MEASURES OF CENTRAL TENDENCY

INTRODUCTION:

In the study of a population with respect to one in which we are interested, we may get a large number of observations. It is not possible to grasp any idea about the characteristic when we look at all the observations. So it is better to get one number for one group. That number must be a good representative one for all the observations to give a clear picture of that characteristic. Such representative number can be a central value for all these observations. This central value is called a measure of central tendency or an average or a measure of locations. There are five averages. Among them mean, median and mode are called simple averages and the other two averages geometric mean and harmonic mean are called special averages.

The meaning of average is nicely given in the following definitions.

“A measure of central tendency is a typical value around which other figures congregate.”

“An average stands for the whole group of which it forms a part yet represents the whole.”

“One of the most widely used set of summary figures is known as measures of location.”

CHARACTERISTICS FOR A GOOD OR AN IDEAL AVERAGE:

The following properties should possess for an ideal average.

1. It should be rigidly defined.

2. It should be easy to understand and compute.

3. It should be based on all items in the data.
4. Its definition shall be in the form of a mathematical formula.

5. It should be capable of further algebraic treatment.

6. It should have sampling stability.

7. It should be capable of being used in further statistical computations or processing.

**ARITHMETIC MEAN OR MEAN:**

Arithmetic mean or simply the mean of a variable is defined as the sum of the observations divided by the number of observations. If the variable x assumes n values \(x_1, x_2, \ldots x_n\) then the mean, \(x\), is given by

\[
x = \frac{1}{n} \sum_{i=1}^{n} x_i
\]

This formula is for the ungrouped or raw data.

**Grouped Data: (Discrete & Continuous Case)**

The mean for grouped data is obtained from the following formula:

\[
x = \frac{\sum fx}{N}
\]

where \(x\) = the mid-point of individual class

\(f\) = the frequency of individual class
N = the sum of the frequencies or total frequencies.

**MERITS AND DEMERITS OF ARITHMETIC MEAN:**

**Merits:**

1. It is rigidly defined.
2. It is easy to understand and easy to calculate.
3. If the number of items is sufficiently large, it is more accurate and more reliable.
4. It is a calculated value and is not based on its position in the series.
5. It is possible to calculate even if some of the details of the data are lacking.
6. Of all averages, it is affected least by fluctuations of sampling.
7. It provides a good basis for comparison.

**Demerits:**

1. It cannot be obtained by inspection nor located through a frequency graph.
2. It cannot be in the study of qualitative phenomena not capable of numerical measurement i.e. Intelligence, beauty, honesty etc.,
3. It can ignore any single item only at the risk of losing its accuracy.
4. It is affected very much by extreme values.
5. It cannot be calculated for open-end classes.
6. It may lead to fallacious conclusions, if the details of the data from which it is computed are not given.

calculate the arithmetic mean for the following

1600, 1590, 1560, 1610, 1640, 10

\[
\frac{1600 + 1590 + 1560 + 1610 + 1640 + 10}{6} = \frac{8010}{6} = 1335
\]

1. Calculate Arithmetic mean

<table>
<thead>
<tr>
<th>S.No.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales in 1000’s(x)</td>
<td>34</td>
<td>55</td>
<td>45</td>
<td>62</td>
<td>48</td>
<td>57</td>
<td>28</td>
<td>57</td>
<td>62</td>
<td>78</td>
</tr>
</tbody>
</table>

\[
\frac{34 + 55 + 45 + 62 + 48 + 57 + 28 + 57 + 62 + 78}{10} = \frac{526}{10} = 52.6 \text{ (average sales)}
\]

Discrete data

Let \(x_1, x_2, x_3 \ldots \ x_n\) be the \(n\) values of the variable \(x\) with corresponding frequency \(f_1, f_2, f_3 \ldots \ f_n\), then

\[
x_1 \cdot f_1 + x_2 \cdot f_2 + x_3 \cdot f_3 + \ldots + x_n \cdot f_n
\]

the arithmetic mean \(x\) bar = \[
\frac{f_1 + f_2 + f_3 + \ldots + f_n}{\sum f_i}
\]

\[
\frac{\sum x_i \cdot f_i}{\sum f_i}
\]

2. Calculate the arithmetic mean
<table>
<thead>
<tr>
<th>x</th>
<th>f</th>
<th>xf</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4</td>
<td>2x4=8</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>24</td>
</tr>
<tr>
<td>6</td>
<td>10</td>
<td>60</td>
</tr>
<tr>
<td>8</td>
<td>12</td>
<td>96</td>
</tr>
<tr>
<td>10</td>
<td>8</td>
<td>80</td>
</tr>
<tr>
<td>12</td>
<td>7</td>
<td>84</td>
</tr>
<tr>
<td>14</td>
<td>3</td>
<td>42</td>
</tr>
<tr>
<td></td>
<td>Σ fᵢ = 50</td>
<td>Σ xᵢ fᵢ = 394</td>
</tr>
</tbody>
</table>

\[
\text{Arithmetic mean, } X \text{ bar } = \frac{\sum xᵢ fᵢ}{\sum fᵢ} = \frac{394}{50} = 78.8
\]

Continuous Series
Type I: Exclusive Class Intervals
3. Calculate the Arithmetic Mean for the following data

<table>
<thead>
<tr>
<th>Marks</th>
<th>20-30</th>
<th>30-40</th>
<th>40-50</th>
<th>50-60</th>
<th>60-70</th>
<th>70-80</th>
</tr>
</thead>
<tbody>
<tr>
<td>No of Students</td>
<td>5</td>
<td>8</td>
<td>12</td>
<td>15</td>
<td>6</td>
<td>4</td>
</tr>
</tbody>
</table>

Solution

<table>
<thead>
<tr>
<th>Marks</th>
<th>f</th>
<th>m</th>
<th>fm</th>
</tr>
</thead>
<tbody>
<tr>
<td>20-30</td>
<td>5</td>
<td>25</td>
<td>125</td>
</tr>
<tr>
<td>30-40</td>
<td>8</td>
<td>35</td>
<td>280</td>
</tr>
<tr>
<td>40-50</td>
<td>12</td>
<td>45</td>
<td>540</td>
</tr>
<tr>
<td>50-60</td>
<td>15</td>
<td>55</td>
<td>825</td>
</tr>
<tr>
<td>60-70</td>
<td>6</td>
<td>65</td>
<td>390</td>
</tr>
<tr>
<td>70-80</td>
<td>4</td>
<td>75</td>
<td>300</td>
</tr>
</tbody>
</table>

\[
\overline{X} = \frac{\sum fm}{\sum f} = \frac{2460}{50} = 49.20
\]
4. From the following data, compute Arithmetic mean

<table>
<thead>
<tr>
<th>Marks Obtained</th>
<th>0-10</th>
<th>10-20</th>
<th>20-30</th>
<th>30-40</th>
<th>40-50</th>
<th>50-60</th>
</tr>
</thead>
<tbody>
<tr>
<td>No of Students</td>
<td>5</td>
<td>10</td>
<td>25</td>
<td>30</td>
<td>20</td>
<td>10</td>
</tr>
</tbody>
</table>

Solution

<table>
<thead>
<tr>
<th>Marks Obtained</th>
<th>f</th>
<th>m</th>
<th>fm</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-10</td>
<td>5</td>
<td>5</td>
<td>25</td>
</tr>
<tr>
<td>10-20</td>
<td>10</td>
<td>15</td>
<td>150</td>
</tr>
<tr>
<td>20-30</td>
<td>25</td>
<td>25</td>
<td>625</td>
</tr>
<tr>
<td>30-40</td>
<td>30</td>
<td>35</td>
<td>1050</td>
</tr>
<tr>
<td>40-50</td>
<td>20</td>
<td>45</td>
<td>900</td>
</tr>
<tr>
<td>50-60</td>
<td>10</td>
<td>55</td>
<td>550</td>
</tr>
</tbody>
</table>

\[ \bar{X} = \frac{\sum fm}{\sum f} = \frac{3300}{100} = 33 \]

MEDIAN

Definition
Median is the value of the middle most items when all the items are in the order of magnitude.
It is denoted by M.

Merits of Median:

1. Median is not influenced by extreme values because it is a positional average.

2. Median can be calculated in case of distribution with open end intervals.

3. Median can be located even if the data are incomplete.

4. Median can be located even for qualitative factors such as ability, honesty etc.
Demerits of Median:

1. A slight change in the series may bring drastic change in median value.

2. In case of even number of items or continuous series, median is an estimated value other than any value in the series.

3. It is not suitable for further mathematical treatment except its use in mean deviation.

4. It is not taken into account all the observations.

Formulae:

Individual Series: Position of $M = \left( \frac{N + 1}{2} \right)^{th}$ term

Discrete Series: Position of $M = \left( \frac{\sum f + 1}{2} \right)^{th}$ term

Continuous Series: $M = L + \left[ \frac{\sum f / 2 - p.c.f}{f} \right]_i$

Where $L =$ Lower limit of the Median class
$f =$ Frequency of the median class
$p.c.f =$ Cumulative frequency preceding the median class
$i =$ Length of the class interval

Individual Series:

1. Find the Median for the following data

   6, 9, 21, 5, 7, -2, 0, 32, 9

Solution:

$N = 9$ (Odd number)

Ascending Order: -2, 0, 5, 6, 7, 9, 9, 21, 32,

Position of Median = $\left( \frac{N + 1}{2} \right)^{th}$ term
\[ = \left( \frac{9 + 1}{2} \right)^{th} \text{ term} \]
\[ = \left( \frac{10}{2} \right)^{th} \text{ term} \]
\[ = 5^{th} \text{ term} \]

\[ \therefore M = 7 \]

2. Find the Median

57, 58, 61, 42, 38, 65, 72, 66

Solution:

N = 8 (Even number)

Ascending Order: 38, 42, 57, 58, 61, 65, 66, 72,

\[ \text{Position of Median} = \left( \frac{N + 1}{2} \right)^{th} \text{ term} \]
\[ = \left( \frac{8 + 1}{2} \right)^{th} \text{ term} \]
\[ = \left( \frac{9}{2} \right)^{th} \text{ term} \]
\[ = 4.5^{th} \text{ term} \]

\[ \therefore M = \text{average of 4}^{th} \text{ and 5}^{th} \text{ number} \]
\[ = 58 + 61 / 2 \]
\[ = 119 / 2 \]
\[ \therefore M = 59.5 \]

Discrete Series:

3. The marks (out of a maximum of 10) scored by the students of a class are given below. Find the Median mark

<table>
<thead>
<tr>
<th>Mark</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>No of Students</td>
<td>1</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>10</td>
<td>15</td>
<td>10</td>
<td>5</td>
<td>59</td>
</tr>
</tbody>
</table>

Solution:

<table>
<thead>
<tr>
<th>Marks</th>
<th>No of Students</th>
<th>Cumulative frequency</th>
</tr>
</thead>
</table>
4. Find the Median from the following data

<table>
<thead>
<tr>
<th>Wages</th>
<th>50</th>
<th>75</th>
<th>100</th>
<th>150</th>
<th>250</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>No of Labourers</td>
<td>8</td>
<td>14</td>
<td>10</td>
<td>5</td>
<td>3</td>
<td>40</td>
</tr>
</tbody>
</table>

Solution:

\[
\begin{array}{|c|c|c|}
\hline
x & f & \text{cf} \\
\hline
3 & 1 & 1 \\
4 & 5 & 5+1 = 6 \\
5 & 6 & 6+6 = 12 \\
6 & 7 & 19 \\
7 & 10 & 29 \\
8 & 15 & 44 \\
9 & 10 & 54 \\
10 & 5 & 59 \\
\hline
\sum f = 59 \\
\hline
\end{array}
\]

\[
\text{Solution:}
\]

\[
N = 59\text{(Odd Number)}
\]

\[
\text{Position of } M = \left( \frac{\sum f + 1}{2} \right)^{th} \text{ term}
\]

\[
= \left( \frac{59 + 1}{2} \right)^{th} \text{ term}
\]

\[
= \left( \frac{60}{2} \right)^{th} \text{ term}
\]

\[
= 30^{th} \text{ term}
\]

\[
\therefore M = 8
\]

4. Find the Median from the following data

<table>
<thead>
<tr>
<th>Wages</th>
<th>50</th>
<th>75</th>
<th>100</th>
<th>150</th>
<th>250</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>No of Labourers</td>
<td>8</td>
<td>14</td>
<td>10</td>
<td>5</td>
<td>3</td>
<td>40</td>
</tr>
</tbody>
</table>

Solution:

\[
\begin{array}{|c|c|c|}
\hline
x & f & \text{cf} \\
\hline
50 & 8 & 8 \\
75 & 14 & 22 \\
100 & 10 & 32 \\
\hline
\end{array}
\]
\[ \sum f = 40 \]

N = 40 (Even Number)

Position of \( M \) = \( \left( \frac{\sum f + 1}{2} \right)^{th} \) term

\[ = \left( \frac{40 + 1}{2} \right)^{th} \text{ term} \]

\[ = \left( \frac{41}{2} \right)^{th} \text{ term} \]

\[ = 20.5^{th} \text{ term} \]

\[ \therefore M = \text{Average of } 20^{th} \text{ term and the } 21^{st} \text{ term} \]

\[ = 75 + 75/2 \]

\[ = 150/2 \]

\[ \therefore M = 75 \]

5. Find the Median from the following data

<table>
<thead>
<tr>
<th>No of cars sold in a day</th>
<th>10</th>
<th>15</th>
<th>17</th>
<th>18</th>
<th>21</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>No of Days</td>
<td>4</td>
<td>16</td>
<td>12</td>
<td>5</td>
<td>3</td>
<td>40</td>
</tr>
</tbody>
</table>

Solution:

<table>
<thead>
<tr>
<th>Wages x</th>
<th>No of Labourers f</th>
<th>Cumulative frequency cf</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>15</td>
<td>16</td>
<td>20</td>
</tr>
<tr>
<td>17</td>
<td>12</td>
<td>32</td>
</tr>
<tr>
<td>18</td>
<td>5</td>
<td>37</td>
</tr>
<tr>
<td>21</td>
<td>3</td>
<td>40</td>
</tr>
</tbody>
</table>

\[ \sum f = 40 \]

Solution:

N = 40 (Even Number)
Position of \( M \) = 
\[
\left( \sum_{i=1}^{n} f_i \cdot i \right) + 1 \cdot \frac{1}{2}
\]

\[
= \left( \frac{40 + 1}{2} \right)^{th} \text{ term}
\]

\[
= \left( \frac{41}{2} \right)^{th} \text{ term} = 20.5^{th} \text{ term}
\]

\[ \therefore M = \text{Average of } 20^{th} \text{ term and the } 21^{st} \text{ term} \]

\[ = 15 + 17/2 = 32/2 \]

\[ \therefore M = 16 \]

Continuous Series:
Type I: Exclusive Class Interval

6. Calculate the Median Height.

<table>
<thead>
<tr>
<th>Height (cms.)</th>
<th>145-150</th>
<th>150-155</th>
<th>155-160</th>
<th>160-165</th>
<th>165-170</th>
<th>170-175</th>
</tr>
</thead>
<tbody>
<tr>
<td>No of Students</td>
<td>2</td>
<td>5</td>
<td>10</td>
<td>8</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>Solution:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Height (cms.)</th>
<th>No of Students</th>
<th>Cumulative frequency cf</th>
</tr>
</thead>
<tbody>
<tr>
<td>145-150</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>150-155</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>155-160</td>
<td>10</td>
<td>17</td>
</tr>
<tr>
<td>160-165</td>
<td>8</td>
<td>25</td>
</tr>
<tr>
<td>165-170</td>
<td>4</td>
<td>29</td>
</tr>
<tr>
<td>170-175</td>
<td>1</td>
<td>30</td>
</tr>
<tr>
<td>( \sum f = 30 )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Solution:

\[
\frac{\sum f}{2} = \frac{30}{2} = 15
\]

Median Class = 155-160

\( L = 155 \)

p.c.f = 7
f = 10
i = 5

\[ M = L + \left( \frac{\sum f}{2} - \text{p.c.f.} \right) i \]

\[ = 155 + \left( \frac{15 - 7}{10} \right) 5 = 155 + \left( \frac{8}{10} \right) 5 = 155 + 4 = 159 \text{ cms.} \]

\[ \therefore M = 159 \text{ cms.} \]

7. Calculate the Median from the following data:

<table>
<thead>
<tr>
<th>Marks</th>
<th>10-25</th>
<th>25-40</th>
<th>40-55</th>
<th>55-70</th>
<th>70-85</th>
<th>85-100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>6</td>
<td>20</td>
<td>44</td>
<td>26</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>

Solution:

<table>
<thead>
<tr>
<th>Marks</th>
<th>frequency f</th>
<th>Cumulative frequency cf</th>
</tr>
</thead>
<tbody>
<tr>
<td>10-25</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>25-40</td>
<td>20</td>
<td>26</td>
</tr>
<tr>
<td>40-55</td>
<td>44</td>
<td>70</td>
</tr>
<tr>
<td>55-70</td>
<td>26</td>
<td>96</td>
</tr>
<tr>
<td>70-85</td>
<td>3</td>
<td>99</td>
</tr>
<tr>
<td>85-100</td>
<td>1</td>
<td>100</td>
</tr>
</tbody>
</table>

\[ \frac{\sum f}{2} = \frac{100}{2} = 50 \]

Median Class = 40-55

L = 40

p.c.f = 26

f = 44

i = 15

\[ M = L + \left( \frac{\sum f / 2 - \text{p.c.f.}}{f} \right) i \]
\[= 40 + \left[ \frac{50 - 26}{44} \right] \times 15 = 40 + \left[ \frac{24}{44} \right] \times 15 = 40 + 8.18\]

\[\therefore M = 48.18\]

**MODE**

**Definition:**
Mode is the value which has the greatest frequency density. It is denoted by ‘\(Z\).

**Formulae:**

**Individual Series:** Most repeated numbers.

**Discrete Series:** The value straight to the greatest frequency.

**Continuous Series:**
\[Z = L + \left[ \frac{D_1}{D_1 + D_2} \right] \times i\]

Where \(L\) = Lower limit of the modal class.

\[D_1 = f_1 - f_0\]

= Frequency of the modal class – frequency of the class preceding the modal class.

\[D_2 = f_1 - f_2\]

= Frequency of the modal class – frequency of the class succeeding the modal class.

\(i\) = Size of the Modal class.

**Individual Series:**

1. Determine the Mode.

   (i) \(320, 395, 342, 444, 551, 395, 425, 417, 395, 401, 390, 400\).

   Mode = 395

   (ii) \(3, 6, 7, 5, 8, 4, 9\).

   Mode = No mode

   (iii) \(25, 32, 24, 27, 32, 27, 25, 32, 24, 27, 25, 24\).

   Mode = No Mode

   (iv) \(0, 2, 5, 6, 9, 5, 6, 14, 6, 15, 5, 6, 5\).

   Mode = 5 and 6

**Discrete Series:**

1. Determine the mode:
<table>
<thead>
<tr>
<th>Size of the dress (x)</th>
<th>18</th>
<th>20</th>
<th>22</th>
<th>24</th>
</tr>
</thead>
<tbody>
<tr>
<td>No of sets produced(f)</td>
<td>55</td>
<td>120</td>
<td>108</td>
<td>45</td>
</tr>
</tbody>
</table>

Solution: Greatest frequency = 120
Mode = 20

3. Determine the modal size:

<table>
<thead>
<tr>
<th>Size of Shoes (x)</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>No of pairs sold (f)</td>
<td>10</td>
<td>25</td>
<td>32</td>
<td>38</td>
<td>61</td>
<td>47</td>
<td>34</td>
</tr>
</tbody>
</table>

Solution: Greatest frequency = 61
Mode = 7

Continuous Series:
Type I: Exclusive Class Interval

4. Calculate the Mode:

<table>
<thead>
<tr>
<th>Daily wages in Rs.</th>
<th>50-60</th>
<th>60-70</th>
<th>70-80</th>
<th>80-90</th>
<th>90-100</th>
</tr>
</thead>
<tbody>
<tr>
<td>No of Labourers</td>
<td>40</td>
<td>62</td>
<td>75</td>
<td>100</td>
<td>65</td>
</tr>
</tbody>
</table>

Solution:

<table>
<thead>
<tr>
<th>Daily wages in Rs.</th>
<th>No of Labourers</th>
</tr>
</thead>
<tbody>
<tr>
<td>50-60</td>
<td>40</td>
</tr>
<tr>
<td>60-70</td>
<td>62</td>
</tr>
<tr>
<td>70-80</td>
<td>75 = f_0</td>
</tr>
<tr>
<td>80-90</td>
<td>100 = f_1</td>
</tr>
<tr>
<td>90-100</td>
<td>65 = f_2</td>
</tr>
</tbody>
</table>

Greatest Frequency = 100
Modal class = 80-90
L = 80
D_1 = f_1 - f_0 = 100-75 = 25
D_2 = f_1 - f_2 = 100-65 = 35
i = 10
\[ Z = L + \left[ \frac{D_1}{D_1 + D_2} \right] \times i = 80 + \left[ \frac{25}{25 + 35} \right] 10 = 80 + \frac{25}{60} 10 = 80 + 4.17 \]

\[ Z = \text{Rs. 84.17} \]

**Merits of Mode:**

1. It is easy to calculate and in some cases it can be located mere inspection

2. Mode is not at all affected by extreme values.

3. It can be calculated for open-end classes.

4. It is usually an actual value of an important part of the series.
   
   In some circumstances it is the best representative of data.

**Demerits of mode:**

1. It is not based on all observations.

2. It is not capable of further mathematical treatment.

3. The Mode is ill-defined, generally, it is not possible to find mode in some cases.

4. As compared with the mean, mode is affected to a great extent, by sampling fluctuations.

5. It is unsuitable in cases where the relative importance of items has to be considered.

**Geometric Mean**

Definition:

Geometric mean of N values is the N\(^{th}\) root of the product of the N values. It is abbreviated as G.M.

Formulae:

- Individual Series: \( G.M = \text{Antilog} \left( \frac{\sum \log X}{N} \right) \)
Discrete Series: \( G.M = \text{Antilog} \left( \frac{\sum f \log X}{\sum f} \right) \)

Continuous Series: \( G.M = \text{Antilog} \left( \frac{\sum f \log m}{\sum f} \right) \)

1. Find the Geometric Mean of 3, 6, 24, 48

Solution:

<table>
<thead>
<tr>
<th>( X )</th>
<th>( \log X )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.4771</td>
</tr>
<tr>
<td>6</td>
<td>0.7782</td>
</tr>
<tr>
<td>24</td>
<td>1.3802</td>
</tr>
<tr>
<td>48</td>
<td>1.6812</td>
</tr>
</tbody>
</table>

\( \sum \log X = 4.3167 \)

\( G.M = \text{Antilog} \left( \frac{\sum \log X}{N} \right) \)

\( = \text{Antilog} \left( \frac{4.3167}{4} \right) = \text{Antilog}(1.0792) = 12 \)

2. Calculate the Geometric Mean for the following data:

<table>
<thead>
<tr>
<th>( X )</th>
<th>( f )</th>
<th>( \log X )</th>
<th>( f \log X )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>4</td>
<td>1.0000</td>
<td>4.0000</td>
</tr>
<tr>
<td>15</td>
<td>6</td>
<td>1.1761</td>
<td>7.0566</td>
</tr>
<tr>
<td>25</td>
<td>10</td>
<td>1.3979</td>
<td>13.9790</td>
</tr>
<tr>
<td>40</td>
<td>7</td>
<td>1.6021</td>
<td>11.2147</td>
</tr>
<tr>
<td>50</td>
<td>3</td>
<td>1.6990</td>
<td>5.0970</td>
</tr>
</tbody>
</table>

\( \sum f = 30 \)

\( \sum f \log X = 41.3473 \)

\( G.M = \text{Antilog} \left( \frac{\sum f \log X}{\sum f} \right) \)

\( = \text{Antilog} \left( \frac{41.3473}{30} \right) = \text{Antilog}(1.3782) = 23.89 \)

3. Compute the Geometric Mean for the following series:
<table>
<thead>
<tr>
<th>Marks</th>
<th>No of Students</th>
<th>m</th>
<th>Log m</th>
<th>f log m</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-10</td>
<td>5</td>
<td>5</td>
<td>0.6990</td>
<td>3.4950</td>
</tr>
<tr>
<td>10-20</td>
<td>7</td>
<td>15</td>
<td>1.1761</td>
<td>8.2327</td>
</tr>
<tr>
<td>20-30</td>
<td>15</td>
<td>25</td>
<td>1.3979</td>
<td>20.9685</td>
</tr>
<tr>
<td>30-40</td>
<td>25</td>
<td>35</td>
<td>1.5441</td>
<td>38.6025</td>
</tr>
<tr>
<td>40-50</td>
<td>8</td>
<td>45</td>
<td>1.6532</td>
<td>13.2256</td>
</tr>
</tbody>
</table>

\[ \sum f = 60 \quad \sum f \log m = 84.5243 \]

G.M = Antilog\left( \frac{\sum f \log m}{\sum f} \right)

\[ = \text{Antilog} \left( \frac{84.5243}{60} \right) = \text{Antilog}(1.4087) = 25.63. \]

**Merits of GM**
- It is based on all the items of the data.
- It is rigidly defined. It means different investigators will find the same result from the given set of data.
- It is a relative measure and given less importance to large items and more to small ones unlike the arithmetic mean.
- Geometric mean is useful in ratios and percentages and in determining rates of increase or decrease.
- It is capable of algebraic treatment. It mean we can find out the combined geometric mean of two or more series.

**Demerits or Limitations of GM**
- It is not easily understood and therefore is not widely used.
- It is difficult to compute as it involves the knowledge of ratios, roots, logs and antilog.
- It becomes indeterminate in case any value in the given series happen to be zero or negative.
- With open-end class intervals of the data, geometric mean cannot be calculated.
- Geometric mean may not correspond to any value of the given data.
HARMONIC MEAN

Definition:
Harmonic mean is the reciprocal of the mean of the reciprocals of the values. It is abbreviated as H.M.

Formulae:

- Individual Series: \( H.M = \frac{N}{\sum \frac{1}{X}} \)
- Discrete Series: \( H.M = \frac{\sum f}{\sum \frac{f}{X}} \)
- Continuous Series: \( H.M = \frac{\sum f}{\sum \frac{f}{m}} \)

1. Find the Harmonic Mean for the following individual data:

\[
\begin{array}{|c|c|}
\hline
X & 1/X \\
\hline
6 & 0.1667 \\
15 & 0.0667 \\
35 & 0.0286 \\
40 & 0.0250 \\
900 & 0.0011 \\
520 & 0.0019 \\
300 & 0.0033 \\
400 & 0.0025 \\
1800 & 0.0006 \\
2000 & 0.0005 \\
\hline
\end{array}
\]

\[
\sum \frac{1}{X} = 0.2969
\]

\[
H.M = \frac{N}{\sum \frac{1}{X}} = \frac{10}{0.2969} = 33.68
\]

2. Calculate the Harmonic mean from the following data:
The harmonic mean has the following merits.

1. It is rigidly defined.
2. It is based on all the observations of a series i.e. it cannot be calculated ignoring any item of a series.
3. It is capable of further algebraic treatment.
4. It gives better result when the ends to be achieved are the same for the different means adopted.
5. It gives the greatest weight to the smallest item of a series.
6. It can be calculated even when a series contains any negative value.
7. It makes a skewed distribution a normal one.
8. It gives a curve straighter than that of the arithmetic and geometric mean.

**Demerits**

However, the harmonic mean suffers from the following demerits.

1. It is not easy to understand by a man of ordinary prudence.
2. Its calculation is cumbersome as it involves finding out of the reciprocals of the numbers.
3. It does not give better and accurate results when the means adopted are the same for the different ends achieved.
4. Its algebraic treatment is very much limited and not far and wide as that of the arithmetic mean.
5. It is greatly affected by the values of the extreme items.
6. It can not be calculated, if any, of the items is zero.