$

$$\frac{343P - 216P}{216} = 1270$$

$$\frac{127P}{216} = 1270$$

$$P = \frac{1270 \times 216}{127}$$

$$P = 2160$$

$$S.I = \frac{P \times R \times T}{100}$$

$$= \frac{2160 \times \frac{50}{3} \times 3}{100}$$

$$= \frac{2160 \times 50 \times 1}{100}$$

$$= \frac{2160}{2} = 1080 \text{ Rs}$$

9) The difference between the Compound interest and Simple interest on a Certain Sum at 10% per annum for 2 years is Rs. 631. Find the Sum.

Soln: Given $C.I - S.I = 631 \rightarrow (1)$

$$S.I = \frac{P \times R \times T}{100}$$

$$S.I = \frac{P \times 10 \times 2}{100}$$

$$S.I = \frac{P}{5}$$

$$\text{Amount} = P + C.I = P\left(1 + \frac{R}{100}\right)^n$$

$$P + C.I = P\left(1 + \frac{10}{100}\right)^2$$

$$= P\left(1 + \frac{1}{10}\right)^2$$

$$= P\left(\frac{11}{10} \times \frac{11}{10}\right)$$

$$P + C.I = \frac{121}{100}P$$

$$C.I = \frac{121P}{100} - P$$

$$= \frac{121P - 100P}{100}$$

$$C.I = \frac{21P}{100}$$

$$C.I - S.I = \frac{21P}{100} - \frac{P}{5} = 631$$

$$\frac{21P - 20P}{100} = 631$$

$$\frac{P}{100} = 631$$

$$P = 63100 \text{ Rs}$$

Logarithms

Formulae

$a^m = x$ can be written as

$$\log_a x = m$$

a is any positive real number other than

Examples:

$10^4 = 10000 \Rightarrow \log_{10} 10000 = 4$

$3^4 = 81 \Rightarrow \log_3 81 = 4$

$2^{-3} = \frac{1}{8} \Rightarrow \log_2 \frac{1}{8} = -3$

properties:

1. $\log_a xy = \log_a x + \log_a y$

2. $\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$

3. $\log_x x = 1$

4. $\log_a 1 = 0$ (a may be any number)

5. $\log_a x^p = p \log_a x$

6. $\log_a x = \dfrac{1}{\log_x a}$

7. $\log_a x = \dfrac{\log_b x}{\log_b a}$

Problems:

1. Evaluate (i) $\log_3 27$

$$= \log_3 3^3 = 3 \log_3 3$$

$$= 3$$

(ii) $\log_7 \left(\frac{1}{343}\right) = \log_7 343^{-1}$

$$= -1 \log_7 343$$

$$= -1 \log_7 7^3 = -3 \log_7 7$$

$$= -3$$

(iii) $\log_{100} 0.01 = \log_{100} \left(\frac{1}{100}\right)$

$$= \log_{100} 100^{-1}$$

$$= -1 \log_{100} 100$$

$$= -1$$

Another Method.

$\log_{100} 0.01 = x$

$100^x = 0.01$

$= \frac{1}{100}$

$= 100^{-1}$

$x = -1$

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2) Evaluate

(i) $\log_5 1 = 0$

(ii) $\log_7 7 = 1$

(iii) $36^{\log_6 4}$

$\log_6 4 = x$

$6^x = 4$

$6^x \; 6^x = 4 \times 4$

$(36)^x = 16$ // Ans.

$36^{\log_6 4} = 16$ //

3) If $\log_{\sqrt{8}} x = 3\frac{1}{3}$, find x

$\log_{\sqrt{8}} x = \frac{10}{3}$

$(\sqrt{8})^{\frac{10}{3}} = x$

$\left(8^{1/2}\right)^{10/3} = x$

$\left(\left(2^3\right)^{1/2}\right)^{10/3} = x$

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$2^{3 \times \frac{1}{2} \times \frac{10}{3}} = x$

$2^5 = x$

$x = 32$ //

4) Evaluate (i) $\log_5 3 \times \log_{27} 25$

$= \frac{\log_a 3}{\log_a 5} \times \frac{\log_a 25}{\log_a 27}$

$= \frac{\cancel{\log_a 3}}{\cancel{\log_a 5}} \times \frac{2 \cancel{\log_a 5}}{3 \cancel{\log_a 3}}$

$= \frac{2}{3}$ //

(ii) $\log_9 27 - \log_{27} 9$

$= \frac{\log_a 27}{\log_a 9} - \frac{\log_a 9}{\log_a 27}$

$= \frac{\log_a 3^3}{\log_a 3^2} - \frac{\log_a 3^2}{\log_a 3^3}$

$= \frac{3 \log_a 3}{2 \log_a 3} - \frac{2 \log_a 3}{3 \log_a 3}$

$= \frac{3}{2} - \frac{2}{3}$

$= \frac{9-4}{6} = \frac{5}{6}$ // Ans

5. Simplify : $\log \frac{75}{16} - 2\log \frac{5}{9} + \log \frac{32}{243}$

$= \log \frac{3\times 25}{2^4} - 2\log \frac{5}{3^2} + \log \frac{2^5}{3^5}$

$= \log \frac{3\times 5^2}{2^4} - \log \frac{5^2}{3^4} + \log \frac{2^5}{3^5}$

$= \log \left(\frac{3\times 5^2}{2^4} \times \frac{3^4}{5^2} \times \frac{2^5}{3^5} \right)$

$= \log \frac{3^5 \times 5^2 \times 2^5}{2^4 \times 5^2 \times 3^5}$

$= \log 2$

Since $\log a - \log b + \log c = \log\left(\frac{ac}{b}\right)$

6. Find the value of x which satisfies the relation

$\log_{10} 3 + \log_{10}(4x+1) = \log_{10}(x+1) + 1$

$\log_{10} 3(4x+1) = \log_{10}(x+1) + \log_{10} 10$

$= \log_{10}(x+1)10$

$\therefore 3(4x+1) = 10(x+1)$

$12x + 3 = 10x + 10$

$12x - 10x = 10 - 3$

$2x = 7$

$x = 7/2$

7. Simplify : $\frac{1}{\log_{xy} xyz} + \frac{1}{\log_{yz} xyz} + \frac{1}{\log_{zx} xyz}$

Soln: We know that $\log_b a = \frac{1}{\log_a b}$

$\therefore \frac{1}{\log_{xy} xyz} = \log_{xyz} xy.$

$\frac{1}{\log_{yz} xyz} = \log_{xyz} yz$

$\frac{1}{\log_{zx} xyz} = \log_{xyz} zx$

$\frac{1}{\log_{xy} xyz} + \frac{1}{\log_{yz} xyz} + \frac{1}{\log_{zx} xyz} = \log_{xyz} xy + \log_{xyz} yz + \log_{xyz} zx$

$= \log_{xyz}(xy \times yz \times zx)$

$= \log_{xyz} x^2y^2z^2$

$= 2\log_{xyz} xyz$

$= 2$

8. If $\log_{10} 2 = 0.30103$, find the value of $\log_{10} 50$

Soln: $\log_{10} 50 = \log_{10}\left(\frac{100}{2}\right)$

$= \log_{10} 100 - \log_{10} 2$

$\log_{10} 50 = \log_{10} 10^2 - \log_{10} 2$

$= 2 - 0.30103$

$= 1.69897$

9. If $\log 2 = 0.3010$ and $\log 3 = 0.4771$ Find the value of (i) $\log 25$ (ii) $\log 4.5$

(i) $\log 25 = \log \frac{100}{4}$

$= \log 100 - \log 4$

$= 2\log_{10} 10 - 2\log_{10} 2$

$= 2 - 2 \times 0.3010$

$= 1.398$

(ii) $\log 4.5 = \log \frac{9}{2}$

$= \log 9 - \log 2$

$= 2\log 3 - \log 2$

$= 2 \times 0.4771 - 0.3010$

$= 0.6532$

10) If $\log 2 = 0.30103$, find the number of digits in 2^{56}.

Soln: $\log_{10} 2^{56} = 56 \log_{10} 2$

$= 56 \times 0.30103$

$= 16.85768$

Characteristic of 2^{56} is 16. Hence the number of digits in 2^{56} is 17.

11) Find $2\log_{10} 5 + \log_{10} 8 - \frac{1}{2}\log_{10} 4$.

$\log_{10} 5^2 + \log_{10} 8 - \log_{10} 4^{1/2}$

$= \log_{10}\left(\frac{5^2 \times 8}{4^{1/2}}\right)$

$= \log_{10} \frac{25 \times 8}{2}$

$= \log_{10} 100$

$= \log_{10} 10^2$

$= 2\log_{10} 10$

$= 2$

12) If $\log_2[\log_3(\log_2 x)] = 1$ then x is equal to

$\log_2 [\log_3(\log_2 x)] = 1$

$\Rightarrow \log_3 \log_2 x = 2$

$\log_3(\log_2 x) = 2\log_3 3$

$= \log_3 3^2$

$= \log_3 9$

$\log_2 x = 9$

$\log_2 x = 9\log_2 2$

$\log_2 x = \log_2 2^9 \Rightarrow x = 2^9 = 512$

13) If $\log_a ab = x$ then find $\log_b ab$

Soln:

$\log_a ab = \log_a a + \log_a b$

$x = 1 + \log_a b$

$\log_a b = x - 1 \Rightarrow \log_a b = \frac{1}{\log_b a} = x - 1$

$\log_b a = \frac{1}{x-1}$

$\log_b ab = \log_b a + \log_b b$

$= \frac{1}{x-1} + \log_b b$

$= 1 + \frac{1}{x-1}$

$= \frac{x-1+1}{x-1}$

$= \frac{x}{x-1}$ //

14) If $\log_4 x + \log_2 x = 6$, then x is equal to

Soln: $\log_4 x + \log_2 x = \frac{1}{\log_x 4} + \frac{1}{\log_x 2}$

$\frac{\log_x 2 + \log_x 4}{\log_x 2^2 \log_x 2} = 6$

$\frac{\log_x 2 + 2\log_x 2}{2\log_x 2 \log_x 2} = 6$

$\frac{3\log_x 2}{2(\log_x 2)^2} = 6$

$\frac{3}{2\log_x 2} = 6$

$3 = 6 \times 2\log_x 2$

$1 = 4\log_x 2$

$\log_x 2 = \frac{1}{4}$

$x^{\frac{1}{4}} = 2$

$x = 2^4$

$x = 16$

Another Method:

$\log_4 x + \log_2 x = 6$

$\Rightarrow \frac{\log x}{\log 4} + \frac{\log x}{\log 2} = 6$

$\Rightarrow \frac{\log x}{2\log 2} + \frac{\log x}{\log 2} = 6$

$\frac{\log x + 2\log x}{2\log 2} = 6$

$3\log x = 6 \times 2\log 2$

$\log x = 4\log 2$

$\log x = \log 2^4$

$x = 2^4$

$x = 16$

15. If $\log 3 = 0.477$ and $(1000)^x = 3$ then find x.

Soln: $\log 3 = 0.477$

$\Rightarrow \log_{10} 3 = 0.477$

Given $(1000)^x = 3$

Taking log on both sides

$\log (1000)^x = \log 3$

$x \log 1000 = \log 3$

$x \log_{10} 10^3 = \log 3$

$3x \log_{10} 10 = \log 3$

$3x = \log 3$

$x = \frac{\log 3}{3}$

$x = \frac{0.477}{3}$

$x = 0.159$

16. If $\log 2 = 0.3010$ and $\log 3 = 0.4771$, ~~then~~ $\log 7 = 0.8451$, then find $\log 2.8$ and $\log 1.5$.

Soln: $\log 2.8 = \log \frac{28}{10}$

$= \log 28 - \log_{10} 10$

$= \log (4 \times 7) - 1$

$= \log 4 + \log 7 - 1$

$= \log 2^2 + \log 7 - 1$

$= 2\log 2 + \log 7 - 1$

$= 2(0.3010) + (0.8451) - 1$

$= 0.6020 + 0.8451 - 1$

$= 1.4471 - 1$

$\log 2.8 = 0.4471$

To find $\log 1.5$

$\log 1.5 = \log \frac{3}{2}$

$= \log 3 - \log 2$

$= 0.4771 - 0.3010$

$\log 1.5 = 0.1761$

AREA

Formulae

1. Area of rectangle $= l \times b$

2. Perimeter of rectangle $= 2(l+b)$

3. Area of Square $= a^2$ (or) $\frac{(\text{diagonal})^2}{2}$

4. Area of 4 walls of a room $= 2(l \times h) + 2(b \times h)$

5. Area of a triangle $\Delta = \frac{1}{2}$ base $\times$ height

6. Area of a triangle $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$
   where a, b, c are sides of the triangle and
   $s = \frac{a+b+c}{2}$

7. Area of an equilateral triangle $= \frac{\sqrt{3}}{4} \times (\text{Side})^2$

8. Radius of incircle of an equilateral triangle of side a $= \frac{a}{2\sqrt{3}}$.

9. Radius of Circum Circle of an equilateral triangle of Side a $= \frac{a}{\sqrt{3}}$

10. Radius of incircle of a triangle of area Δ and Semi-perimeter s $= \frac{\Delta}{s}$.

11. Area of parallelogram $=$ Base $\times$ height

12. Area of a rhombus $= \frac{\text{Product of diagonals}}{2}$

13. Area of a trapezium $= \frac{1}{2}$ (Sum of ||el Sides) $\times$ distance between them

14. Area of a Circle $= \pi R^2$ (R is radius)

15. Circumference of a Circle $= 2\pi R$

16. Length of an arc $= \frac{2\pi R\theta}{360}$ where θ is the Central angle.

17. Area of a Sector $= \frac{1}{2} \times (\text{arc} \times R) = \frac{\pi R^2 \theta}{360}$

18. Area of a Semi Circle $= \frac{\pi R^2}{2}$

19. Circumference of Semi Circle $= \pi R + 2R$

Results on Triangles

1. Sum of the angles of a triangle is 180°.

2. Sum of any two sides of a triangle is greater than the third side.

3. Pythagoras Theorem: In a right-angled triangle, $(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Height})^2$

4. The line joining the mid-point of a side of a triangle to the opposite vertex is called Median.

5. The Point where the medians of a triangle meet is called Centroid.
   The Centroid divides each of the median in the ratio 2:1.

6. In an isosceles triangle, the altitude from the vertex bisects the base.

7. The Median of a triangle divides it into two triangles of the same area.

8. The area of the triangle formed by joining the mid-points of the sides of a given triangle is one-forth of the area of the given triangle.

Results on Quadrilateral

1. Diagonals of a rectangle are equal and bisect each other.

2. Diagonals of a Square are equal and bisect each other at right angles

3. Diagonals of Parallelogram ▱ bisect each other

4. Each diagonal of a llelogram divides it into two triangles of the same area.

5. Diagonals of a rhombus ◇ are unequal and bisect each other at right angles.

6. A parallelogram and a rectangle on the same base and between the same parallels are equal in area.

7. Of all the parallelograms of given sides, the ~~pall~~ parallelogram which is a rectangle has the greatest area.

Problems:

1. One side of a rectangular field is 15 m and one of its diagonals is 17 m. Find the area of the field.

Soln: Given

ABD is a right angled $\triangle$le

$\therefore$ By Pythagoras Theorems

$15^2 + b^2 = 17^2$

$b^2 = 17^2 - 15^2$

$= 289 - 225$

$b^2 = 64$

$b = \sqrt{64} = 8$

Therefore breadth of the rectangle is 8 m

Area $= l \times b$

$= 15 \times 8$

$= 120\ m^2$

2. A lawn is in the form of a rectangle having its sides in the ratio 2:3. The area of the lawn is $\frac{1}{6}$ hectares. Find the length and breadth of the lawn.

Soln: Given

Area $= 3x \times 2x = 6x^2 = \frac{1}{6}$ hectares

$1 \text{ hectare} = 10000 m^2$

$\frac{1}{6}\text{ hectare} = \frac{1}{6} \times 10000 m^2.$

$\therefore$ Area $= 6x^2 = \frac{1}{6} 10000 m^2$

$x^2 = \frac{1}{36} 10000 m^2$

$= \frac{2500}{9} m^2$

$x = \sqrt{\frac{2500}{9}} = \frac{5}{3}\sqrt{100}\, m$

$x = \frac{50}{3} m$

$\therefore$ length $= 3x = 3 \times \frac{50}{3} = 50 m$

breadth $= 2x = 2 \times \frac{50}{3} = \frac{100}{3} m$

3. Find the Cost of Carpeting a room 13 m long and 9 m broad with a carpet 75 cm wide at the rate of Rs. 12.40 per ~~square~~ metre.

Soln: Area of the room $= 13 \times 9 m^2$

$= 117 m^2$

Carpet breadth $= 75 cm = 75 \times \frac{1}{100} m$

$= 0.75 m$

Area of Carpet needed $= 117 m^2$

$0.75 \times$ length of Carpet $= 117$

length of Carpet $= \frac{117}{0.75}$

length of the Carpet $= 156 m$

Cost of 1 metre of Carpet $=$ Rs. 12.40

Cost of 156 m of Carpet $= 12.4 \times 156$

$= 1934.4$ Rs.

4. If the diagonal of a rectangle is 17 cm long and its perimeter is 46 cm, find the area of the rectangle.

Soln:

Given diagonal $= 17$ cm

perimeter $= 46$ cm

$2l + 2b = 46$

$l + b = 23 \rightarrow (1)$

From diagonal $\Rightarrow l^2 + b^2 = 17^2 \rightarrow (2)$

$(l+b)^2 = l^2 + b^2 + 2lb$

$23^2 = 17^2 + 2lb$

$2lb = 23^2 - 17^2$

$= 529 - 289$

$2lb = 240$

$lb = 120 cm^2$

5. The length of a rectangle is twice its breadth. If its length is decreased by 5cm and breadth is increased by 5cm, the area of the rectangle is increased by 75 sq.cm. Find the length of the rectangle.

Soln:

In the given rectangle $l = 2b \rightarrow (1)$

In new rectangle $l_1 = l - 5$

$b_1 = b + 5$

$A_1 = l_1 b_1 = A + 75$

$\therefore l_1 b_1 = lb + 75$

$(l-5)(b+5) = lb + 75$

Also by (1) $l = 2b \Rightarrow b = \frac{l}{2}$

$\therefore (l-5)\left(\frac{l}{2}+5\right) = l\frac{l}{2} + 75$

$(l-5)\left(\frac{l+10}{2}\right) = \frac{l^2}{2} + 75$

$(l-5)(l+10) = 2\left[\frac{l^2}{2} + 75\right]$

$l^2 - 5l + 10l - 50 = l^2 + 150$

$l^2 + 5l - 50 - l^2 - 150 = 0$

$5l - 200 = 0$

$5l = 200$

$l = 40cm$ //

6. In measuring the sides of a rectangle, one side is taken 5% in excess and the other 4% in deficit. Find the error percent in the area calculated from these measurements.

Soln. In original rectangle

length $= l$

breadth $= b$

Area $= lb$

By Mistake [illegible]

New rectangle

length $= l + \frac{5}{100} l$

breadth $= b - \frac{4}{100} b$

(ie) $l_1 = \left(1 + \frac{5}{100}\right) l$

[illegible]

$b_1 = \left(1 - \frac{4}{100}\right) b$

[illegible]

Area of new rectangle $= l_1 b_1$

[illegible]

[illegible] lb

Error percentage $= \frac{\text{Area of New rectangle} - \text{Area of Original rectangle}}{\text{Area of original rectangle}}$

$= \frac{l_1 b_1 - lb}{lb} \times 100$

$= \dfrac{\left(\frac{105 \times 96}{10^{?}}\right) lb - lb}{lb} \times 100$

$= \dfrac{\left(\frac{10080}{10000} - 1\right) lb}{lb} \times 100$

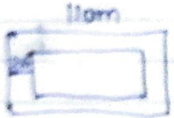
$= \dfrac{10080 - 10000}{10000} \times 100$

$= \dfrac{80}{100}$

$= 0.8\%$

7. A rectangular grassy plot 110m by 65m has a gravel path 2.5m wide all round it on the inside. Find the cost of gravelling the path at 80 paise per sq. metre.

Soln:



Area of gravel path = Area of A + B + C + D

$= (65 \times 2.5) + (65 \times 2.5) + (105 \times 2.5) + (105 \times 2.5)$

$= 2.5\,[65 + 65 + 105 + 105]$

$= 2.5\,[130 + 210]$

$= 2.5\,[340]$

$= 850\ m^2$

Cost of gravelling $1\ m^2$ = 0.80 Rs.

Cost of gravelling $850\ m^2$ = 0.80×850

= 680 Rs

8. The perimeters of two squares are 40 cm and 32 cm. Find the perimeter of a third square whose area is equal to the difference of the areas of the two squares.

Soln:

Perimeter 40cm, Side $= \frac{40}{4} = 10$ — Perimeter $= 32$cm, Side $= \frac{32}{4} = 8$ — Perimeter = ?

~~Area of Square 3 = [Area of Square] - [Area of Square]~~

Area of Square 3 = [Area of Square 1] - [Area of Square 2]

$= 10^2 - 8^2$

$= 100 - 64$

$= 36\ cm^2$

Side of Square 3 $= \sqrt{36} = 6$ cm

Perimeter of Square 3 $= 4 \times 6$

$= 24$ cm

9. A room 5m 44cm long and 3m 74cm broad is to be paved with square tiles. Find the least number of square tiles required to cover the floor.

Soln: Length of the room = 5 m 44 cm

= (500 + 44) cm

= 544 cm

Breadth of the room = 3 m 74 cm

= (300 + 74) cm

$= 374$ cm

Area of the room $= (544 \times 374)\ cm^2$

Size of largest tile = HCF of 544, 374

$$
\begin{array}{r|l}
374 & 544 \ (1 \\
 & 374 \\ \hline
170 & 374 \ (2 \\
 & 340 \\ \hline
34 & 170 \ (5 \\
 & 170 \\ \hline
 & 0
\end{array}
$$

$\therefore$ 34 is HCF of 544, 374

Area of 1 tile of side 34cm $= (34 \times 34)$ cm

$= (34 \times 34)\ cm^2$

Number of tiles required $= \dfrac{544 \times 374}{34 \times 34}$

$= 16 \times 11$

$= 176$ tiles //.

10. Find the area of a Square, one of whose diagonals is 3.8m long.

Soln:

diagonal $= 3.8$ m

$a^2 + a^2 = 3.8^2$

$2a^2 = 3.8^2$

$a^2 = \dfrac{3.8^2}{2}$

$a^2 = \dfrac{14.44}{2}$

$a^2 = 7.22$

Area of the Square $= 7.22\ m^2$

11. The diagonals of two Squares are in the ratio of 2:5. Find the ratio of their areas

Soln.

Square 1

diagonal $= 2x$

$a^2 + a^2 = (2x)^2$

$2a^2 = 4x^2$

$a^2 = \dfrac{4x^2}{2}$

Area $= 2x^2$

Square 2

diagonal $= 5x$

$a^2 + a^2 = (5x)^2$

$2a^2 = 25x^2$

$a^2 = \dfrac{25x^2}{2}$

Area $= \dfrac{25}{2}x^2$

Ratio $2x^2 : \dfrac{25}{2}x^2$

$2 : \dfrac{25}{2}$

$4 : 25$ //

12. If each side of a Square is increased by 25%. Find the percentage change in its area.

Soln:

Square 1

side $= a$

Area $= a^2$

Square 2

side $= a + \dfrac{25}{100}a$

$= a + \dfrac{1}{4}a$

$= \dfrac{4a + a}{4}$

$= \dfrac{5a}{4}$

Area $= \left(\dfrac{5}{4}a\right)^2$

$= \dfrac{25}{16}a^2$

Ratio $a^2 : \dfrac{25}{16}a^2$

$1 : \dfrac{25}{16} \Rightarrow 16 : 25$

Percentage increase = $\frac{\text{Area of sq2} - \text{Area of sq1}}{\text{Area of sq1}}$

$= \frac{\frac{25}{16}a^2 - a^2}{a^2} \times 100$

$= \frac{\frac{25}{16} - 1}{1} \times 100$

$= \frac{25-16}{16} \times 100$

$= \frac{9}{16} \times 100$

$= \frac{9}{4} \times 25$

$= \frac{225}{4}$

$= 56\frac{1}{4}\%$

13. If the length of a certain rectangle is decreased by 4 cm and the width is increased by 3 cm, a square with the same area as the original rectangle would result. Find the perimeter of the original rectangle.

Soln: Original rectangle

length = l

breadth = b

Area = lb

New square

length = $l-4$

breadth = $b+3$

$a = l-4 = b+3$ ①

Area = $(l-4)(b+3)$

They have same Area

$\therefore lb = (l-4)(b+3)$

$= lb - 4b + 3l - 12$

$lb - lb + 4b - 3l + 12 = 0$

$4b - 3l + 12 = 0$

From ① $l - 4 = b + 3$

$l = b + 3 + 4$

$l = b + 7$

$\therefore 4b - 3(b+7) + 12 = 0$

$4b - 3b - 21 + 12 = 0$

$b - 9 = 0$

$b = 9$

$l = b + 7$

$= 9 + 7$

$= 16$

$\therefore$ length = 16 cm, breadth = 9 cm

Perimeter = $2l + 2b$ = $2 \times 16 + 2 \times 9$

$= 32 + 18$

$= 50$ cm //

~~15~~ 14. A room is half as long again as it is breadth. The cost of carpeting the room at Rs. 5 per sq.m is Rs. 270 and the cost of papering the four walls at Rs. 10 per m^2 is Rs. 1728. If a door and 2 windows occupy 8 sq.m. Find the dimensions of the room.

Soln: Breadth of the room $= b$ m

Length of the room $= \left(b + \frac{b}{2}\right)$ m

$= \frac{3b}{2}$ m

$\therefore$ Area of the floor of the room $= b \times \frac{3b}{2}$

$= \frac{3b^2}{2}$ m^2

Cost of carpeting $1 m^2$ = Rs. 5

Cost of carpeting $\frac{3b^2}{2} = \frac{3b^2}{2} \times 5 = 270$

$b^2 = \frac{270 \times 2}{3 \times 5}$

$= \frac{90 \times 2}{5}$

$= 18 \times 2 = 36$

$b^2 = 36 \Rightarrow b = 6$ m

$\therefore$ length of the room $= 6 + \frac{6}{2} = 9$ m

breadth of the room $= 6$ m

height of the room $= h$ m

Area of 4 walls $= 2lh + 2bh$

$= 2 \times 9 \times h + 2 \times 6 \times h$

$= 18h + 12h = 30h$ m^2

Area of 1 door and 2 windows $= 8$ m^2

Area of remaining space of 4 walls $= 30h - 8$

Cost of papering $1 m^2$ = Rs. 10

Cost of papering $(30h - 8) m^2 = (30h - 8) \times 10$

Given cost = 1720 Rs.

$\therefore (30h - 8) 10 = 1720$

$30h - 8 = 172$

$30h = 172 + 8$

$30h = 180$

$h = \frac{180}{30}$

$h = 6$ m

$\therefore$ Dimensions of the room

length = 9 m

breadth = 6 m

height = 6 m //

15. Find the area of a triangle whose sides measure 13 cm, 14 cm and 15 cm.

Soln: Area of the $\triangle$le $= \sqrt{s(s-a)(s-b)(s-c)}$

$s = \frac{a+b+c}{2} = \frac{13+14+15}{2}$

$= \frac{42}{2} = 21$

Area $= \sqrt{21(21-13)(21-14)(21-15)}$

$= \sqrt{21 \times 8 \times 7 \times 6}$

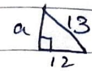
$= \sqrt{7 \times 3 \times 2 \times 2 \times 2 \times 7 \times 3 \times 2}$

$= 7 \times 3 \times 2 \times 2$

$= 84$ cm^2 //

16. Find the area of a right-angled triangle whose base is 12 cm and hypotenuse 13 cm.

Soln: Given is a right angled triangle



$\therefore a^2 + 12^2 = 13^2$

$a^2 = 13^2 - 12^2$

$= 169 - 144$

$a^2 = 25$

$a = \sqrt{25} = 5$

Area $= \frac{1}{2} \times$ base $\times$ height

$= \frac{1}{2} \times 12 \times 5$

$= 30\ cm^2$

17. The base of a triangular field is three times its altitude. If the cost of cultivating the field at Rs. 24.68 per hectare be Rs. 333.18, find its base and height.

Soln: altitude $= x$

base $= 3x$

Area of the $\Delta^{le} = \frac{1}{2} \times 3x \times x$

$= \frac{3x^2}{2}\ m^2$

1 hectare $= 10000\ m^2$

Cost of Cultivating 1 hectare $= 24.68$

Cos of Cultivating 10000 m^2 $= 24.68$

Cost of Cultivating 1 m^2 $= \frac{24.68}{10000}$

Cost of Cultivating $\frac{3x^2}{2}\ m^2 = \frac{24.68}{10000} \times \frac{3x^2}{2}$

$= \frac{12.34}{10000} \times 3x^2$

$\frac{12.34}{10000} \times 3x^2 = 333.18$

$x^2 = \frac{333.18 \times 10000}{3 \times 12.34}$

$= \frac{33318 \times 10000}{3 \times 1234}$

$= \frac{11106 \times 10000}{1234}$

$x^2 = 90000$

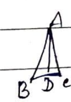
$x = 300\ m$

Altitude $= 300\ m$

Base $= 3 \times 300 = 900\ m$

18. The altitude drawn to the base of an isosceles triangle is 8 cm and the perimeter is 32 cm. Find the area of the triangle.

Soln:



$AD = 8\ cm$

$AB + BC + CA = 32 \rightarrow (1)$

In Isoceles $AB = AC$ and $BD = DC$

ΔADC is right angled triangle.

$AD^2 + DC^2 = AC^2$

$8^2 + DC^2 = AC^2 \rightarrow (2)$

From (1) $2AB + 2DC = 32$

$AB + DC = 16$

$DC = 16 - AB$

From (2) $8^2 + DC^2 = AC^2$

$8^2 + DC^2 = AB^2$ (since $AB = AC$)

$8^2 + (16-AB)^2 = AB^2$

$8^2 + 16^2 + AB^2 - 2\times 16 \times AB = AB^2$

$8^2 + 16^2 - 32AB = 0$

$32AB = 8^2 + 16^2$

$= 64 + 256$

$32AB = 320$

$AB = \frac{320}{32}$

$AB = 10$

$$\boxed{\begin{aligned} DC &= 16-10 \\ &= 6 \\ BC &= 2\times 6 \\ &= 12 \end{aligned}}$$

Area of the Δ^{le} $= \frac{1}{2} \times$ base $\times$ height

$= \frac{1}{2} \times BC \times AD$

$= \frac{1}{2} \times 12 \times 8$

$= 48\ cm^2$

19. Find the length of the ~~altitude~~ side of an equilateral triangle of ~~side~~ altitude $3\sqrt{3}$ cm.

Soln:

$AD = 3\sqrt{3}$ cm

In equilateral Δ^{le} $AB = BC = CA = x$

$BD = \frac{1}{2} BC$

$= \frac{1}{2} AB = \frac{1}{2} x$

ΔADC is an right angled Δ^{le}.

$AC^2 = AD^2 + DC^2$

$x^2 = (3\sqrt{3})^2 + \left(\frac{x}{2}\right)^2$

$x^2 = 9\times 3 + \frac{x^2}{4}$

$x^2 - \frac{x^2}{4} = 27$

$\frac{4x^2 - x^2}{4} = 27$

$\frac{3x^2}{4} = 27$

$x^2 = \frac{27\times 4}{3}$

$x^2 = 9\times 4$

$x = 3\times 2 = 6$ cm

length of the side $= 6$ cm

20. Find the length of the altitude of an equilateral triangle of side $3\sqrt{3}$ cm.

Soln.

$AB = AC = BC = 3\sqrt{3}$

$BD = DC = \frac{1}{2} BC$

$= \frac{3\sqrt{3}}{2}$

$AD^2 + DC^2 = AC^2$

$AD^2 + \left(\frac{3\sqrt{3}}{2}\right)^2 = (3\sqrt{3})^2$

$$AD^2 = 27 - \frac{27}{4}$$

$$= \frac{4 \times 27 - 27}{4}$$

$$= \frac{3 \times 27}{4}$$

$$AD^2 = \frac{9 \times 9}{2 \times 2}$$

$$AD = \frac{9}{2} = 4.5\text{cm}$$

21. In two triangles, the ratio of the areas is 4:3 and the ratio of their heights is 3:4. Find the ratio of their bases.

Soln: Triangle 1   Triangle 2

Area $A_1 = \frac{1}{2} h_1 b_1$ Area $A_2 = \frac{1}{2} h_2 b_2$

Area Ratio 4 : 3 $\Rightarrow \frac{A_1}{A_2} = \frac{4}{3}$

$$\frac{\frac{1}{2} h_1 b_1}{\frac{1}{2} h_2 b_2} = \frac{4}{3}$$

$$\frac{h_1 b_1}{h_2 b_2} = \frac{4}{3} \rightarrow (1)$$

Height Ratio 3 : 4 $\Rightarrow \frac{h_1}{h_2} = \frac{3}{4}$

$$h_1 = \frac{3h_2}{4}$$

Substituting in (1)

$$\frac{3h_2 b_1}{4h_2 b_2} = \frac{4}{3}$$

$$\frac{3b_1}{4b_2} = \frac{4}{3}$$

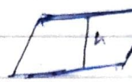
$$\frac{b_1}{b_2} = \frac{4}{3} \times \frac{4}{3}$$

$$\frac{b_1}{b_2} = \frac{16}{9}$$

Ratio of Bases 16 : 9

22. The base of a parallelogram is twice its height. If the area of the parallelogram is 72 sqcm, Find its height.

Soln:



Area of ||elogram $= b \times h$

Given base $= 2h$ $\therefore$ Area $= 2h \times h$

$$= 2h^2$$

$$2h^2 = 72$$

$$h^2 = 36$$

$$h = 6$$

height = 6cm

23. Find the area of a rhombus one side of which measures 20cm and one diagonal 24cm.

Soln:

ABCD is a Rhombus

AED is a right angled Δle.

$$x^2 + 12^2 = 20^2$$

$$x^2 = 20^2 - 12^2$$

$x^2 = 400 - 144$

$x^2 = 256$

$x = 16$ cm

$\therefore$ Diagonals of the rhombus measures 24 cm and 32 cm

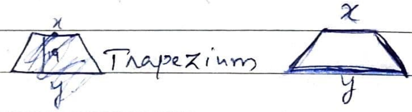
Area of the rhombus $= \frac{1}{2} \times$ Product of diagonals

$= \frac{1}{2} \times 24 \times 32$

$= 384\ cm^2$

24. The difference between two parallel sides of a trapezium is 4 cm. The perpendicular distance between them is 19 cm. If the area of the trapezium is $475\ cm^2$. Find the length of the parallel sides.

Soln:



Given $y - x = 4$ cm

$\perp r$ distance $h = 19$ cm

Area of the trapezium $= \frac{1}{2}$ (Sum of $\parallel$ sides) $\times h$

$= \frac{1}{2}(x+y)h$

$\frac{1}{2}(x+y)h = 475$

$(x+y)h = 475 \times 2$

$= 950$

$(x+y)19 = 950$

$x+y = \frac{950}{19}$

$x + y = 50$

$y - x = 4$

$2y = 54$

$y = \frac{54}{2}$

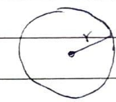
$y = 27$

$x + 27 = 50 \Rightarrow x = 50 - 27 \Rightarrow x = 23$

$\therefore$ length of the sides of trapezium are 23 cm and 27 cm.

25. Find the length of a rope by which a cow must be tethered in order that it may be able to graze an area of 9856 sq. metres.

Soln:



If the length of the rope $= r$ m

Then the cow can tether πr^2 (area of circle)

$\pi r^2 = 9856$

$r^2 = \frac{9856}{\pi}$

$= \frac{9856}{22} \times 7$ (Since $\pi = \frac{22}{7}$)

$= \frac{896}{2} \times 7 = 448 \times 7$

$r^2 = 448 \times 7 = 4 \times 112 \times 7$

$= 4 \times 7 \times 16 \times 7$

$r^2 = 3136$

Length of the rope $r = 56$ m

26. The area of a circular field is 13.86 hectares. Find the cost of fencing it at the rate of Rs. 4.40 per metre.

Soln:

Area of the circular field = 13.86 hec

$= 13.86 \times 10000\ m^2$

$= 138600\ m^2$

$\pi r^2 = 138600$

$r^2 = \frac{138600}{\pi}$

$= 138600 \times \frac{7}{22}$ (Since $\pi = \frac{22}{7}$)

$= \frac{12600}{2} \times 7$

$= 6300 \times 7$

$r^2 = 44100$

$r = \sqrt{44100} = 210\ m$

Perimeter (or) Circumference of Circle $= 2\pi r$

$= 2 \times \frac{22}{7} \times 210$

$= 44 \times 30$

$= 1320\ m$

Cost of fencing 1 m = 4.40

Cost of fencing 1320 m $= 4.40 \times 1320$

= Rs. 5808

27. The diameter of the driving wheel of a bus is 140 cm. How many revolutions per minute must the wheel make in order to keep a speed of 66 kmph?

Soln: Diameter of the wheel $= 2r = 140$ cm

$2r = 140$

$r = \frac{140}{2} = 70$ cm

Circumference of the wheel $= 2\pi r$

$= 2 \times \frac{22}{7} \times 70$

$= 440$ cm

Speed = 66 km/hr

Distance covered in one hour = 66 km

in 60 mins = 66 km

$= 66 \times 1000$ m

= 66000 m

$= 66000 \times 100$ cm

in 60 mins = 6600000 cm

distance covered in 1 minute $= \frac{6600000}{60}$ cm

= 110000 cm

Number of revolutions per minute $= \frac{\text{distance in 1 min}}{\text{Circumference of wheel}}$

$= \frac{110000}{440}$

$= \frac{1000}{4}$

= 250

28. A wheel makes 1000 revolutions in covering a distance of 88 Km. Find the radius of the wheel.

Soln:

$$\text{No. of revolutions} = \frac{\text{distance}}{\text{Circumference}}$$

$$1000 = \frac{88}{2\pi r}$$

$$2\pi r = \frac{88}{1000}$$

$$\pi r = \frac{88}{1000 \times 2}$$

$$r = \frac{88}{1000 \times 2} \cdot \frac{1}{\pi}$$

$$= \frac{88}{1000 \times 2} \times \frac{7}{22}$$

$$= \frac{4 \times 7}{1000 \times 2}$$

$$= \frac{14}{1000} \text{ km} = 0.014 \text{ km}$$

$$= \frac{14}{1000} \times 1000 \text{ m}$$

$$r = 14 \text{ m}$$

29. The inner circumference of a circular race track, 14m wide, is 440 m. Find the radius of the outer circle.

Soln:

Inner Circumference $= 2\pi r$

$$2\pi r = 440 \text{ m}$$

$$r = \frac{440}{2\pi}$$

$$= \frac{440}{2} \times \frac{7}{22}$$

$$r = 70 \text{ m}$$

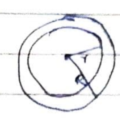
Race track 14m wide

$$\therefore R = (70 + 14) \text{ m}$$

Radius of outer circle $\}$ $R = 84 \text{ m}$ //

30. Two concentric circles form a ring. The inner and outer circumferences of the ring are $50\frac{2}{7}$ m and $75\frac{3}{7}$ m respectively. Find the width of the ring.

Soln:



Let radius of inner circle $= r$

radius of outer circle $= R$

width of the ring $= R - r$

Given Circumferences

$2\pi r = 50\frac{2}{7}$ & $2\pi R = 75\frac{3}{7}$

$$2 \times \frac{22}{7} \times r = \frac{352}{7}$$

$$r = \frac{352}{7} \times \frac{7}{22} \times \frac{1}{2}$$

$$= \frac{352}{22 \times 2}$$

$$= \frac{32}{2 \times 2}$$

$$r = 8 \text{ m}$$

$$2 \times \frac{22}{7} \times R = \frac{528}{7}$$

$$R = \frac{528}{7} \times \frac{7}{22} \times \frac{1}{2}$$

$$= \frac{528}{22 \times 2}$$

$$= \frac{48}{2 \times 2}$$

$$= 12 \text{ m}$$

$$\therefore \text{width} = 12 - 8 = 4 \text{ m} //$$