

Unit V

Clocks Important facts

1. Face of a clock is a circle whose circumference is divided into 60 equal parts called minutes.
2. Clock has 2 hands - Short hand called hour hand and long hand called min hand.
3. In 60 minutes, the minute hand gains 55 minutes on the hour hand.
4. In every hour, ~~both~~ the hands coincide once.
5. The hands are in the same straight line when they are coincident or opposite to each other.
6. When the two hands are at right angles, they are 15 minutes spaces apart.

7. When the hands are in opposite directions, they are 30 minute spaces apart.
8. Angle traced by hour hand in 12 hrs
 $= 360^\circ$
9. Angle traced by minute hand in 60 min
 $= 360^\circ$
10. If a watch or a clock indicates 8.15, when the correct time is 8, it is said to be 15 minutes too fast.
11. On the other hand, if it indicates 7.45 when the correct time is 8, it is said to be 15 minutes too slow.

Problems:

1. Find the angle between the hour hand and the minute hand of a clock when the time is 3.25.

Soln:

Angle traced by hour hand } = 360°
in 12 hours }

Angle traced by 3 hrs 25 min } = $\frac{360}{12} \times \frac{41}{12}$
(ie) $(3 + \frac{25}{60})$ hrs = $\frac{41}{12}$ hrs }

$= 102\frac{1}{2}^\circ$

Angle traced by minute hand } = 360°
in 60 min }

Angle traced by minute hand } = $\frac{360}{60} \times 25$
in 25 min }

$= 150^\circ$

$\therefore$ Angle between hr hand } = $150^\circ - 102\frac{1}{2}^\circ$
and min hand }

$= 47\frac{1}{2}^\circ$

2. At what time between 2 and 3 o'clock will the hands of a clock be together?

Soln: At 2 o'clock - hr hand at 2
min hand at 12
(ie) 10 min spaces apart.

55 minutes are gained by it in 60 min

10 minutes will be gained in $(\frac{60}{55} \times 10)$ min

$$\frac{120}{11} \text{ min. } \cancel{34 \frac{2}{11} \text{ min}}$$

$$= \frac{24}{11} = 2\frac{2}{11}$$

$$\frac{120}{11} \text{ min. (ie) } 10\frac{10}{11} \text{ min}$$

The clock will coincide at $10\frac{10}{11}$ min past 2.

3. At what time between 4 and 5 o'clock will the hands of a clock be at right angle?

Soln: At 4 o'clock - hour hand at 4
min hand at 12
(ie) 20 min spaces apart.

When they are at right angles they should be 15 min spaces apart.

It may happen in 2 Cases.

Case 1: When min hand is 15 min spaces behind hour hand.

In this case the min hand
will have to gain $(20-15) = 5$ min
spaces.

55 min spaces are gained
by it in 60 min

5 min spaces are gained by it in

$$\left(\frac{60}{55} \times 5\right) \text{ min}$$

$$= 5\frac{5}{11} \text{ min past 4}$$

Case 2: When the minute hand is 15 min
spaces ahead of the hour hand.

To be in this position, the minute hand will
have to gain $(20+15) = 35$ minute spaces

55 min spaces gained in 60 min

35 min spaces gained in $\left(\frac{60}{55} \times 35\right) \text{ min}$

$$38\frac{2}{11} \text{ min past 4}$$

$\therefore$ At 4 hrs $5\frac{5}{11}$ min & 4 hrs $38\frac{2}{11}$ min

the hr hand & min hand are at
right angles.

4. Find at what time between 8 and 9 o'clock
will the hands of a clock be in the same
straight line but not together.

Soln. At 8 o'clock the hour hand at 8
min hand at 12
20 min spaces apart.

If they were in 30 min spaces apart
they are in straight line.

$\therefore$ It has to gain 10 min spaces.

55 min spaces gained in 60 min
10 min spaces gained in $\left(\frac{60}{55} \times 10\right) \text{ min}$

$$\frac{120}{11} \text{ min}$$

$$= 10\frac{10}{11} \text{ min past 8.}$$

5. At what time between 5 and 6 o'clock
are the hands of a clock 3 min apart?

Soln: At 5 o'clock the hour hand at 5
min hand at 12
ie) 25 min spaces apart

Case 1: Minute hand is 3 min spaces behind the hour hand.

(ie) Min hand has to gain 22 min spaces

55 min spaces gained in 60 min

22 min spaces gained in $\left(\frac{60}{55} \times 22\right)$ min

$$\frac{120}{5} \text{ min}$$

24 min past 5

Case 2: Minute hand is 3 min spaces ahead of hour hand.

(ie) Min hand has to gain 28 min spaces

55 min spaces gained in 60 min

28 min spaces gained in $\left(\frac{60}{55} \times 28\right)$ min

$$\frac{336}{11} \text{ min (ie) } 30\frac{6}{11} \text{ min}$$

past 5 //

Stocks and Shares

1. Stock - Capital - The total amount of money needed to run the Company is called the stock-capital.
2. Shares or Stock: The whole capital is divided into small units called shares or stock.
3. Dividend: The annual profit distributed among share holders is called dividend.
Dividend is paid annually as per share or as a percentage.
4. Face Value: The value of a share or stock printed on the share-certificate is called its face value or nominal value or par value.
5. Market Value: The stocks of different companies are sold and bought in the open market through brokers at stock-exchanges. A share (or stock) is said to be:

(i) At premium or Above par, if its market value is more than its face value.

(ii) At par, if its market value is the same as its face value.

(iii) At discount or Below par, if its market value is less than its face value.

6. Brokerage: The broker's charge is called brokerage.

(i) When stock is purchased, brokerage is added to the cost price.

(ii) When stock is sold, brokerage is subtracted from the selling price.

Note:

1. The face value of a share always remains the same.
2. The market value of a share changes from time to time.
3. Dividend is always paid on the face value of a share.
4. Number of shares held by a person =

$$\frac{\text{Total Investment}}{\text{Investment in 1 share}} = \frac{\text{Total Income}}{\text{Income from 1 share}}$$

$$= \frac{\text{Total Face Value}}{\text{Face Value of 1 share.}}$$

Thus by a Rs. 100, ~~100~~ 9% stock at 120 means

(i) Face Value (F.V.) of stock = Rs. 100

(ii) Market Value (M.V.) of stock = Rs. 120

(iii) Annual dividend on 1 share = 9% of Face value
 $= 9\% \text{ of Rs. } 100 = 9 \text{ Rs.}$

(vi) An investment of Rs. 120 gives an annual income of Rs. 9.

(v) Rate of interest p.a. = Annual income from an investment of Rs. 100

$$= \left(\frac{9}{120} \times 100 \right) \% = 7\frac{1}{2} \%$$

Problems

1. Find the cost of

(i) Rs. 7200, 8% stock at 90.

(ii) Rs. 4500, 8.5% stock at 4 premium

(iii) Rs. 6400, 10% stock at 15 discount

Soln:

(i) Cost of Rs. 100 Stock = Rs. 90.

$$\text{Cost of Rs. 7200 Stock} = \frac{90}{100} \times 7200$$

$$= \text{Rs. } 6480$$

(ii) Cost of Rs. 100 Stock = Rs. 104

$$\text{Cost of Rs. 4500 Stock} = \frac{104}{100} \times 4500$$

$$= \text{Rs. } 4680$$

(iii) Cost of Rs. 100 Stock = Rs. 85

$$\text{Cost of Rs. 6400 Stock} = \frac{85}{100} \times 6400$$

$$= 5440$$

2. Find the cash required to purchase

Rs. 3200, $7\frac{1}{2}\%$ stock at 107

(brokerage $\frac{1}{2}\%$)

Soln:

$$\left. \begin{array}{l} \text{Cash required to purchase} \\ \text{Rs. 100 Stock} \end{array} \right\} = \text{Rs. } 107 + \frac{1}{2}$$

$$= \frac{215}{2} \text{ Rs.}$$

$$\left. \begin{array}{l} \text{Cash required to purchase} \\ \text{Rs. 3200 Stock} \end{array} \right\} = \frac{215}{2 \times 100} \times 3200$$

$$= 3440 \text{ Rs.}$$

3. Find the cash realised by selling

Rs. 2440, 9.5% stock at 4 discount

(brokerage $\frac{1}{4}\%$).

Soln: By selling Rs. 100 Stock cash

$$\text{realised} = 96 - \frac{1}{4}$$

$$= \frac{383}{4} \text{ Rs.}$$

$$\left. \begin{array}{l} \text{By selling Rs. 2440 Stock cash} \\ \text{realised} \end{array} \right\} = \frac{383}{4 \times 100} \times 2440$$

= Rs. 2298

4. Find the annual income derived from the Rs. 2500, 8% Stock at 106.

Soln:

Income from Rs. 100 Stock = 8

$$\text{Income from Rs. 2500 Stock} = \frac{8 \times 2500}{100} \\ = 150 \text{ Rs.}$$

5. Find the annual income derived by investing Rs. 6800 in 10% Stock at 136.

Soln:

By investing Rs. 136, income obtained = Rs. 10

$$\text{By investing Rs. 6800, income obtained} \\ = \frac{10}{136} \times 6800 = \text{Rs. 500.}$$

6. Which is better investment? $7\frac{1}{2}\%$ Stock at 105 or $6\frac{1}{2}\%$ Stock at 94.

Soln:

Case 1: $7\frac{1}{2}\%$ Stock at 105.

On investing Rs. 105 income = Rs. $\frac{15}{2}$

$$\text{On investing Rs. } 105 \times 94 \text{ income} = \frac{15}{2} \times \frac{1}{105} \times 105 \times 94 \\ = 705 \text{ Rs.}$$

Case 2: $6\frac{1}{2}\%$ Stock at 94:

On investing Rs. 94 income = Rs. $\frac{13}{2}$

$$\text{On investing Rs. } (105 \times 94) \text{ income} = \\ \frac{13}{2} \times \frac{1}{94} \times 105 \times 94 \\ = \text{Rs. 682.50}$$

Conclusion:

The investing ~~Rs.~~ $7\frac{1}{2}\%$ Stock at 105 is better.

True Discount

Suppose a man has to pay Rs. 156 after 4 years and the rate of interest is 14% per annum. Clearly Rs. 100 at 14% will amount to Rs. 156 in 4 yrs. So, the payment of Rs. 100 now will clear off the debt of Rs. 156 due 4 years hence we say that:

Sum due = Rs. 156 due 4 years hence
 present worth (P.W) = Rs. 100
 True Discount (T.D) = Rs. (156 - 100)
 = Rs. 56

We define $T.D = \text{Interest on P.W}$
 $\text{Amount} = P.W + T.D$

Interest is reckoned on P.W and true discount is reckoned on the amount.

Formulas: Let Rate = $R\%$ per annum
 Time = T years

$$1. P.W = \frac{100 \times \text{Amt}}{100 + (R \times T)} = \frac{100 \times T.D}{R \times T}$$

$$2. T.D = \frac{P.W \times R \times T}{100} = \frac{\text{Amt} \times R \times T}{100 + (R \times T)}$$

$$3. \text{Sum} = \frac{S.I \times T.D}{S.I - T.D}$$

$$4. S.I - T.D = S.I \text{ on } T.D$$

$$5. \text{When the Sum is put at Compound interest, then } P.W = \frac{\text{Amount}}{\left(1 + \frac{R}{100}\right)^T}$$

Problems:

1. Find the Present worth of Rs. 930 due 3 years hence at 8% per annum. Also find the discount.

Soln:

$$P.W = \frac{100 \times \text{Amt}}{100 + (R \times T)} = \frac{100 \times 930}{100 + (8 \times 3)}$$

$$= \frac{100 \times 930}{124} = \text{Rs. } 750$$

$$T.D = \text{Amt} - P.W = 930 - 750 = \text{Rs. } 180$$

2. The true discount on a bill due 9 months hence at 12% per annum is Rs. 540. Find the amount of the bill and its present worth.

Soln: Let amount be Rs. x . Then

$$\frac{x \times R \times T}{100 + (R \times T)} = T.D$$

$$\frac{x \times 12 \times \frac{3}{4}}{100 + (12 \times \frac{3}{4})} = 540$$

$$x = \frac{540 \times 109}{9} = \text{Rs. } 6540$$

$$\therefore \text{Amt} = \text{Rs. } 6540$$

$$P.W = \text{Rs. } (6540 - 540) = \text{Rs. } 6000$$

3. The true discount on a certain sum of money due 3 years hence is Rs. 250 and the simple interest on the same sum for the same time and at the same rate is Rs. 375. Find the sum and the rate percent.

Soln: T.D = Rs. 250
and S.I = Rs. 375

$$\begin{aligned} \text{Sum due} &= \frac{S.I \times T.D}{S.I - T.D} \\ &= \frac{375 \times 250}{375 - 250} \\ &= \text{Rs. } 750 \end{aligned}$$

$$\text{Rate} = \frac{100 \times 375}{750 \times 3} \% = 16 \frac{2}{3} \% //$$

4. The difference between the Simple interest and true discount on a certain sum of money for 6 months at $12 \frac{1}{2} \%$ per annum is Rs. 25. Find the sum.

Soln: Let the sum be Rs. x .

$$\begin{aligned} \text{Then } T.D &= \frac{x \times \frac{25}{2} \times \frac{1}{2}}{100 + \left(\frac{25}{2} \times \frac{1}{2}\right)} = \frac{x \times \frac{25}{4} \times \frac{1}{2}}{100 + \frac{25}{4}} \\ &= \frac{x}{17} \end{aligned}$$

$$S.I = \left(x \times \frac{25}{2} \times \frac{1}{2} \times \frac{1}{100}\right) = \frac{x}{16}$$

$$\therefore \frac{x}{16} - \frac{x}{17} = 25$$

$$17x - 16x = 25 \times 16 \times 17$$

$$x = 6800 \text{ Rs.}$$

$$\therefore \text{Sum due} = \text{Rs. } 6800 //$$

5. A bill falls due in 1 year. The creditor agrees to accept immediate payment of the half and to defer the payment of the other half for 2 years. By this arrangement he gains Rs. 40. What is the amount of the bill, if the money be worth $12\frac{1}{2}\%$?

Soln: Let the Sum be Rs. x , Then

$$\frac{x}{2} + \frac{\frac{x}{2} + 100}{100 + \left(\frac{25}{2} \times 2\right)} - \frac{x \times 100}{100 + \left(\frac{25}{2} \times 1\right)} = 40$$

$$\frac{x}{2} + \frac{2x}{5} - \frac{8x}{9} = 40$$

$$x = 3600.$$

The Amount of the bill = Rs. 3600