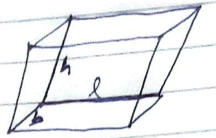
Unit IV
Volume and Surface Area.

1. Cuboid



Length = l
breadth = b
height = h

Volume = $(l \times b \times h)$ cubic units

Surface area = $2lb + 2bh + 2lh$

Diagonal = $\sqrt{l^2+b^2+h^2}$

2. Cube

If $l=b=h$ (ie) side length $=a$

Volume = a^3

Surface area = $6a^2$

Diagonal = $a\sqrt{3}$

3. Cylinder

Radius of base = r
height = h

Volume = $\pi r^2 h$

Curved Surface Area = $2\pi rh$

Total Surface Area = $2\pi rh + 2\pi r^2$

4. Cone

radius of base = r
height of cone = h

Slant height $l = \sqrt{h^2+r^2}$

Volume = $\frac{1}{3}\pi r^2 h$

Curved Surface Area = $\pi r l$

Total Surface Area = $\pi r l + \pi r^2$

5. Sphere

Example: lemon
Ball
etc.

Radius of the Sphere = r

~~Then~~ Volume = $\frac{4}{3}\pi r^3$

Surface Area = $4\pi r^2$

6. Hemisphere

half of sphere

Radius of the Hemisphere = r

Volume = $\frac{2}{3}\pi r^3$

Curved Surface Area = $2\pi r^2$

Total Surface Area = $3\pi r^2$

Note:

Volume measured in Cubic units like m^3, cm^3, etc.

2. Area measured in Square units
Eg m^2, cm^2, km^2, etc

3. length, breadth, height are measured in units Eg: m, cm, km, ...

4. 1 litre $= 1000\ m^3$

Problems

1. Find the Volume and Surface area of a Cuboid 16m long 14m broad and 7m high.

Soln: Volume of Cuboid $= l \times b \times h$

$= 16 \times 14 \times 7$

$= 1568\ m^3$

Surface area $= 2lb + 2bh + 2lh$

$= 2(16\times14) + 2(14\times7) + 2(16\times7)$

$= 2[224 + 98 + 112]$

$= 2[434]$ $= 868\ m^2$

2. Find the length of the longest pole that can be placed in a room 12m long 8m broad and 9m high.

Soln: Diagonal is the longest of the room

Diagonal $= \sqrt{l^2 + b^2 + h^2}$

$= \sqrt{12^2 + 8^2 + 9^2}$

$= \sqrt{144 + 64 + 81}$

$= \sqrt{289}$ $= 17\ m$

3. The volume of a wall, 5 times as high as it is broad and 8 times as long as it is high, is 12.8 cu.metres. Find the breadth of the wall.

Soln:

height $h = 5b$

length $l = 8h$

$= 8(5b)$

$= 40b$

breadth $= b$

Volume of the wall $= 12.8\ m^3$ (given)

$l \times b \times h = 12.8\ m^3$

$40b \times b \times 5b = 12.8$

$200\ b^3 = 12.8$

$b^3 = \frac{12.8}{200}$

$= \frac{0.128}{2}$

$= 0.064$ (or) $\frac{64}{1000}$

$b^3 = \frac{64}{1000} = \frac{4^3}{10^3}$

$b = \frac{4}{10} = 0.4\ m$

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4. Find the number of bricks, each measuring 24cm × 12cm × 8cm required to construct a wall 24m long, 8m high and 60cm thick if 10% of the wall is filled with mortar?

Soln:

$$\text{No. of bricks} = \frac{\text{Volume of the wall}}{\text{Volume of one brick}}$$

Volume of one brick $= 24 \times 12 \times 8 \ cm^3$

Volume of the wall $= (24\times100) \times (8\times100) \times 60$

$= 24 \times 8 \times 60 \times 10^4 \ cm^3$

10% of wall filled with mortar.

∴ ~~Brick~~ Volume of requires brick

$=$ Volume of wall − 10% of Volume of wall

$= 24\times8\times60\times10^4 - \frac{10}{100}\times24\times8\times60\times10^4$

$= 24\times8\times60\times10^4 - 24\times8\times6\times10^4$

$= 24\times8\times6\times10^4\,(10-1)$

$= 24\times8\times6\times10^4\times9$

$$\text{No. of Bricks} = \frac{24\times8\times6\times10^4\times9}{24\times12\times8}$$

$= \frac{10^4\times9}{2} = 10^3\times5\times9$

$= 45000$ bricks / Ans

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5. Water flows into a tank 200m × 150m through a rectangular pipe 1.5m × 1.25m @ 20kmph. In what time (in minutes) will the water rise by 2 meters?

Soln:

Volume of water required in the tank $= 200\times150\times2 \ m^3$

$= 60000 \ m^3$

Speed of water flow $= 20$ km/h

$= \frac{20\times1000}{60}$ m/min

$= \frac{1000}{3}$ m/min

(ie) length of water occupied in 1min $= \frac{1000}{3}$ m

Volume flow in 1min $= \frac{1000}{3}\times1.5\times1.25$

$= 625 \ m^3$

Time required to fill 2 m $= \frac{\text{Volume of tank water}}{\text{Volume flow in 1 min}}$

$= \frac{60000}{625}$

$= 96$ mins.

6. The diamensions of an open box are 50cm, 40cm and 23cm. Its thickness is 3 cm. If 1 Cubic cm of metal used in the box weighs 0.5 gms, find the

Weight of the box.

Soln:

Volume of the metal used in the box } = ~~External~~ volume of the box

$= (\text{External volume of the box}) - (\text{Internal volume of the box})$

External volume of the box $= l \times b \times h$

$= 50 \times 40 \times 23 = 46000 \text{ cm}^3$

Internal Box dimensions

height $= h - 3$
length $= l - 6$
breadth $= b - 6$

$\therefore$ Internal Volume $= (l-6) \times (b-6) \times (h-3)$
$= 44 \times 34 \times 20$
$= 29920 \text{ cm}^3$

$\therefore$ Volume of metal used in the box $= 46000 - 29920$
$= 16080 \text{ cm}^3$

Given that weight of $1 \text{ cm}^3 = 0.5 \text{ gm}$

$\therefore$ Weight of $16080 \text{ cm}^3 = (16080 \times 0.5)$ gms
$= 8040$ gms

[Note:
1000 gms $= 1$ kg.
1 gm $= \frac{1}{1000}$ kg]

Weight of $16080 \text{ cm}^3 = 8040$ gms
$= 8040 \times \frac{1}{1000}$ kg
$= 8.04$ kg // Ans.

7. The diagonal of a Cube is $6\sqrt{3}$ cm. Find its Volume and Surface Area.

Soln:

Diagonal of a Cube of Side a $= \sqrt{a^2+a^2+a^2}$
$= \sqrt{3a^2}$
$= a\sqrt{3}$

Here diagonal $= 6\sqrt{3}$ cm

$\therefore a\sqrt{3} = 6\sqrt{3}$ cm
$a = 6$ cm

Volume of the Cube $= a^3 = 6 \times 6 \times 6$
$= 216 \text{ cm}^3$ //

Surface area $= 6a^2 = 6 \times 6 \times 6$
$= 216 \text{ cm}^2$ //

8. The Surface area of a Cube is 1734 Sq. cm. Find its volume.

Soln:

Give S.A of a Cube $= 1734 \text{ cm}^2$

$6a^2 = 1734$

$a^2 = \frac{1734}{6}$

$a^2 = 289$

$a = \sqrt{289} = 17$ //

Volume $= a^3 = 17 \times 17 \times 17$
$= 4913 \text{ cm}^3$

9. A rectangular block 6cm by 12 cm by 15cm is cut up into an exact number of equal cubes. Find the least possible number of cubes.

Soln:
HCF of 6, 12, 15 is

$6 = 2 \times 3$
$12 = 2 \times 2 \times 3$
$15 = 3 \times 5$

HCF $= 3$

$\therefore$ Cube side length $= 3$ cm

Volume of rect. block $= 6 \times 12 \times 15$
$= 1080 \text{ cm}^3$

Volume of the cube $= 3 \times 3 \times 3$
$= 27 \text{ cm}^3$

Number of cubes cut from rect. block $= \dfrac{\text{Vol. of rect blk}}{\text{Volume of cube}}$

$= \dfrac{1080}{27}$

$= 40$ Cubes //

10. A Cube of edge 15cm is immersed completely in a rectangular vessel containing water. If the dimensions of the base of vessel are 20cm × 15cm find the rise in water level.

Soln:
Increase in Volume = Volume of Cube
$= 15 \times 15 \times 15$
$= 3375 \text{ cm}^3$

Rise in water level $= \dfrac{\text{Volume of the cube}}{\text{Area of the vessel (base)}}$

$= \dfrac{3375}{20 \times 15}$

$= \dfrac{45}{4}$

$= 11.25$ cm

11. Three solid cubes of sides 1cm, 6cm and 8cm are melted to form a new cube. Find the surface area of the cube so formed.

Soln: Volume of 3 cubes $= 1^3 + 6^3 + 8^3$
$= 1 + 216 + 512$
$= 729 \text{ cm}^3$

New cube volume $= 729 \text{ cm}^3$

Side $= \sqrt[3]{729} = \sqrt[3]{9 \times 9 \times 9}$
$= 9$ cm

Surface Area $= 6a^2 = 6 \times 9 \times 9 = 486$ sq cm

12. If each edge of a cube is increased by 50% find the percentage increase in its Surface area.

Soln:

Original Cube.

Side $= a$

Surface Area $= 6a^2$

New Cube

Side $= a + \frac{50}{100} a$

$= a + \frac{a}{2}$

$= \frac{3a}{2}$

Surface Area $= 6\left(\frac{3a}{2}\right)^2$

$= 6 \times \frac{9a^2}{4}$

$= \frac{3 \times 9}{2} a^2$

Percentage increase $= \frac{(S.A\ of\ New\ Cube) - (S.A\ of\ Original\ Cube)}{S.A\ of\ Original\ Cube} \times 100$

$= \frac{\frac{3 \times 9}{2} a^2 - 6a^2}{6a^2} \times 100$

$= \frac{27a^2 - 12a^2}{2 \times 6a^2} \times 100$

$= \frac{15a^2}{12a^2} \times 100$

$= \frac{5}{4} \times 100 = 125\%$

13. Two cubes have their volumes in the ratio 1:27. Find the ratio of their surface areas.

Soln:

Cube 1

Volume $= x$

$a_1^3 = x$

$a_1 = \sqrt[3]{x} = x^{1/3}$

S.A $= 6a_1^2$

$= 6(x)^{2/3}$

Cub 2

Volume $= 27x$

$a_2^3 = 27x$

$a_2 = \sqrt[3]{27x} = (27x)^{1/3}$

S.A $= 6a_2^2$

$= 6(27x)^{2/3}$

$= 6 \times (3^3 x)^{2/3}$

$= 6 \times 3^2 x^{2/3}$

Ratio $= 6x^{2/3} : 6 \times 9 \times x^{2/3}$

$= 1:9$

14. Find the volume, curved S.A and the total S.A of a cylinder with diameter of base 7cm and height 40cm.

Soln: Radius of Base of Cylinder $= \frac{7}{2}$

$r = 3.5cm$

height $h = 40cm$

Volume of the cylinder $= \pi r^2 h$

$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 40\ cm^3$

$= \frac{11 \times 7}{2} \times 40\ cm^3$

$= 1540 cm^3$

Curved Surface area of the Cylinder = $2\pi rh$

$= 2 \times \frac{22}{7} \times \frac{7}{2} \times 40$

$= 880\ cm^2$

Total Surface Area of the Cylinder = $2\pi rh + 2\pi r^2$

$= 880 + 2 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$

$= 880 + 77$

$= 957\ cm^2$ //

15. If the capacity of a Cylinderical tank is $1848\ m^3$ and the diameter of its base is 14 m then find the depth of the tank.

Soln:

Volume of the Cylindrical tank $= 1848\ m^3$

Diameter of base $2r = 14m$

$r = 7m$

Volume $= \pi r^2 h = 1848$

$\frac{22}{7} \times 7 \times 7 \times h = 1848$

$h = \frac{1848 \times 7}{22 \times 7 \times 7}$

$= \frac{924 \times 7}{11 \times 7 \times 7}$

$= \frac{84}{7}$

height $= 12m$ //

16. $2.2\ dm^3$ of lead is to be drawn into a cylindrical wire 0.5cm in diameter. Find the length of the wire in metres

Soln:

Volume $= 2.2\ dm^3$

1 decimeter = 10 cm

$\therefore$ Volume $= 2.2\ dm^3$

$= 2.2 \times 10 \times 10 \times 10\ cm^3$

$= 2200\ cm^3$

diameter $2r = 0.5\ cm$

$r = \frac{0.5}{2}\ cm$

$r = \frac{1}{4}\ cm$

Volume $= \pi r^2 h = 2200$

$\frac{22}{7} \times \frac{1}{4} \times \frac{1}{4} \times h = 2200$

$h = \frac{2200 \times 7 \times 4 \times 4}{22}$

height $= 11200\ cm$ //

17. How many iron rods each of length 7 m and diameter 2 cm can be made out of 0.88 cubic meter of iron?

Soln:

Volume of iron available $= 0.88\ m^3$

Volume of 1 iron rod $= \pi r^2 h$

$= \frac{22}{7} \times \frac{1}{100} \times \frac{1}{100} \times 7$

$= 0.0022\ m^3$

No. of irons made $= \frac{\text{Volume of iron available}}{\text{Volume of one iron rod}}$

$= \frac{0.88}{0.0022} = \frac{8800}{22}$

$= 400$ Ans

18. The radii of two cylinders are in the ratio 3:5 and their heights are in the ratio of 2:3. Find the ratio of their Curved Surface areas.

Soln:

Cylinder 1	Cylinder 2
radius $= r_1$	radius $= r_2$
height $= h_1$	height $= h_2$

Curved Surface Area $= 2\pi r_1 h_1$ C.S.A $= 2\pi r_2 h_2$

Given. $\frac{r_1}{r_2} = \frac{3}{5}$, $\frac{h_1}{h_2} = \frac{2}{3}$

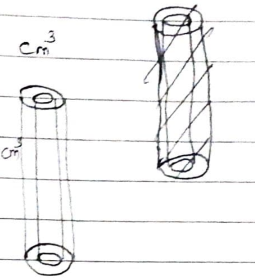
$\therefore \frac{2\pi r_1 h_1}{2\pi r_2 h_2} = \frac{r_1 h_1}{r_2 h_2} = \frac{3}{5} \times \frac{2}{3} = \frac{2}{5}$

Ratio of their Curved Surface Areas $= 2 : 5$

19. If 1 cubic cm of cast iron weighs 21 gms then find the weight of a cast iron pipe of length 1 metre with a bore of 3 cm and in which thickness of the metal is 1 cm.

Soln:

Given 1 cm^3 of Cast iron $= 21$ gms



Volume of inner Cylinder $= \pi r^2 h\ cm^3$

$= \frac{22}{7} \times \frac{3}{2} \times \frac{3}{2} \times 100\ cm^3$

Volume of Outer Cylinder $= \pi r^2 h\ cm^3$

$= \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} \times 100\ cm^3$

Volume of the pipe of length 1 m (100 cm) $=$

(Volume of Outer Cyl. − Vol. of inner cyl)

$= \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} \times 100 - \frac{22}{7} \times \frac{3}{2} \times \frac{3}{2} \times 100$

$= \frac{22}{7} \times \frac{100}{4}(25 - 9)$

$= \frac{22}{7} \times 25 \times 16\ cm^3$

$= \frac{8800}{7}\ cm^3$

$1\ cm^3$ of Cast iron $= 21$ gms

$\frac{8800}{7}\ cm^3$ of Cast iron $= 21 \times \frac{8800}{7}$ gms $= 26400$ gms

Weight of Cast iron required = 26400 gms

$= 26400 \times \frac{1}{1000}$ kg

$= 26.4$ kg//

20. Find the Slant height, volume, Curved Surface area and the whole Surface area of a cone of radius 21 cm and height 28 cm.

Soln: Given radius = 21 cm
height = 28 cm

Slant height of the Cone $l = \sqrt{r^2+h^2}$

$= \sqrt{21^2+28^2}$

$= \sqrt{441+784}$

$= \sqrt{1225}$

$= \sqrt{35 \times 35} = 35$ cm

Volume of the Cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 21 \times 21 \times 28$

$= 22 \times 21 \times 28$

$= 12936$ cm^3

C.S.A of the cone $= \pi r l = \frac{22}{7} \times 21 \times 35$

$= 2310$ cm^2

TSA of the Cone $= \pi r l + \pi r^2$

$= 2310 + \frac{22}{7} \times 21 \times 21$

$= 2310 + 1386$

$= 3696$ cm^2//

21. Find the length of canvas 1.25 m wide required to build a conical tent of base radius 7 meters and height 24 meters.

Soln:

Curved S.A of the cone $= \pi r l$

$l = \sqrt{r^2+h^2}$

$= \sqrt{7^2+24^2}$

$= \sqrt{49+576}$

$= \sqrt{625}$

$l = 25$ m

C.S.A of Cone $= \frac{22}{7} \times 7 \times 25$

$= 550$ m^2

Canvass required for Area $550 m^2$

breadth ~~length~~ of Canvas (b) 1.25 m

Area $= lb = 550$

$1.25 \times l = 550$

$l = \frac{550}{1.25}$

$= \frac{550 \times 100}{125}$

$= 550 \times \frac{4}{5} = 110 \times 4$

length of Canvas req. $= 440$ m. // Ans

22. The heights of two right circular cones are in the ratio 1:2 and the perimeters of their bases are in the ratio 3:4. Find the ratio of their volumes.

Soln:

Right Circular Cone 1	Right Circular Cone 2
height $= h_1$	height $= h_2$
radius $= r_1$	radius $= r_2$

$$\frac{h_1}{h_2} = \frac{1}{2}$$

$$\frac{2\pi r_1}{2\pi r_2} = \frac{3}{4}$$

$$\frac{r_1}{r_2} = \frac{3}{4}$$

Volume of a right circular cone $= \frac{1}{3}\pi r^2 h$

$$\therefore \text{ ratio} = \frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2}$$

$$= \frac{r_1^2 h_1}{r_2^2 h_2}$$

$$= \frac{r_1}{r_2} \frac{r_1}{r_2} \frac{h_1}{h_2}$$

$$= \frac{3}{4} \times \frac{3}{4} \times \frac{1}{2}$$

$$= \frac{9}{32}$$

$\therefore$ Ratio of their volumes $= 9 : 32$

23. The radii of the bases of a cylinder and a cone are in the ratio of 3:4 and their heights are in the ratio 2:3. Find the ratio of their volumes.

Soln:

Cylinder	Cone
radius of base $= r_1$	radius of base $= r_2$
height $= h_1$	heigh $= h_2$

Given $\frac{r_1}{r_2} = \frac{3}{4}$ $\qquad \frac{h_1}{h_2} = \frac{2}{3}$

Volume of the cylinder $= \pi r_1^2 h_1$

Volume of the cone $= \frac{1}{3}\pi r_2^2 h_2$

$$\text{ratio of volumes} = \frac{\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2} = \frac{3 r_1^2 h_1}{r_2^2 h_2}$$

$$= 3 \times \frac{r_1}{r_2} \times \frac{r_1}{r_2} \times \frac{h_1}{h_2} = 3 \times \frac{3}{4} \times \frac{3}{4} \times \frac{2}{3}$$

$$= \frac{9}{8}$$

Ratio of their volumes $= 9:8$ // Ans.

24. A ~~canonical~~ conical vessel, whose internal radius is 12 cm and height 50 cm, is full of liquid. The contents are emptied into a cylindrical vessel with internal radius 10 cm. Find the height to which the liquid rises in the cylindrical vessel.

Soln:

Volume of the Conical Vessel $= \frac{1}{3}\pi r^2 h$

Cone height $= 50cm$

radius of base $= 12cm$

$\therefore$ Volume of the Conical vessel $= \frac{1}{3} \times \frac{22}{7} \times 12^2 \times 50$

$= \frac{1}{3} \times \frac{22}{7} \times 12 \times 12 \times 50 \ cm^3$

Radius of Cylindrical vessel $= 10cm$

height of the Cylindrical vessel $= h$ cm

Volume of the Cylindrical vessel $= \pi r^2 h$

$= \frac{22}{7} \times 10 \times 10 \times h \ cm^3$

Volume of Conical vessel = Volume of Cylindrical vessel.

$\frac{1}{3} \times \frac{22}{7} \times 12 \times 12 \times 50 = \frac{22}{7} \times 10 \times 10 \times h$

$4 \times 12 \times 50 = 10 \times 10 \times h$

$h = \frac{4 \times 12 \times 50}{10 \times 10}$

$h = 24$ cm // Ans.

25. Find the volume and Surface area of a sphere of radius 10.5cm.

Soln: Volume of the Sphere $= \frac{4}{3}\pi r^3$

Surface area $= 4\pi r^2$

Volume of the Sphere $= \frac{4}{3} \times \frac{22}{7} \times 10.5^3$

$= \frac{4 \times 22}{3 \times 7} \times \frac{21}{2} \times \frac{21}{2} \times \frac{21}{2}$

$= \frac{22 \times 21 \times 21}{2}$

$= 11 \times 21 \times 21$

$= 4851 \ cm^3$

Surface area $= 4 \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2}$

$= 22 \times 3 \times 21$

$= 1386 \ cm^2$

26. If the radius of a sphere is increased by 50% find the increase percent in volume and the increase percent to the surface area.

Soln: Sphere 1

radius $= r_1$

Volume $V_1 = \frac{4}{3}\pi r_1^3$

S.A $S_1 = 4\pi r_1^2$

Sphere 2

radius $= r_1 + \frac{50}{100} r_1$

$= r_1 + \frac{r_1}{2}$

$= \frac{3r_1}{2}$

Volume $V_2 = \frac{4}{3}\pi \left(\frac{3}{2} r_1\right)^3$

$= \frac{9}{2}\pi r_1^3$

S.A $S_2 = 4\pi \left(\frac{3}{2} r_1\right)^2$

$= 9\pi r_1^2$

Ratio $V_1 : V_2 = \frac{4}{3}\pi r_1^3 : \frac{9}{2}\pi r_1^3$

$= \frac{4}{3} : \frac{9}{2}$

multiply by 6

$= 8 : 27$ //

$S_1 : S_2 = 4\pi r_1^2 : 9\pi r_1^2$

$= 4 : 9$

Increase percent in Volume $= \frac{V_2 - V_1}{V_1} \times 100$

$= \frac{\frac{9}{2}\pi r_1^3 - \frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_1^3} \times 100$

$= \frac{\left(\frac{9}{2} - \frac{4}{3}\right)}{\frac{4}{3}} \times 100$

$= \left(\frac{9}{2} - \frac{4}{3}\right) \times \frac{3}{4} \times 100$

$= \frac{27 - 8}{6} \times \frac{3}{4} \times 100$

$= \frac{19}{8} \times 100$

$= \frac{1900}{8} = 237.5\%$

Increase percent in S.A $= \frac{S_2 - S_1}{S_1} \times 100$

$= \frac{9\pi r_1^2 - 4\pi r_1^2}{4\pi r_1^2} \times 100$

$= \frac{5}{4} \times 100$

$= 125\%$

27. Find the number of lead balls, each 1cm in diameter that can be made from a sphere of diameter 12cm.

Soln:

Volume of the sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times \frac{12}{2} \times \frac{12}{2} \times \frac{12}{2}$

$= \frac{11 \times 4 \times 12 \times 12}{7}$

Volume of the lead ball $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

$= \frac{11}{3 \times 7}$

No. of lead balls can be made from sphere $\Big\} = \frac{\text{Vol. of sphere}}{\text{Vol. of lead ball}}$

$= \frac{11 \times 4 \times 12 \times 12}{7 \times 11} \times 3 \times 7$

$= 1728$ //

32. Find the Volume, Curved S.A and the total S.A of a hemisphere of radius 10.5cm.

Soln:

Volume of the hemisphere $= \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times 10.5 \times 10.5 \times 10.5 \ cm^3$

$= \frac{44}{21} \times \frac{21}{2} \times \frac{21}{2} \times \frac{21}{2} \ cm^3$

$= \frac{11 \times 21 \times 21}{2} \ cm^3$

$= 2425.5 \ cm^3$ //

Curved S.A. $= 2\pi r^2 = 2 \times \frac{22}{7} \times 10.5 \times 10.5$

$= \frac{44}{7} \times \frac{21}{2} \times \frac{21}{2} \ cm^2$

$= 11 \times 3 \times 21 \ cm^2$

$= 693 \ cm^2$ //

Total S.A $= 3\pi r^2 = 3 \times \frac{22}{7} \times 10.5 \times 10.5$

$= \frac{66}{7} \times \frac{21}{2} \times \frac{21}{2} \ cm^2$

$= \frac{66 \times 3 \times 21}{4} \ cm^2$

$= \frac{33 \times 3 \times 21}{2} \ cm$

$= 1039.5 \ cm^2$ //

33. A hemispherical bowl of internal radii 9 cm contains a liquid. This liquid is to be filled into cylindrical shaped small bottles of diameter 3 cm and height 4 cm. How many bottles will be needed to empty the bowl?

Soln: Volume of the Hemisphere $= \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times 9 \times 9 \times 9$

$= \frac{44 \times 81 \times 3}{7} \ cm^3$

Volume of Cylindrical bottles $= \pi r^2 h$

$= \frac{22}{7} \times \frac{3}{2} \times \frac{3}{2} \times 4$

No. of bottles needed $= \frac{\text{Volume of Hemisphere}}{\text{Volume of Cyl. bottles}}$

$= \frac{44 \times 81 \times 3 \times 7 \times 2 \times 2}{7 \times 22 \times 3 \times 3 \times 4}$

$= \frac{2 \times 27}{1}$

$= 54$ bottles.

34. A Cone, a hemisphere and a cylinder stand on equal bases and have the same height. Find the ratio of their volumes.

Soln: Volume of Cone $= \frac{1}{3}\pi r^2 h$

Volume of hemisphere $= \frac{2}{3}\pi r^3$

Volume of Cylinder $= \pi r^2 h$

here h is also r

∴ Ratio of volumes of Cone : hemisphere : Cylinder

$\frac{1}{3}\pi r^3 : \frac{2}{3}\pi r^3 : \pi r^3$

$\Rightarrow 1:2:3$ Ans

Races and Games of Skill

Races: A Contest of speed in running, riding, driving, sailing or rowing is called a race.

Race Course: The ground or path on which contests are made is called a race course.

Starting point: The point from which a contest begins is known as a starting point.

Winning point or Goal: The point set to bound a race is called a winning point or a goal.

Winner: The person who first reaches the winning point is called a winner.

Start: Suppose A and B are two contestants in a race if before the start of the race A is at the starting point and B is ahead of A by 12 metres then we say that A gives B a start of 12 metres.

To cover a race of 100 metres in this case, A will have to cover 100 metres while B will have to cover only $(100-12) = 88$ metres

Games: A game of 100, means that the person among the ~~contest~~ contestants who scores 100 points first is the winner.

If A scores 100 points while B scores only 80 points, then we say that A can give B 20 points.

Problems:

1. In a km race, A beats B by 28 metres or 7 seconds. Find A's time over the course.

Soln:

A beats B by 28 metres or 7 secs

$\therefore$ B covers 28 metres in 7 secs

B covers 1000 metres in $\left(\frac{7}{28} \times 1000\right)$ secs

250 secs

A covers 1000 metres in $(250-7)$ secs

$= 243$ secs

$= \frac{243}{60}$ mins

$= 4$ mins 3 secs

2. A runs $1\frac{3}{4}$ times as fast as B. If A gives B a start of 84 m, how far must the winning post be so that A and B might reach it at the same time?

Speed of ~~A~~ B = x

Speed of A = $1\frac{3}{4}x = \frac{7}{4}x$

$\therefore$ Ratio of A : B = $\frac{7}{4}x : x$

$\times$ by x

$= \frac{7}{4} : 1$

$\times$ by 4

$= 7 : 4$

Therefore In a race of 7 metres A gains 3 metres.

In a race of 84m A gains $\left(\frac{3}{7} \times 84\right)$m

~~= 36 metres~~

3 metres gained by A in 7 metre Race

84 metres gained by A in $\left(\frac{7}{3} \times 84\right)$ metres

= 196 metres

Winning post must be 196 m away from the starting point.

3. A can run 1 Km in 3 min 10 sec and B can cover the same distance in 3 min 20 sec. By what distance can A beat B?

Soln: A beats B by 10 secs.

Distance covered by B in 3 min 20 sec (ie) 200 sec = 1 km

Distance covered by B in 10 sec $\left(\frac{1000}{200} \times 10\right)$ m

= 50 m

$\therefore$ A beats B by 50 metres.

4. In a 100m race, A runs at 8 km per hour, If A gives B a start of 4m and still beats him by 15 seconds what is the speed of B?

Soln:

Speed of A = 8 km/hr.

$= \frac{8 \times 1000}{3600}$ m/sec

$= \frac{80}{36}$ m/sec

$= \frac{20}{9}$ m/sec

(ie) A covers $\frac{20}{9}$ m in 1 sec

A covers 100m in $\left(\frac{1}{20/9} \times 100\right)$ sec

$= \frac{900}{20}$ sec = 45 secs.

Therefore B covers 96 m in 60 secs

Speed of B = $\frac{\text{dist}}{\text{Time}} = \frac{96}{60}$ m/sec

= 1.6 m/s //

$= 1.6\ m/s$

$= 1.6 \times \frac{3600}{1000}\ km/hr$

$= \frac{16 \times 36}{100}\ km/hr.$

$= 5.76\ km/hr$

5. A, B and C are three Contestants in a km race. If A can give B a start of 40 m and A can give C a start of 64 m, how many metres start can B give C?

Soln:

A Covers a distance of 1000 m

B Covers a distance of 960 m

C Covers a distance of 936 m

When B Covers 960 m C Covers 936 m

If B Covers 1000m C Covers $\frac{936}{960} \times 1000$

$= 975$ metres

(ie) B can give C a start of $(1000-975)$ m

(ie) 25 metres.

6. In a game of 80 points, A can give B 5 points and C 15 points. Then how many points B can give C in a game of 60?

Soln:

If A Covers 80 points
B Covers 75 points
C Covers 65 points.

(ie) $A : B = 80 : 75$

$A : C = 80 : 65$

$\frac{B}{C} = \frac{B}{A} \times \frac{A}{C} = \frac{75}{80} \times \frac{80}{65}$

$= \frac{75}{65} = \frac{15}{13}$

If B Covers 15 points C Covers 13 points.

If B Covers 60 points then C Covers $\left(\frac{13}{15} \times 60\right)$ points

52 points

∴ In a game of 60 points, B can give C 8 points.

Calendar.

Leap year : A year divisible by 4 is called a leap year (except centuries)
In century, if the year is divisible by 400 is a leap year

Ordinary year : A year which is not a leap year is called a ordinary year.

Eg : 1984, 1996, 1992, 1740, 1652 are all divisible by 4 ∴ leap year.

Years 1985, 1997, 1999, 1982, 1742 are ordinary years (Not divisible by 4)

Centuries : 1600, 1700, 1800, 1900, 2000, 2100, 2500, 2800 are all Centuries

In this 1600, 2000, 2800 are leap years (divisible by 400)

1700, 1800, 1900, 2100, 2500 are Ordinary Years

Note: 1. Leap year has 366 days

2. Ordinary year has 365 days.

3. In 366 days, it has 52 weeks + 2 days

4. In 365 days, it has 52 weeks + 1 days

5. In 100 years there are 76 Ordinary years + 24 leap years
(ie) 5217 weeks + 5 days.

6. 100 years has 5 odd days

7. 200 years has 3 odd days

8. 300 years has 1 odd day

9. 400 years has (20+1) (ie) 0 odd day.

(ie) 400, 800, 1200, 1600, 2000, etc Contains 0 odd days.

10.

No. of odd days	0	1	2	3	4	5	6
Day	Sun	Mon	Tue	Wed	Thur	Fri	Sat

Problems

1. What was the day of the week on 16th July 1776?

Soln : upto 1600 years - 0 odd days
next 100 years - 5 odd days

Next 75 years Contains 18 leap years
57 ordinary years

(ie) $(18\times2) = 36$ = 1 odd day
$57\times1 = 57$ = 1 odd day

1776 is a leap year
∴ $31+29+31+30+31+30+16 = 198$ days

198 days = 28 weeks 2 odd days

$\therefore$ Total odd days = 0+5+1+1+2

= 9 days

= 2 odd days

$\therefore$ July 16^{th} 1776 was Tuesday.

2. What was the day of the week on 15^{th} August, 1947?

Soln: upto 1600 years – 0 odd days
next 300 years – 1 odd day
next 46 years
Contains 11 leap year & 35 ordinary
(ie) 2×11 = 22 days – 1 odd day
1×35 = 35 days – 0 odd day
up to August 15^{th} in 1947

31+28+31+30+31+30+31+15 = 227 days

(ie) 3 odd days

Total odd days = 1+1+3 = 5 odd days

$\therefore$ August 15^{th} 1947 was a friday.

3. What was the day of the week on 16^{th} April 2000?

Soln: upto 1600 years – 0 odd days
next 300 years – 1 odd day
next 99 years has 24 leap years
75 ordinary years

$\therefore$ 24×2 = 48 = 6 odd days
75×1 = 75 = 5 odd days
In 2000 (leap year)

31+29+31+16 = 107 days

= 2 odd days

Total odd days = 0+1+6+5+2

= 0 odd days

$\therefore$ April 16^{th} 2000 was a Sunday.

4. On what dates of July 2004 did monday fall?

Soln: upto 2000 – 0 odd days
Next 3 years – 3 odd days
(ordinary yrs)
In 2004 upto July 1

31+29+31+30+31+30+1 = 183 days

= 1 odd day

Total odd days = 3+1 = 4 odd days

$\therefore$ July 1^{st} 2004 is Thursday

July 5^{th} 2004
July 12 2004
July 19 2004
July 26 2004
$\Big\}$ ie the are Monday.

5. Prove that the Calendar for the year 2003 will Serve for the year 2014.

Soln:

First 2003 & 2014 are Ordinary Years.

Now it is enough to prove

Jan 1 2003 & Jan 1 2014 fall on Same day.

To find Jan 1 2003

upto 2000 - 0 odd days

Next 2 years - 2 odd days

Jan 1 2003 - 1 odd day

Total odd days = 0+2+1 = 3 odd days

∴ Jan 1 2003 was ~~Monday~~ Wednesday.

To find Jan 1 2014.

upto 2000 - 0 odd days

~~next 1 year - 1 odd day~~

~~Jan 1 2014~~

up to 2013 - 3 leap year & 10 ordinary years

= (3×2) = 6 odd days

= 10×1 = 3 odd days

Jan 1 2014 = 1 odd day

Total odd days = 0 + 6+3+1 = 10 days

= 3 odd days

∴ Jan 1 2014 was Wednesday.

∴ Calendar 2003 will Serve for the year 2014.

6. Prove that any date in March of a year is the Same day of the week as the Corresponding date in Nov that year.

Soln:

Let us Calculate the Number of days from 1 March to 31 October

March	Apr	May	June	July	Aug	Sep	Oct
31 +	30 +	31 +	30 +	31 +	31 +	30 +	31

= 245 days

$\frac{245}{7}$ = 35 weeks 0 odd days

∴ From 1st March to 31st October 35 weeks Complete, the next day

1st November will be Same as 1st March

∴ True.