



Procedure DELETED(Q, n, front, rear, item)

if front = rear then call QUEUE EMPTY endif

front  $\leftarrow$  front + 1 mod n

item  $\leftarrow$  Q[front]

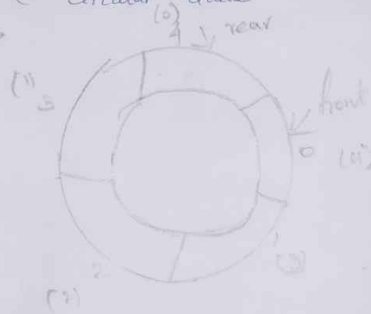
end DELETED

### Circular Queue:

(i) Circular Queue is an Ordered List.

(ii) Last position to be connected back to the front position to make a circular. This is called 'Ring Buffer'.

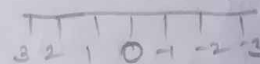
Example: (i) Circular Queue is also FIFO manner.



### Circular Queue:

We can perform two operations.

(i) Insertion (ii) Deletion



Procedure ADD(Q, n, front, rear, item)

if front = 0 and rear = n-1 (or) front = rear + 1, then call CIRCULAR QUEUE IS FULL endif

if front = -1, then front = 0 and rear = 0.

else

if rear = n-1, then, rear = 0.

else

rear  $\leftarrow$  rear + 1

Q[rear]  $\leftarrow$  item

Delete:

Procedure DELETE @ C @ (n, front, rear, item)

if front = -1 Then all CIRCULAR QUEUE IS EMPTY ends if.

else

item ← Q(front)

if front = rear then front and rear will be -1.

else

if front = n-1, then front = 0

else

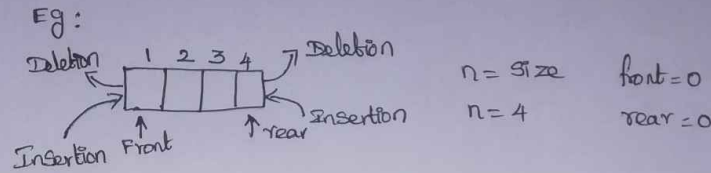
front ← front + 1

end DELETE @.



Double-Ended Queue

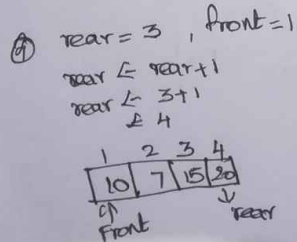
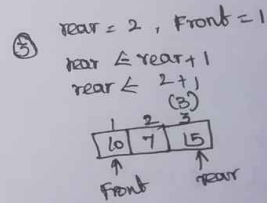
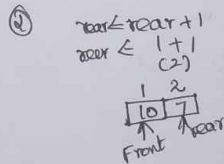
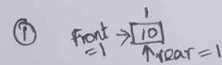
Double Ended Queue means Insertions and deletions are performed at both ends, which means you can insert an element at front end and rear end, similarly you can delete an element at front end and rear end.



Algorithm for Insertion at rear end:

Procedure INSERTREAR (Q, n, front, rear, element)  
 if  $\text{rear} = n$  then call "Queue is Full" end if  
 else  
      $\text{rear} \leftarrow \text{rear} + 1$   
      $Q(\text{rear}) \leftarrow \text{element}$   
     if  $\text{rear} = 0$  then  $\text{rear} = 1$   
     if  $\text{front} = 0$  then  $\text{front} = 1$   
 return  
 end INSERTREAR

Eg: elements are: 10, 7, 15, 20.



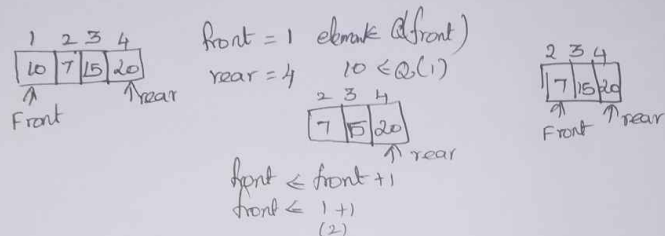
⑤  $\text{rear} = 4, \text{front} = 1$   
 if  $\text{rear} = n$  then Queue Full Condition is true  
 $4 = 4$   
 end if  
 So Procedure End.



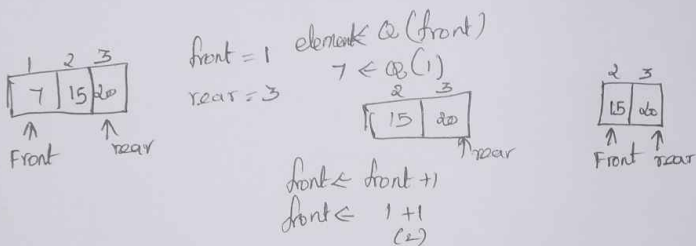
Algorithm for Deletion at front and:

Procedure DELETEFRONT (Q, n, front, rear, elements)  
 if front = 0 then Call "Queue is <sup>Empty</sup> ~~Full~~" end if  
 else  
 element ← Q(front)  
 if front = rear then  
 front = 0  
 rear = 0  
 else  
 front ← front + 1  
 return  
 end DELETEFRONT

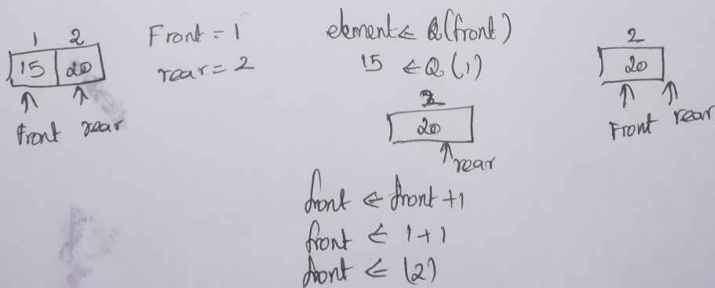
eg: ①



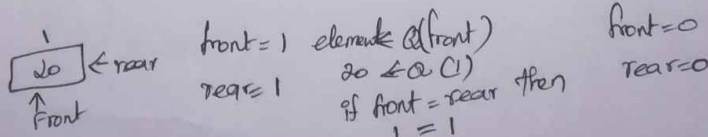
②



③



④



⑤ if front = 0  
 then  
 Queue Empty  
 0 = 0  
 Condition is true end

Algorithm for Insertion at front end:

Procedure INSERTFRONT (Q, n, front, rear, elements)

if front = 1 then call "Queue is Full" and if

else

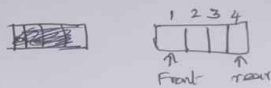
front ← front - 1

Q(front) ← element

return

end INSERTFRONT

Eg:

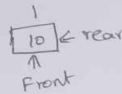


front = 0  
rear = 0

So, before an element is inserted into Q we cannot find front

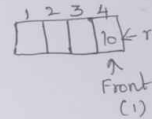
So first element to be inserted at rear end.

①



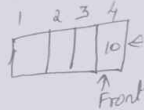
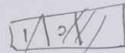
front = 1  
rear = 1

Consider if rear = 1 means that is the 1<sup>st</sup> position from front.

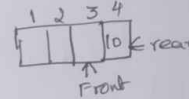


So, front = 4.

②

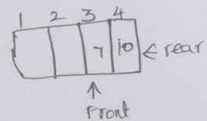


front = 4.  
front ← front - 1  
front ← 4 - 1  
front ← 3



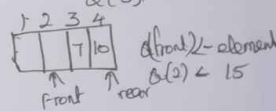
Q(front) ← element  
Q(2) ← 2

③



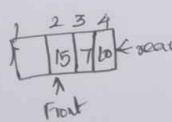
front = 3  
rear = 4

front ← front - 1  
front ← 3 - 1  
front ← 2



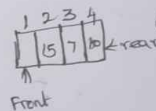
Q(front) ← element  
Q(3) ← 7

④



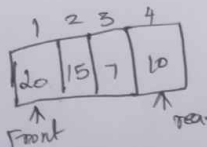
front = 2  
rear = 4

front ← front - 1  
front ← 2 - 1  
front ← 1



Q(front) ← element  
Q(4) ← 15

⑤



front = 1  
rear = 4

if front = 1 then call "Queue Full"  
1 = 1 condition True  
So Procedure End.

## Linked List

- (x) A Linked List is a sequence of data structures, which are connected together via links.
- (x) Linked List is a sequence of links which contains items. Each link contains a connection link to another link.

### Important Terms to understand linked List:

- Link  $\Rightarrow$  Each link of a linked list can store a data called an "Element".
- Next  $\Rightarrow$  Each link of a linked list contains a link to the next link called "Next".
- Linked List  $\Rightarrow$  A linked list contains a connection link to the first link called "First (Head)".

### Linked List Representation:

linked list can be visualized as a chain of nodes, where every node links to the next node.



- (x) linked list contains a link element called first/head.
- (x) Each link carries a data field and a link field called "next".
- (x) Each link is linked with its next link by using its "next".
- (x) last link carries a link as NULL to indicate the end of the list.

### Types of Linked List:

- (i) Simple / single linked list:  
elements can be navigated only forward approach.
- (ii) Doubly linked list:  
(x) Elements can be navigated forward and backward approach.
- (iii) Circular linked list:  
(x) last element contains a link of the first element as next, and the first element has a link to the last element as previous.

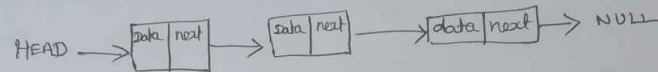
## Operations:

- (i) Insertion (ii) Deletion (iii) Display (iv) Search (v) Delete

## (i) Single Simple Linked List:

- (i) Each link of a single linked list can store a data element at data field, and contains a link to another link in the next field.  
(ii) Contains a connection link (first/head) to the first link of a single linked list.

## Representation of single linked list:



Forward approach →

Backward approach ←

## Node is represented as:

struct node {

int data;

struct node \*next;

}

## A three member single linked list can be created as.

/\* Initialize nodes \*/

struct node \*head;

struct node \*one = NULL;

struct node \*two = NULL;

struct node \*three = NULL;

/\* Allocate memory \*/

one = malloc (size of (struct node));

two = malloc (size of (struct node));

three = malloc (size of (struct node));

/\* Assign data values \*/

one → data = 10;

two → data = 20;

three → data = 30;

/\* connect nodes \*/

one → next = two;

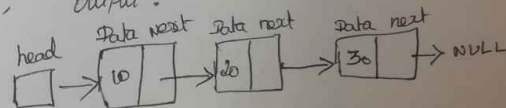
two → next = three;

three → next = NULL;

/\* Save the address of first node in the head \*/

head = one;

output:



## Insertion Algorithm for Singly linked List

Procedure INSERT (link, data, key)

```
struct node *link = (struct node *) malloc (sizeof (struct node));
```

```
link -> data = data;
```

```
link -> key = key;
```

```
if (isempty()) {
```

```
    last = link;
```

```
    link -> next = NULL;
```

```
    head = link;
```

```
}
```

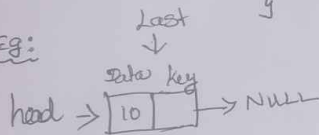
```
else {
```

```
    link -> next = head;
```

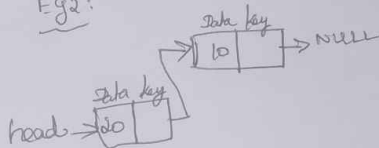
```
    link = head
```

```
}
```

Eg:



Eg2:



## deletion Operation:

Procedure DELETE (link, data, key)

```
struct node *temp = head;
```

```
if (head -> next == NULL)
```

```
{
```

```
    last = NULL;
```

```
}
```

```
else {
```

```
    {
```

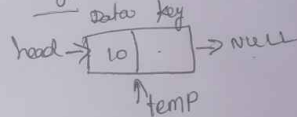
```
        head -> next = head -> next -> next;
```

```
    }
```

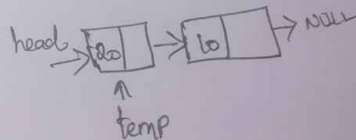
```
    return temp;
```

```
}
```

Eg:

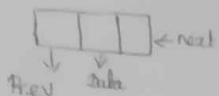


Eg2:



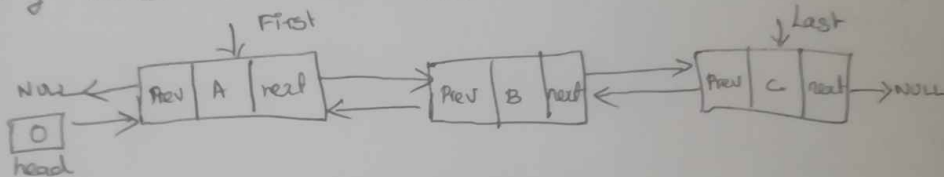
## Doubly linked list

A node/link contains three parts



- (\*) Each link of a linked list can store an element at Data field.
- (\*) Each link of a linked list contains a connection link to the next link by using next field.
- (\*) Each link of a linked list contains a connection link to the previous link by using prev field.
- (\*) A linked list contains the connection link to the first link called First/Head, and contains the connection link to the last link called Last.

Doubly linked list Representation:



Insertion algorithm at First:

Procedure INSERT FIRST (link, prev, next, data)

// create a link

```
struct node *link = (struct node*) malloc(size of (struct node));
```

```
link->data = data;
```

```
link->key = key;
```

```
if (isEmpty())
```

```
{
```

```
last = link;
```

```
}
```

```
else
```

```
{ head->prev = link;
```

```
link->next = head;
```

```
link->prev = link;
```

### Deletion at Front / first:

Procedure DELETEFIRST (link, Prev, next, data)

```
struct node * deleteFirst ()
{
    struct node * temp = head ;
    if (head > next == NULL)
    {
        last = null ;
    }
    else {
        head > next -> Prev = NULL ;
    }
    head = head -> next ;
    return temp ;
}
```

### Insertion algorithm at last:

Procedure INSERTLAST (link, Prev, next, data)

// create a link

```
struct node * link = (struct node *) malloc (size of (struct node));
```

```
link -> data = data ;
```

```
link -> key = key ;
```

```
if (!isEmpty ())
```

```
{
    last = link ;
}
```

```
else
```

```
{
    last -> next = link ;
}
```

```
link -> Prev = last
```

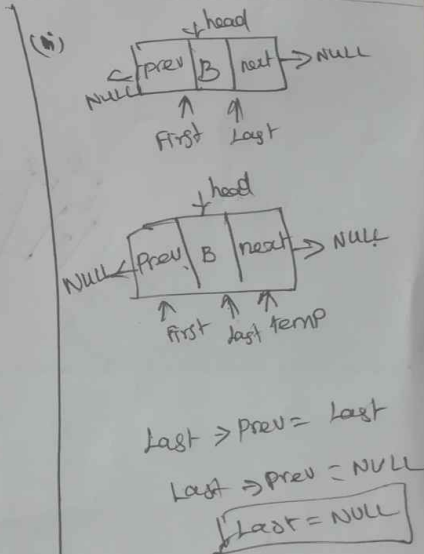
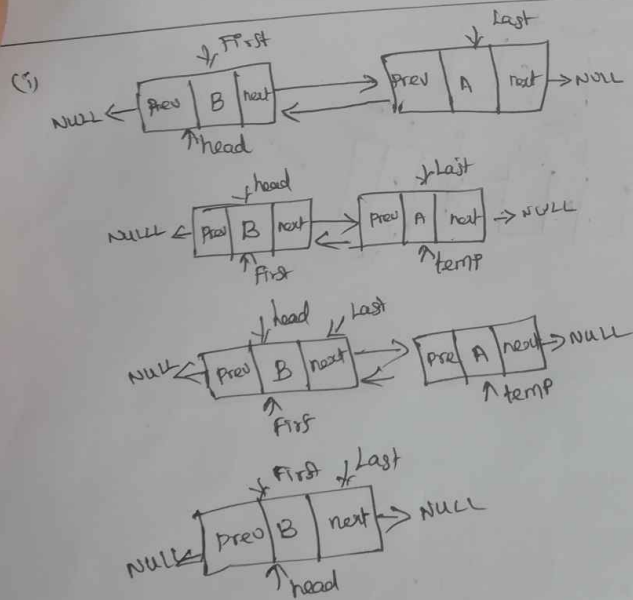
```
last = link ;
```

```
}
```

Deletion at Last:

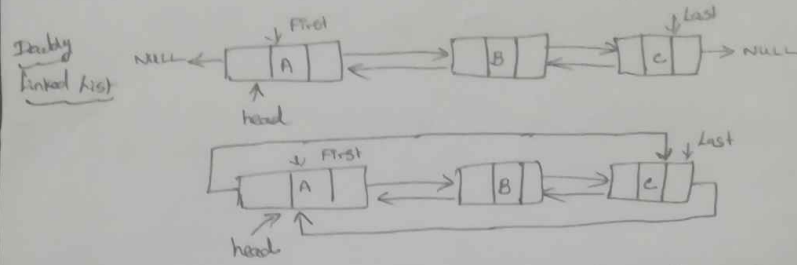
Procedure DELETELAST (link, Prev, next, data)

```
struct node* deleteLast()  
{  
    struct node *temp = last;  
    if (last->next == NULL)  
    {  
        last->prev = last;  
        return temp;  
    }  
}
```





Representation of Doubly Linked List into Circular:



Insertion Algorithm for Singly linked list to make Circular linked list:

(→)

Procedure INSERT (link)

// create a link

struct node \*link = (struct node \*) malloc (sizeof (struct node));

link->data = data ;

link->next = next ;

if (is Empty ())

{

last = link ; link->next = link ; head = link ;

}

else

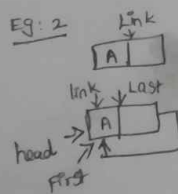
{ link->next = head ; link = head ; last->next = head ;

}

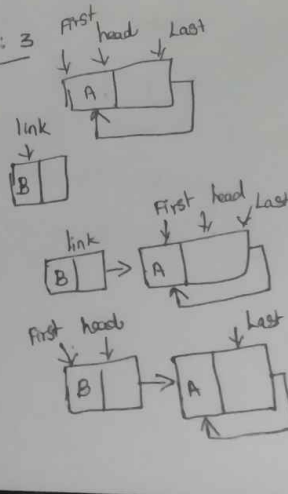
Eg: 1

empty

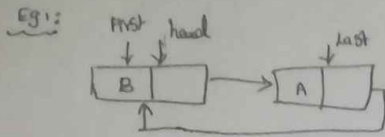
Eg: 2



Eg: 3



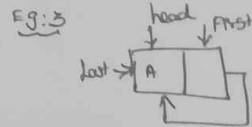
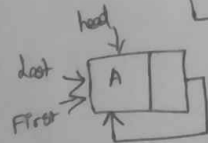
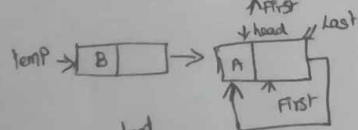
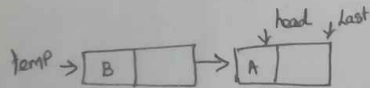
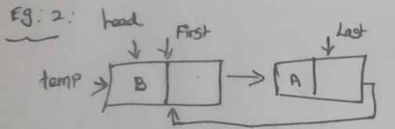
Deletion Algorithm for Singly Linked List to make Circular.



Procedure DELETE (link)

```

struct node * temp = head;
if (head -> next == NULL)
{
    last = NULL;
}
else
{
    head -> next = head;
}
last -> next = head;
return temp;
}
    
```



last = NULL;

return temp;

NO link in linked list.

Insertion algorithm for Doubly linked list at first into to make

Circular linked list:

Procedure INSERTFIRST (link)

↵ create a link

struct node \*link = (struct node \*) malloc (size of (struct node));

link → data = data;

link → Prev = Prev;

link → next = next;

If (isempty())

{

last = link;

link → next = first;

link → Prev = last;

}

else {

head → Prev = link; link → next = head;

link → Prev = last; last → next = first;

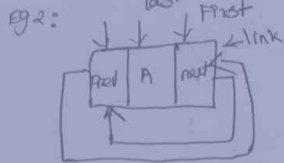
}

head = link;

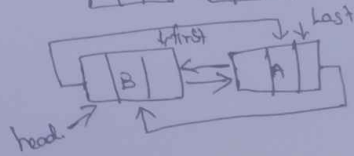
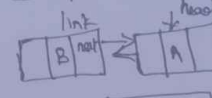
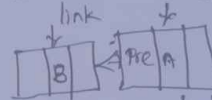
}

Eg:1

Empty



Eg:3:



Deletion Algorithm for Doubly linked list at first into to make a circular linked list:

Procedure DELETEFIRST (link)

Struct node \* deleteFirst ()

{  
 struct node \* temp = head;

~~if (head == NULL)~~

~~head = next = head;~~

if (head -> next == head)

{

last = NULL;

}

else

{

head -> next = head;

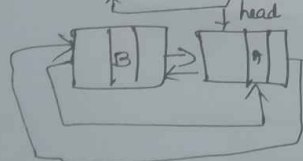
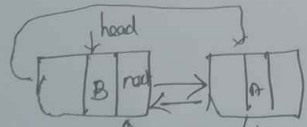
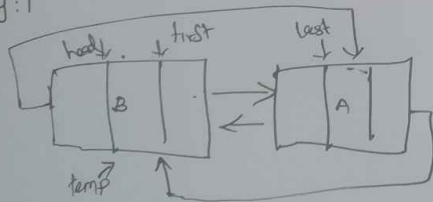
}

return temp;

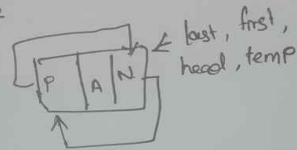
}

end DELETEFIRST.

Eg: 1



Eg: 2



### Arithmetic Expressions:

- ∞ Arithmetic Expressions is also known as "Notation".
- ∞ Arithmetic Expressions can be written in three different ways but all are produce same result.

There are three types of Notations.

- (i) Infix Notation
- (ii) Prefix Notation
- (iii) Postfix Notation.

### Infix Notation:

< Operand > < Operator > < Operand >

eg:  $A + B \rightarrow$  Operand.  
 $\downarrow \rightarrow$  Operator  
 Operand

when an Operator is placed between the two Operand is called Infix Notation.

### Prefix Notation:

< Operator > < Operand >

eg:  $+ A$

when an Operator is placed before an Operand is called Prefix Notation.

## Postfix Notation:

$\langle \text{Operand} \rangle \langle \text{Operator} \rangle$

eg:

A+

When an operator is placed after the operand is called Postfix Notation.

## Operator Precedence:

When an operand is placed between two different operators means which operator can take the operand for processing first is decided by the Operator Precedence Rule.

Eg:

$A+B * C$

Consider B operand, it placed between two different operators + and \*, which operator can take B operand as first for processing is decided by Operator Precedence Rule.

Symbol of Operator	Name of Operator	Precedence
( )	Parenthesis	Highest Operator Precedence
$\uparrow, \wedge$	Exponent	Second highest
$*$ , /	Multiplication, Division	Third
$+$ , -	Addition, Subtraction	Last

## Associativity:

When an Operand is Placed between two different Operator but both the Operator have a Same Operator Precedence value means, which Operator can take the Operand first is decided by Associativity Rule.

Eg:

$$A + B - C$$

here Operator + and - have Same Operator Precedence value.

Operator Symbol	Associativity Rule
()	-
↑, ^	Right to Left
*, /	Left to Right
+, -	Left to Right

EX:

Convert Infix Expression to Prefix Expression

Step 1: Reverse Infix Expression and also reverse. '(' becomes ')' and ')' will become '('.

Step 2: Obtain the Postfix Expression. of the modified expression.

Step 3: Reverse the Postfix Expression. we can get Prefix Expression.

Eg 1:

Convert Infix Expression into ~~Post~~ Prefix Expression

$$A + B * C$$

Step 1: Reverse the Infix Expression

$$C * B + A$$

You can use stack for this Conversion

Expression	Stack	Output	Description
C	-	C	Print
*	*	C	Push
B	*	CB	Print
+	+	CB*	Pop * and Push +
A	+	CB*A	Print
End	-	CB*A+	Pop +



Step 2: Obtained modified Postfix Expression is

$$CB * A +$$

Step 3: Reverse the Postfix Expression to get Prefix Expression

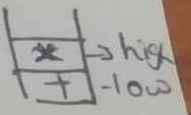
$$\boxed{+ A * B C}$$

Eg 2:

Infix to Postfix

A + B \* C

Expression	stack	Output	Description
A	-	A	Print
+	+	A	Push
B	+	AB	Print
*	+, *	AB	Push
C	+, *	ABC	Print
End	+	ABC*	POP *
End	Empty	ABC*+	POP +



ABC\*+

is the Postfix Expression.



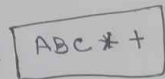
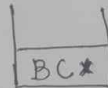
Eg 4: Prefix to Postfix Conversion:

$+A * BC$

Step 1: Reverse Prefix Expression

$C B * A +$

Expression	Stack	Description
C	<div style="border: 1px solid black; padding: 2px; width: 30px; margin: 0 auto;">C</div>	Push
B	<div style="border: 1px solid black; padding: 2px; width: 30px; margin: 0 auto;">B C</div>	Push
*		Pop the top two operands and concatenate $\langle 1^{st} \text{ op} \rangle \langle 2^{nd} \text{ op} \rangle \langle \text{operator} \rangle$ B C *
A	<div style="border: 1px solid black; padding: 2px; width: 30px; margin: 0 auto;">A BC*</div>	Push, After pushing A consider BC * A
		Pop the top two operands and concatenate $\langle 1^{st} \text{ operand} \rangle \langle 2^{nd} \text{ operand} \rangle \langle \text{operator} \rangle$ A BC *
	<div style="border: 1px solid black; padding: 2px; width: 30px; margin: 0 auto;">ABC*</div>	Push the result ABC* into stack
+	<div style="border: 1px solid black; padding: 2px; width: 30px; margin: 0 auto;">ABC* +</div>	Push. Result in the stack is Postfix



(i) If character is an Operand  
Push into stack.

(ii) If character is an Operator  
Pop the top two operand  
from stack.

Concatenate:

$\langle \text{operand } 1^{st} \text{ top} \rangle \langle \text{operand } 2^{nd} \text{ top} \rangle$

$\langle \text{operator} \rangle$

(iii) Push the new result into  
stack.

Eg 5: Postfix to Infix

ABC\*+

Expression	Stack	Description			
A	[A]	Push			
B	<table border="1"><tr><td>B</td></tr><tr><td>A</td></tr></table>	B	A	Push	
B					
A					
C	<table border="1"><tr><td>C</td></tr><tr><td>B</td></tr><tr><td>A</td></tr></table>	C	B	A	Push
C					
B					
A					
*	<table border="1"><tr><td>B * C</td></tr><tr><td>A</td></tr></table>	B * C	A	Pop the top two operands from stack.	
B * C					
A					

- (i) If character is Operand Push into stack.
- (ii) If character is operator, Pop the top two operands from stack, Concatenate  
 $\langle 2^{nd} \text{ top operand} \rangle \langle \text{operator} \rangle \langle 1^{st} \text{ top operand} \rangle$
- (iii) Push the result back into stack.

Concatenate:  
 $\langle 2^{nd} \text{ top} \rangle \langle op \rangle \langle 1^{st} \text{ top} \rangle$   
 B \* C

(ii) Result B \* C Push back to the stack

Consider Stack: 

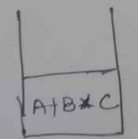
C
*
B
A

  
 A B \* C

Pop the top two operands from stack  
 B \* C  $\langle 1^{st} \text{ top operand} \rangle$   
 A  $\langle 2^{nd} \text{ top operand} \rangle$

Concatenate:  
 $\langle 2^{nd} \text{ top operand} \rangle \langle \text{operator} \rangle \langle 1^{st} \text{ top operand} \rangle$   
 A + B \* C

(iii) Result A + B \* C Push back into stack.



End.




Finally Result in the stack 

A + B * C
-----------

  
 is infix

Eg 6: Postfix to Prefix

ABC\*+

Expression	Stack	Description
A		Push
B		Push
C		Push
*		

Pop the top two operands from stack

C <1st top operand>  
B <2nd top operand>



Concatenate:

<operator> <2nd top> <1st top>  
\* B C

(i) Push the result **\*BC** into stack

Consider stack:



Pop the top two operands from stack

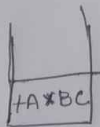
\*BC <1st top operand>  
A <2nd top operand>

Concatenate:

<operator> <2nd top> <1st top>  
+ A \*BC

(ii) Push the result **+A\*BC** ~~from~~ back to push stack

End



So, Result **+A\*BC** in the stack are prefix expression.

(i) if character is operand

Push into stack.

(ii) if character is an operator.

Pop the top two operands from stack and

Concatenate

<operator> <2nd top operand>  
<1st top operand>

(iii) Push the new result back into stack.

## Stack applications:-

### Stack:

Stack is an abstract data type and a data structure that follows LIFO (Last In First Out) strategy. It means the element added last will be removed first.

### Applications:

1. Stack data structure is used for evaluating the expression.

Ex:  $5 * (6 + 2) - 12 / 4$

$\rightarrow (6 + 2) = 8$

$\rightarrow 5 * 8 = 40$

$\rightarrow 12 / 4 = 3$

$\rightarrow 40 - 3 = 37$

2. Stacks can be used to check parenthesis matching in an expression.

Ex:  $(a + b) * (c + d) \rightarrow$  Valid expression

$(a + b) * [c + d] \rightarrow$  Not valid expression.

3. Stacks are used for conversion from one form of expression to another.

Ex:

Infix to prefix, postfix.

Prefix to Infix, postfix.

Postfix to Infix, prefix.

Infix	prefix	postfix
$a + b$	$+ ba$	$ab +$
$(a + b) * (c + d)$	$* + dc + ba$	$ab + cd + *$
$b * b - 4 * a * c$	$- * c * a 4 * bb$	$bb * 4 a * c *$

4. Syntax parsing :- Many compilers use a stack for parsing the syntax of expressions, program blocks etc. before translating into low level code.

5. Back tracking : Suppose we are finding a path for solving maze problem. We choose a path and after following it we realize that it is wrong. Now we need to go back to the beginning of the path to start with new path. This can be done with the help of stack.

6. Parenthesis checking :

Stack is used to check the proper opening and closing of parenthesis.

7. String Reversal :

Stack is used to reverse a string. We push the characters of string one by one into stack and then pop character from stack.

8. Function call :

Stack is used to keep information about the active functions or subroutines.

9. Stack is used to Redo-undo features at many places like editors, photoshop.

10. It is used to do forward and back-ward features in web browsers.

11. In memory management any modern computer uses stack as the primary - management for a running purpose. Each program that is running in a computer system has its own memory allocations.

12. In graph algorithms like Topological Sorting and Strongly connected components, are using Stack data structure.

13. In Language processing, space for parameters and local variables is created internally using a stack.

14. The stack pattern is also used to keep track of the 'most recently used' feature.

15. In real life stack is using many places. like, a stack of plates, pile of books in the cupboard, Wearing and removing bangles.

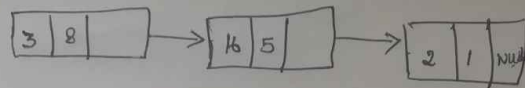
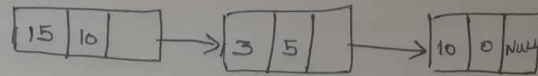


Eg:

$$\text{Poly}_1 (P) = 15x^{10} + 3x^5 + 10$$

$$\text{Poly}_2 (Q) = 3x^8 + 16x^5 + 2x$$

To be represented as follows.



Procedure for Addition:

Procedure ADDITION (Poly, Poly1, Poly2)

Poly1 = P; Poly2 = Q; r = newnode(); Poly = r;

while (P → next and Q → next)

{  
if (P → exp > Q → exp)

{  
r → coef = P → coef;

r → exp = P → exp;

P → next = P;

}

else if (P → exp < Q → exp)

{  
r → coef = Q → coef;

r → exp = Q → exp;

Q → next = Q;

}

else

{  
r → coef = P → coef + Q → coef;

r → exp = P → exp;

P → next = P;

Q → next = Q;

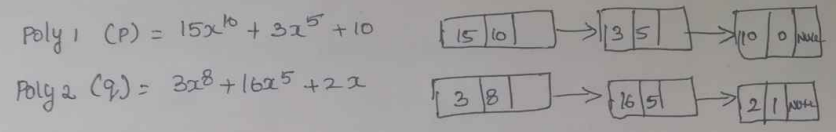
}

```

if (p->next == null)
d
r->coef = p->coef;
r->exp = p->exp;
}
else
d
r->coef = q->coef;
r->exp = q->exp;
}
}

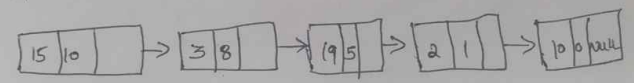
```

Ex:

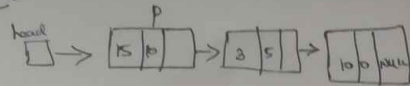


Addition:

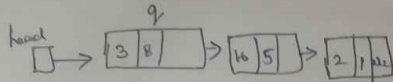
Poly (r) (P+Q) =  $15x^{10} + 3x^8 + 19x^5 + 2x + 10$



Eg (1)

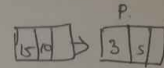


if ( $P \rightarrow \text{exp} > Q \rightarrow \text{exp}$ ) true

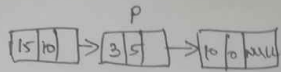


$r = [15 | 10]$

$P \rightarrow \text{next} = P;$

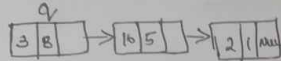


Eg (2):



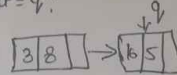
if ( $P \rightarrow \text{exp} > Q \rightarrow \text{exp}$ ) x false

else if ( $P \rightarrow \text{exp} < Q \rightarrow \text{exp}$ ) true



$r = [15 | 10] \rightarrow [3 | 8]$

$Q \rightarrow \text{next} = Q;$



Eg (3):



if ( $P \rightarrow \text{exp} > Q \rightarrow \text{exp}$ ) not consider true or false

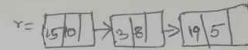
else if ( $P \rightarrow \text{exp} < Q \rightarrow \text{exp}$ ) not consider true or false

else

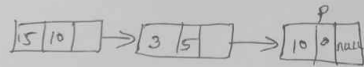
$r \rightarrow \text{cof} = P \rightarrow \text{cof} + Q \rightarrow \text{cof}$

$r \rightarrow \text{exp} = P \rightarrow \text{exp}$

$P \rightarrow \text{next} = P; Q \rightarrow \text{next} = Q;$



Eg (4):



if ( $P \rightarrow \text{exp} > Q \rightarrow \text{exp}$ ) false

else if ( $P \rightarrow \text{exp} < Q \rightarrow \text{exp}$ ) true

$r \rightarrow \text{cof} = Q \rightarrow \text{cof};$

$r \rightarrow \text{exp} = Q \rightarrow \text{exp};$

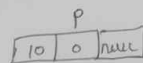
$Q \rightarrow \text{next} = Q;$

$Q \rightarrow \text{next} = \text{NULL};$

$Q = \text{NULL};$



Eg (5)



$Q = \text{NULL}$

$P \rightarrow \text{next} = \text{NULL}$

if ( $P \rightarrow \text{next} = \text{NULL}$ ) is true

d

$r \rightarrow \text{cof} = P \rightarrow \text{cof};$

$r \rightarrow \text{ext} = P \rightarrow \text{exp};$

3



### Applications of Queue :

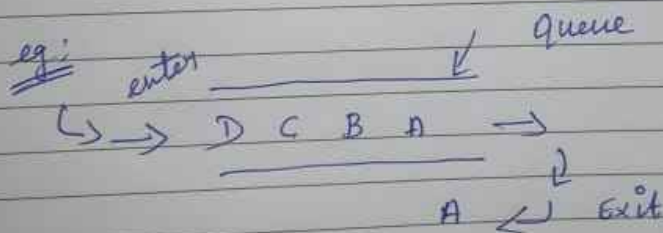
Queue is used when things don't have to be processed immediately, but have to be processed in "First in First out" order like 'Breath first search'.

### Real world examples :

Patients waiting outside the doctor's clinic :-

⊙ The patient who comes first visits the doctor first, and the patients who comes last visits the doctor last.

⊙ Therefore, it follows the FIFO manner.



⊙ And Ticket counter also the example of queue.

calls the phone last  
last gets the phone last  
front gets the response  
front gets a response from the system.  
The person who answers the system gets the response last.

Bank line where people who come will done his transaction front.

Key press sequence in keyboard.

ATM both line.

single one-way road, where the vehicle enters front, exits front.

Queue of air planes waiting for landing instruction.

Job scheduling.

# Chapter 1

## INTRODUCTION

### 1.1 WHAT IS AN ALGORITHM?

The word algorithm comes from the name of a Persian author, Abu Ja'far Mohammed ibn Musa al Khwarizmi (c. 825 A.D.) who wrote a textbook on mathematics. An examination of the latest edition of Webster's dictionary defines its meaning as "any special method of solving a certain kind of problem." But this word has taken on a special significance in computer science, where *algorithm* has come to refer to a precise method useable by a computer for the solution of a problem. This is what makes the notion of an algorithm different from words such as process, technique or method.

An algorithm is composed of a finite set of steps, each of which may require one or more operations. The possibility of a computer carrying out these operations necessitates that certain constraints be placed on the type of operations an algorithm can include. For example, each operation must be *definite*, meaning that it must be perfectly clear what should be done. Directions such as "compute  $5/0$ " or "add 6 or 7 to  $x$ " are not permitted because it is not clear what the result is or which of the two possibilities should be done. Another important property each operation should have is that it be *effective*; each step must be such that it can, at least in principle, be done by a person using pencil and paper in a finite amount of time. Performing arithmetic on integers is an example of an effective operation, but arithmetic with real numbers is not, since some values may be expressible only by an infinitely long decimal expansion. Adding two such numbers would violate the effectiveness property. An algorithm produces one or more *outputs* and may have zero or more *inputs* which are externally supplied.

Another important criterion we will assume about algorithms in this book is that they *terminate* after a finite number of operations. There is another word for an algorithm which obeys all of the above properties ex-

## Chapter 2

# ELEMENTARY DATA STRUCTURES

63/643

Now that we have presented the fundamental methods we need to express and analyze algorithms you might feel all set to begin. But alas we need to make one last diversion to which we devote this chapter, and that is a discussion of data structures. One of the basic techniques for improving algorithms is to structure the data in such a way that the resulting operations can be efficiently carried out. Though we can't possibly survey here all of the techniques that are known, in this chapter we have selected several which we feel occur most frequently. Maybe you have already seen these techniques in a course on data structures (hopefully having used *Fundamentals of data structures*). If so, you may either skip this chapter or scan it briefly. If you haven't been exposed to the ideas of stack, queues, sets, trees, graphs, heaps, or hashing then lets begin our study of algorithms right now with some interesting problems from the field of data structures.

### 2.1 STACKS AND QUEUES

One of the most common forms of data organization in computer programs is the ordered or linear list, which is often written as  $A = (a_1, a_2, \dots, a_n)$ . The  $a_i$ s are referred to as *atoms* and they are chosen from some set. The null or empty list has  $n = 0$  elements. A *stack* is an ordered list in which all insertions and deletions are made at one end, called the *top*. A *queue* is an ordered list in which all insertions take place at one end, the *rear*, while all deletions take place at the other end, the *front*.

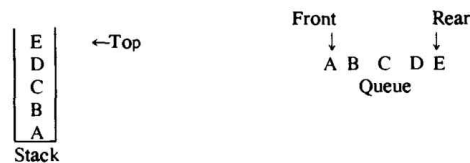


Figure 2.1 Example of a stack and a queue

The operations of a stack imply that if the elements A,B,C,D,E are inserted into a stack, in that order, then the first element to be removed/deleted must be E. Equivalently we say that the last element to be inserted into the stack will be the first to be removed. For this reason stacks are sometimes referred to as **Last In First Out (LIFO)** lists. The operations of a queue require that the first element which is inserted into the queue will be the first one to be removed. Thus queues are known as **First In First Out (FIFO)** lists. See Figure 2.1 for an example of a stack and a queue each containing the same five elements inserted in the same order. Note that the data object queue as defined here need not necessarily correspond to the concept of queue which is studied in queuing theory.

The simplest way to represent a stack is by using a one-dimensional array, say  $STACK(1:n)$ , where  $n$  is the maximum number of allowable entries. The first or bottom element in the stack will be stored at  $STACK(1)$ , the second at  $STACK(2)$  and the  $i$ th at  $STACK(i)$ . Associated with the array will be a variable, typically called  $top$ , which points to the top element in the stack. To test if the stack is empty we ask “if  $top = 0$ ”. If not, the top-most element is at  $STACK(top)$ . Two more substantial operations are inserting and deleting elements. The corresponding procedures are given as algorithms 2.1(a) and (b).

```
procedure ADD(item, STACK, n, top)
  //insert item into the STACK of maximum size n; top is the//
  //number of elements currently in STACK//
  if top ≥ n then call STACKFULL endif
  top ← top + 1
  STACK(top) ← item
end ADD
```

(a) Insertion of an element

```
procedure DELETE(item, STACK, top)
  //remove the top element of STACK and store it//
  //in item unless STACK is empty//
  if top ≤ 0 then call STACKEMPTY endif
  item ← STACK(top)
  top ← top - 1
end DELETE
```

(b) Deletion of an element

**Algorithm 2.1** Stacking operations