15/03/2021.

Fuzzy Relation :-

Fuzzy relation is a fuzzy set defined on the cartesian product, of $x_1, x_2 \ldots x_n$. where n-tuples may be verifying the degree of membership in the relation.

eg:

consider R be a relation between two fuzzy sets $\tilde{A}$ $\tilde{B}$ is a set of symptoms and represent columns R relation is defined as,

$\tilde{A}$ = {typhoid, viral fever, cold}

$\tilde{B}$ = {Running nose, high Temperature, shivering}

$$R = \begin{array}{c|ccc} & \text{Running nose} & \text{High temperature} & \text{shivering} \\ \hline \text{typhoid} & 0.2 & 0.9 & 0.7 \\ \text{viral fever} & 0.3 & 0.9 & 0.8 \\ \text{cold} & 0.9 & 0.5 & 0.7 \end{array}$$

Cartesian product of fuzzy set :-

$\tilde{A}$ be a fuzzy set defined on the universal set X, $\tilde{B}$ be a fuzzy set defined on the universal set Y.

To form a relation R.

R is defined as

$$R = \tilde{A} \times \tilde{B}$$

$$R = x \times y.$$

Membership function of R is defined as,

$$\boxed{\mu_R(x,y) = \min(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(y))}$$

eg. 1

$\tilde{A} = \{(x_1, 0.2), (x_2, 0.4), (x_3, 0.7)\}$

$\tilde{B} = \{(y_1, 0.5), (y_2, 0.6)\}$

$\mu_R(x_1, y_1) = \min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{B}}(y_1))$

$= \min(0.2, 0.5)$

$= 0.2$

$\mu_R(x_1, y_2) = \min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{B}}(y_2))$

$= \min(0.2, 0.6)$

$= \min = 0.2.$

$\mu_R(x_2, y_1) = \min(\mu_{\tilde{A}}(x_2), \mu_{\tilde{B}}(y_1))$

$= \min(0.4, 0.5)$

$= 0.4.$

$\mu_R(x_2, y_2) = \min(\mu_{\tilde{A}}(x_2), \mu_{\tilde{B}}(y_2))$

$= \min(0.4, 0.6)$

$= 0.4$

$\mu_R(x_3, y_1) = \min(\mu_{\tilde{A}}(x_3), \mu_{\tilde{B}}(y_1))$

$= \min(0.7, 0.5)$

$= 0.5$

$$\mu_R(x_3, y_2) = \min(\mu_{\tilde{A}}(x_3), \mu_{\tilde{B}}(y_2))$$

$$= \min(0.7, 0.6)$$

$$= 0.6.$$

O/P: $R = \begin{array}{c|cc} & y_1 & y_2 \\ \hline x_1 & 0.2 & 0.2 \\ x_2 & 0.4 & 0.4 \\ x_3 & 0.5 & 0.6 \end{array}$

Relations on fuzzy relations:

Q: $S = \begin{array}{c|ccc} & y_1 & y_2 & y_3 \\ \hline x_1 & 0.5 & 0.7 & 0.3 \\ x_2 & 0.8 & 0.1 & 0.4 \end{array}$

$R = \begin{array}{c|cc} & y_1 & y_2 \\ \hline x_1 & 0.2 & 0.2 \\ x_2 & 0.4 & 0.4 \\ x_3 & 0.5 & 0.6 \end{array}$

RUS:

1)

$$\mu_{RUS}(x_1, y_1) = \max(\mu_R(x_1, y), \mu_S(x, y_1))$$

$$= \max(\{0.2, 0.5\}), ($$

$$= \max(0.5, 0.8)$$

$$= 0.8$$

$$\mu_{R\cup S}(x_1, y_1) = \max\left(\mu_R(x_1, y), \mu_S(x, y_1)\right)$$

$$= \max\left((0.\cancel{4}, 0.7), (0.2, 1)\right)$$

$$= \max(0.7, 1)$$

$$= \cancel{0}\, 1$$

$$R \cup S = \begin{array}{c|ccc} & y_1 & y_2 & y_3 \\ \hline x_1 & 0.8 & 1 & 0.4 \\ x_2 & 0.8 & 1 & 0.4 \\ x_3 & 0.8 & 1 & 0.6 \end{array}$$

(i) $R \cap S$:

$$\mu_{R\cap S}(x, y) = \min\left(\mu_R(x, y), \mu_S(x, y)\right)$$

$$R \cap S = \begin{array}{c|ccc} & y_1 & y_2 & y_3 \\ \hline x_1 & 0.2 & 0.2 & 0.2 \\ x_2 & 0.4 & 0.4 & 0.3 \\ x_3 & 0.5 & 0.5 & 0.3 \end{array}$$

$$\mu_{R\cap S}(x_1, y_1) = \min\left(\mu_R(x_1, y), \mu_S(x, y_1)\right)$$

$$= \min\left((0.2, 0.5), (0.2, 0.8)\right)$$

$$= 0.2$$

$$\mu_{R\cap S}(x_1, y_2) = \min\left(\mu_R(x_1, y), \mu_S(x, y_2)\right)$$

$$= \min\left((0.2, 0.7), (0.2, 1)\right)$$

$$= 0.2$$

$$\mu_{R\cap S}(x_1, y_3) = \min(\mu_R(x_1, y), \mu_S(x_1, y_3))$$

$$= \min((0.2, 0.3), (0.2, 0.4))$$

$$= 0.2$$

Complement :

$$R = \begin{array}{c c} & \begin{array}{cc} y_1 & y_2 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array} & \begin{bmatrix} 0.2 & 0.2 \\ 0.4 & 0.4 \\ 0.5 & 0.6 \end{bmatrix} \end{array} , \quad S = \begin{array}{c c} & \begin{array}{ccc} y_1 & y_2 & y_3 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \end{array} & \begin{bmatrix} 0.5 & 0.7 & 0.3 \\ 0.8 & 0.1 & 0.4 \end{bmatrix} \end{array}$$

$$\boxed{R^c(x, y) = 1 - \mu_R(x, y)}$$

$$R^c(x_1, y_1) = 1 - \mu_R(x_1, y_1)$$

$$= \cancel{0}\ 1 - 0.2$$

$$= 0.8.$$

$$R^c(x_1, y_2) = 1 - \mu_R(x_1, y_2)$$

$$= 1 - 0.2$$

$$= 0.8$$

$$R^c(x_2, y_1) = 1 - \mu_R(x_2, y_1)$$

$$= 1 - 0.4$$

$$= 0.6$$

$$R^c(x_2, y_2) = 1 - \mu_R(x_2, y_2)$$

$$= 1 - 0.4$$

$$= 0.6$$

$$R^c(x_3, y_1) = 1 - \mu_R(x_3, y_1)$$

$$= 1 - 0.5$$

$$= 0.5$$

$$R^{c}(x_3, y_2) = 1 - \mu_R(x_3, y_2)$$
$$= 1 - (0.6, \cancel{0.4})$$
$$= 1 - 0.6 = 0.4$$

$R^{c}(x, y)$

$$\begin{bmatrix} 0.8 & 0.8 \\ 0.6 & 0.6 \\ 0.5 & 0.4 \end{bmatrix}$$

<u>composition operation :</u>

R be a relation of $X \times Y$.

S be a relation of $Y \times Z$.

RoS to be composition relation of $X \times Z$.

Membership function : (max, min composition)

$$RoS = \max(\min(\mu_R(x, y), \mu_S(y, z)))$$

$= y \in Y$.

$$R = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{cc} y_1 & y_2 \end{array} \\ \begin{bmatrix} 0.2 & 0.2 \\ 0.4 & 0.4 \\ 0.5 & 0.6 \end{bmatrix} \end{array} , \quad S = \begin{array}{c} \\ y_1 \\ y_2 \end{array} \begin{array}{c} \begin{array}{ccc} z_1 & z_2 & z_3 \end{array} \\ \begin{bmatrix} 0.5 & 0.7 & 0.3 \\ 0.8 & 0.1 & 0.4 \end{bmatrix} \end{array}$$

$$R \circ S = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{ccc} z_1 & z_2 & z_3 \end{array} \\ \begin{bmatrix} 0.2 & 0.2 & 0.2 \\ 0.4 & 0.4 & 0.4 \\ 0.6 & 0.5 & 0.4 \end{bmatrix} \end{array}$$

$$\mu_{R \circ S}(x_1, z_1) = \max\left(\min\left[\mu_R(x_1, y_1), \mu_S(y_1, z_1)\right], \min\left[\mu_R(x_1, y_2), \mu_S(y_2, z_1)\right]\right)$$

$$= \max(\min(0.2, 0.5), \min(0.2, 0.8))$$

$$= \max(0.2, 0.2)$$

$$= 0.2$$

$$\mu_{R \circ S}(x_1, z_2) = \max(\min(\mu_R(x_1, y_1), \mu_S(y_1, z_2)), \min(\mu_R(x_1, y_2), \mu_S(y_2, z_2)))$$

$$= \max(\min(0.2, 0.7), \min(0.2, 0.1))$$

$$= \max(0.2, 0.1)$$

$$= 0.2$$

$$\mu_{R \circ S}(x_1, z_3) = \max(\min(\mu_R(x_1, y_1), \mu_S(y_1, z_3)), (\min(\mu_R(x_1, y_2), \mu_S(y_2, z_3)))$$

$$= \max(\min(0.2, 0.3), \min(0.2, 0.4))$$

$$= \max(0.2, 0.2)$$

$$= \max(0.2)$$

$\mu_{ROS}(x_2, z_1) = \max(\min(\mu_R(x_2,y_1), \mu_S(y_1,z_1)), \min(\mu_R(x_2,y_2), (\mu_S(y_2,z_1)))$

$= \max(\min(0.4, 0.5), \min(0.4, 0.8))$

$= \max(0.4, 0.4)$

$= \max(0.4)$

$= 0.4.$

$\mu_{ROS}(x_2, z_2) = \max(\min(\mu_R(x_2,y_1), \mu_S(y_1,z_2)), \min((\mu_R(x_2,y_2), \mu_S(y_2,z_2)))$

$= \max(\min(0.4, 0.7), \min(0.4, 0.1))$

$= \max(0.4, 0.1)$

$= 0.4.$

$\mu_{ROS}(x_2, z_3) = \max(\min(\mu_R(x_2,y_1), \mu_S(y_1,z_3)), \min(\mu_R(x_2,y_2), \mu_S(y_2,z_3)))$

$= \max(\min(0.4, 0.3), \min(0.4, 0.4))$

$= \max(0.3, 0.4)$

$= 0.4$

$\mu_{ROS}(x_3, z_1) = \max(\min(\mu_R(x_3,y_1), \mu_S(y_1,z_1)), \min(\mu_R(x_3,y_2), \mu_S(y_2,z_1)))$

$= \max(\min(0.5, 0.5), \min(0.6, 0.8))$

$= \max(0.5, 0.6)$

$= \max(0.6).$

$\mu_{ROS}(x_3, z_2) = \max(\min(0.5, 0.7), \min(0.6, 0.1))$

$= \max(0.5, 0.1)$

$= \max(0.5)$

$\mu_{ROS}(x_3, z_3) = \max(\min(0.5, 0.3), \min(0.6, 0.4))$

$= \max(0.3, 0.4)$

$= \max(0.4).$

Fuzzy Rule:

Fuzzy rule is also known as Fuzzy implication, conditional statement, fuzzy if then rule.

if x is A then y is B,

x is A ← always fuzzy set.

→ antecedent (or) premise

y is B ← either a fuzzy set (or) crisp number.

→ consequence (or) conclusion.

A is a linguistic value

x is a linguistic variable.

eg: consider a fuzzy sets $\tilde{A}$ derived from universal set x, $\tilde{B}$ derived from universal set y. then form a relation

($\tilde{A} \rightarrow \tilde{B}$ it is fuzzy implication rule.)

$R = \tilde{A} \times \tilde{B} = \tilde{A} \rightarrow \tilde{B}$

Relation using fuzzy rule:

i) continuous form:

$$R = \tilde{A} \to \tilde{B} = \tilde{A} \times B = \int_{x \times y} \mu_R^{(x,y)} / (x,y) \quad \forall x, y \in x \times y$$

2) Discrete form:

$$R = \tilde{A} \to \tilde{B} = \tilde{A} \times \tilde{B} = \sum_{x \times y} \mu_R^{(x,y)} / (x,y) \; \forall x, y \in x \times y.$$

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Fuzzy Relation Using Fuzzy Rule:

i) continuous form:

$$R = \tilde{A} \times \tilde{B} = \tilde{A} \to \tilde{B}$$

$$\int_{x \times y} \mu_{\tilde{A}}^{(x)}, \mu_{\tilde{B}}^{(y)} / (x,y) \; \forall x, y \in x \times y$$

ii) Discrete form:

$$R = \tilde{A} \times \tilde{B} = \tilde{A} \to B$$

$$\sum_{x \times y} \mu_{\tilde{A}}^{(x)}, \mu_{\tilde{B}}^{(y)} / (x,y) \; \forall x, y \in x \times y.$$

Fuzzy Relation interpreted by using Fuzzy Rule:

There are 2 ways to interpret fuzzy relation by using fuzzy rule.

i) A coupled with B

ii) A entails B

A coupled with B :- T-nom operator

A T-nom operator is used,

continuous form :

$$R = \tilde{A} \times \tilde{B} = \tilde{A} \rightarrow \tilde{B}$$

$$\int_{X \times Y} T\left[\mu_{\tilde{A}}^{(x)}, \mu_{\tilde{B}}^{(y)}\right] (x, y) \ \forall x, y \in X \times Y$$

Discrete form :

$$R = \tilde{A} \times \tilde{B} = \tilde{A} \rightarrow \tilde{B}$$

$$\sum_{X \times Y} T\left[\mu_{\tilde{A}}^{(x)}, \mu_{\tilde{B}}^{(y)}\right] / (x, y) \ \forall x, y \in X \times Y.$$

There are 4 types of T-nom operators.

1. A coupled with B using Mminimum T-nom operator.
2. A coupled with B using Algebric product T-nom operator.
3. A coupled with B using Bounded product T-nom operator.
4. A coupled with B Drastic product T-nom operator.

1. A coupled with B using Mminimium T-nom operator :

min → intersection → ∩ → ∧ (and)

max → union → ∪ → ∨ (or)

1) continuous form :

$$R = \tilde{A} \times \tilde{B} = \tilde{A} \to \tilde{B}$$

$$\int (\mu_{\tilde{A}}^{(x)} \wedge \mu_{\tilde{B}}^{(y)}) / (x,y) \ \forall x,y \in X \times Y.$$

2) Discrete form :

$$R = \tilde{A} \times \tilde{B} = \tilde{A} \to \tilde{B}$$

$$\sum_{X \times Y} (\mu_{\tilde{A}}^{(x)} \wedge \mu_{\tilde{B}}^{(y)} / (x,y) \ \forall x,y \in X \times Y.$$

eg 1:

$\tilde{A} = \{(20, 0.2), (25, 0.4), (30, 0.6), (45, 0.8), (50, 1)\}$

$\tilde{B} = \{(1, 0.4), (2, 0.6), (3, 0.7), (4, 0.8)\}$

$$R = \tilde{A} \times \tilde{B} = \tilde{A} \to \tilde{B}$$

$$\begin{array}{c|cccc} & 1 & 2\ (y) & 3 & 4 \\ \hline 20 & 0.2 & 0.2 & 0.2 & 0.2 \\ 25 & 0.4 & 0.4 & 0.4 & 0.4 \\ 30 & 0.4 & 0.6 & 0.6 & 0.6 \\ 45 & 0.4 & 0.6 & 0.7 & 0.8 \\ 50 & 0.4 & 0.6 & 0.7 & 0.8 \end{array}$$

$= \mu_{\tilde{A}}^{(20)} \wedge \mu_{\tilde{B}}^{(1)}$

$= 0.2 \wedge 0.4$

$= 0.2$

2) A coupled with B using Algebraic product T norm operator:

1) Continuous form:

$$R = \tilde{A} \times \tilde{B} = \tilde{A} \rightarrow \tilde{B}$$

$$\int_{X \times Y} \left(\mu_{\tilde{A}}^{(x)} \times \mu_{\tilde{B}}^{(y)} \right) / (x, y) \ \forall x, y \in X \times Y.$$

2) Discrete form:

$$R = \tilde{A} \times \tilde{B} = \tilde{A} \rightarrow \tilde{B}$$

$$\sum_{X \times Y} \left(\mu_{\tilde{A}}^{(x)} \times \mu_{\tilde{B}}^{(y)} \right) / (x, y) \ \forall x, y \in X, Y.$$

eg:

$\tilde{A} = \{(20, 0.2), (25, 0.4), (30, 0.6), (45, 0.8), (50, 1)\}$

$\tilde{B} = \{(1, 0.4), (2, 0.6), (3, 0.7), (4, 0.8)\}$

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 20 & 0.08 & 0.12 & 0.14 & 0.16 \\ 25 & 0.16 & 0.24 & 0.28 & 0.32 \\ 30 & 0.24 & 0.36 & 0.42 & 0.48 \\ 45 & 0.32 & 0.48 & 0.56 & 0.64 \\ 50 & 0.4 & 0.6 & 0.7 & 0.8 \end{array}$$

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3. A coupled with B using Algebraic / Bounded Product T-norm operator:-

i) Continuous form:-

$$\int_{X\times Y} \mu_{\tilde{A}}(x) + \mu_{\tilde{B}}(y) - 1) \,/\, (x,y),\ \forall\, x,y \in X \times Y.$$

$$R = \tilde{A} \times \tilde{B} = \tilde{A} \rightarrow \tilde{B}$$

ii) Discrete form:-

$$R = \tilde{A} \times \tilde{B} = \tilde{A} \rightarrow \tilde{B}$$

$$\sum_{X\times Y} 0 \vee (\mu_{\tilde{A}}(x) + \mu_{\tilde{B}}(y) - 1) \,/\, (x,y)\ \forall\, x,y \in X \times Y.$$

ex:)

$\tilde{A} = \{(20, 0.2), (25, 0.4), (30, 0.6), (45, 0.8), (50, 1)\}$

$\tilde{B} = \{(1, 0.4), (2, 0.6), (3, 0.7), (4, 0.8)\}$

0.10 - 1.0

0.0

$$R = \tilde{A} \times \tilde{B} = \tilde{A} \rightarrow \tilde{B}$$

$0 \vee (\mu_{\tilde{A}}(x) + \mu_{\tilde{B}}(y) - 1)$

$0.2 + 0.4 - 1$

$0.6 - 1 \Rightarrow -0.4$

$0 \vee -0.4 \Rightarrow 0$

$$R = \begin{array}{c} \\ 20 \\ 25 \\ 30 \\ 45 \\ 50 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0.1 & 0.2 \\ 0 & 0.2 & 0.3 & 0.4 \\ 0.2 & 0.4 & 0.5 & 0.6 \\ 0.4 & 0.6 & 0.7 & 0.8 \end{bmatrix} \end{array}$$

A coupled with B using Drastic product T_norm operator:

1) continuous form :-

$$R = \tilde{A} \times \tilde{B} = \tilde{A} \rightarrow \tilde{B}$$

$$\int_{x \times y} \mu_{Rdp}(x,y) / (x,y), \forall\ x,y \in X \times Y.$$

2) Discrete form :-

$$R = \tilde{A} \times \tilde{B} = \tilde{A} \rightarrow \tilde{B}$$

$$\sum_{x \times y} \mu_{Rdp}(x,y) / (x,y), \forall\ x,y \in x \times y.$$

eg:1

$\tilde{A} = \{(20, 0.2), (25, 0.4), (30, 0.8), (45, 0.8), (50, 1)\}$

$\tilde{B} = \{(1, 0.4), (2, 0.6), (3, 0.7), (4, 0.8)\}$

$$R = \begin{array}{c} \\ 20 \\ 25 \\ 30 \\ 45 \\ 50 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0.4 & 0.6 & 0.7 & 0.8 \end{bmatrix} \end{array}$$

...rators

$$\mu_{adp}(x,y) = \begin{cases} \mu_{\tilde{A}}(x) & \text{if } \mu_{\tilde{B}}(y) = 1 \\ \mu_{\tilde{B}}(y) & \text{if } \mu_{\tilde{A}}(x) = 1 \\ 0 & \text{otherwise} \end{cases}$$

eg:2 $\tilde{A} = \{(20,0.2),(25,0.4),(30,0.8),(45,0.8),(50,1)\}$

$\tilde{B} = \{(1,0.4),(2,0.6),(3,0.7),(4,0.8)\}$

$$R = \begin{array}{c} \\ 20 \\ 25 \\ 30 \\ 45 \\ 50 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 0 & 0.2 & 0 \\ 0 & 0 & 0.4 & 0 \\ 0 & 0 & 0.8 & 0 \\ 0 & 0 & 0.8 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

A entails B :-

$R = \tilde{A} \times \tilde{B} = \tilde{A} \to B$

1) material implecation method. (MI)
2) propositional calculas (PC)
3) Extended propositional calculas (EPC
4) Generalization of Modus Ponens (GMP)

1) Material implication Method:

i) continuous form:

$$\int_{x \times y}^{1 \wedge} \left((1 - \mu_{\tilde{A}}(x)) + (\mu_{\tilde{B}}(y)) \right) \Big| (x,y), \forall x, y \in X \times Y$$

2) Discrete form:

$$\sum_{x \times y} 1 \wedge \left((1 - \mu_{\tilde{A}}(x)) + \mu_{\tilde{B}}(y) \right) / (x, y), \forall x, y \in X \times Y.$$

"Zadheh Arithmetic Rule" by using $\neg A \cup B$, Union follows the Arithmetic rule.

eg:

$\tilde{A} = \{(20, 0.2), (25, 0.4), (30, 0.6), (45, 0.8), (50, 0.9)\}$

$\tilde{B} = \{(1, 0.4), (2, 0.6), (3, 0.7), (4, 0.8), (4, 0.8)\}$

$$R = \begin{array}{c|c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 20 \\ 25 \\ 30 \\ 45 \\ 50 \end{array} & \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0.8 & 1 & 1 & 1 \\ 0.6 & 0.8 & 0.9 & 1 \\ 0.5 & 0.7 & 0.8 & 0.9 \end{bmatrix} \end{array}$$

$1 \wedge ((1 - \mu_{\tilde{A}}(x)) + \mu_{\tilde{B}}(y))$

$1 - \mu_{\tilde{A}}(20)$

$1 - 0.2$

$0.8 + \mu_{\tilde{B}}(1)$

$0.8 + 0.4$

1.2

$1 \wedge 1.2$

$1 - 0.9 + 0.4$

$0.1 + 0.4$

0.5

$1 - 0.8$

$0.2 + 0.6$

0.8

Propositional calculus :- (pc)

continuous form:

$R_{pc} = \tilde{A} \times \tilde{B} = \tilde{A} \to \tilde{B} = \neg A \cup (A \cap B)$

$$\int_{xy} (1 - \mu_{\tilde{A}}(x)) \vee (\mu_{\tilde{A}}(x) \wedge \mu_{\tilde{B}}(y)) \,/\, (x, y), \forall\, x, y \in X \times Y.$$

Discrete form:

$$\sum_{X \times Y} (1 - \mu_{\tilde{A}}(x)) \vee (\mu_{\tilde{A}}(x) \wedge \mu_{\tilde{B}}(y)) \,/\, (x, y), \forall\, x, y \in X \times Y.$$

① $\tilde{A} = \{(20, 0.2), (25, 0.4), (30, 0.6), (45, 0.8), (50, 0.9)\}$

$\tilde{B} = \{(1, 0.4), (2, 0.6), (3, 0.7), (4, 0.8)\}$

$$\begin{array}{c|c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 20 \\ 25 \\ 30 \\ 45 \\ 50 \end{array} & \begin{bmatrix} 0.8 & 0.8 & 0.8 & 0.8 \\ 0.6 & 0.6 & 0.6 & 0.6 \\ 0.4 & 0.6 & 0.6 & 0.6 \\ 0.4 & 0.6 & 0.7 & 0.8 \\ 0.4 & 0.6 & 0.7 & 0.8 \end{bmatrix} \end{array}$$

"Zadheh max-min rule"

by using $\neg A \vee (A \cap B)$

max follows union

min follows intersection.

$1 - \mu_{\tilde{A}}(x) \vee (\mu_{\tilde{A}}(x) \wedge \mu_{\tilde{B}}(y))$

$1 - \mu_{\tilde{A}}(x) \vee (0.2 \wedge 0.4)$

$1 - \mu_{\tilde{A}}(x) \vee (0.4)$

$1 - 0.2 \vee (0.4)$

$0.8 \vee 0.4$

0.8

$1 - 0.2 \vee (0.6)$

$0.8 \vee (0.6)$

0.8

$1 - 0.2 \vee (0.7)$

$0.8 \vee 0.7$

0.8

$0.8 \vee 0.8$

0.8

$1 - 0.4 \vee (0.4 \wedge 0.4)$

$0.6 \vee (0.4)$

$1 - 0.4 \vee (0.4)$

$0.6\ (0.4)$

0.6 max

Extended propositional calculus :- EPC

$$R_{EPC} = \tilde{A} \times \tilde{B} = \tilde{A} \to \tilde{B} = (\neg A \cap \neg B) \cup B$$

continuous :-

$$\int_{X \times Y} \left(\left((1 - \mu_{\tilde{A}}(x)) \wedge (1 - \mu_{\tilde{B}}(y)) \right) \vee \mu_{\tilde{B}}(y) \right) / (x, y), \ \forall x, y \in X \times Y.$$

Discrete form :

$$\sum_{X \times Y} \left(\left((1 - \mu_{\tilde{A}}(x)) \wedge (1 - \mu_{\tilde{B}}(y)) \right) \vee \mu_{\tilde{B}}(y) \right) / (x, y), \ \forall, x, y \in X \times Y.$$

eg :-

$\tilde{A} = \{(20, 0.2), (25, 0.4), (30, 0.6), (45, 0.8), (50, 0.9)\}$

$\tilde{B} = \{(1, 0.4), (2, 0.6), (3, 0.7), (4, 0.8)\}$

$$R = \begin{array}{c} \\ 20 \\ 25 \\ 30 \\ 45 \\ 50 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0.6 & 0.6 & 0.7 & 0.8 \\ 0.6 & 0.6 & 0.7 & 0.8 \\ 0.4 & 0.6 & 0.7 & 0.8 \\ 0.4 & 0.6 & 0.7 & 0.8 \\ 0.4 & 0.6 & 0.7 & 0.8 \end{bmatrix} \end{array}$$

R.W

$\left((1 - \mu_{\tilde{A}}(x)) \wedge 1 - \mu_{\tilde{B}}(y) \right) \vee \mu_{\tilde{B}}(y)$

$\left(1 - \mu_{\tilde{A}}(20) \right) \wedge \left(1 - \mu_{\tilde{B}}(1) \right) \vee \mu_{\tilde{B}}(1)$

$\left((1 - 0.2) \wedge (1 - 0.4) \right) \vee 0.4$

$0.8 \wedge (0.6) \vee 0.4$

$0.6 \wedge 0.4$

generalization of modus ponens:

$$R_{gmp} = \tilde{A} \times \tilde{B} = \tilde{A} \to \tilde{B} = \tilde{A} \Sigma \tilde{B}$$

1) continuous form:

$$\int_{x,y} \mu_{R_{gmp}}(x,y) \Big/ (x,y), \ \forall\, x,y \in X \times Y.$$

2) Discrete form:

$$\sum_{x,y} \mu_{R_{gm}}(x,y) \Big/ (x,y), \ \forall\, x,y \in X \times Y.$$

---

eg. 2

$\tilde{A} = \{(20, 0.2), (25, 0.4), (30, 0.6), (45, 0.8), (50, 0.9)\}$

$\tilde{B} = \{(1, 0.4), (2, 0.6), (3, 0.7), (4, 0.8)\}$

① $$R_{gmp}(x,y) = \begin{cases} 1 & \text{if } \mu_{\tilde{A}}(x) \le \mu_{\tilde{B}}(y) \\ \dfrac{\mu_{\tilde{B}}(y)}{\mu_{\tilde{A}}(x)} & \text{if } \mu_{\tilde{A}}(x) > \mu_{\tilde{B}}(y) \end{cases}$$

$$R_{gmp} = \begin{array}{c} \\ 25 \\ 30 \\ 35 \\ 45 \\ 50 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0.2 & 1 & 1 & 1 \\ 0.67 & 0.75 & 0.875 & 1 \\ 0.44 & 0.67 & 0.78 & 0.89 \end{bmatrix} \end{array}$$

22/02/2021 :

T norm and S norm :

T norm :-

Let consider $T[0,1] \times [0,1] \rightarrow [0,1]$

1. Mapping the membership functions of two fuzzy sets $\tilde{A}$ and $\tilde{B}$ with membership function of T_norm fuzzy sets of $\tilde{A}$ and $\tilde{B}$ is defined as

$$T[\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)] = \mu_{\tilde{A} \cap \tilde{B}}(x) = \mu_{\tilde{A}}(x) \cap \mu_{\tilde{B}}(x)$$

The function of T is an intersection and must be satisfy the following 4 requirements :

i) Minimum condition :

$$T[0.0] = \min[0,0] = 0 \cap 0 = 0$$

$$T[\mu_{\tilde{A}}(x), 1] = T[1, \mu_{\tilde{A}}(x)] = \mu_{\tilde{A}}(x)$$

2) commutativity :

$$T = [\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)] = T[\mu_{B}(x), \mu_{\tilde{A}}(x)]$$

3) Non - Decreasing :

if $\mu_{\tilde{A}}(x) \leq \mu_{\tilde{B}}(x)$ and

$\mu_{\tilde{C}}(x) \leq \mu_{\tilde{D}}(x)$ then

$$T\left[\mu_{\tilde{A}}^{(x)}, \mu_{\tilde{C}}^{(x)}\right] \leq T\left[\mu_{\tilde{B}}^{(x)}, \mu_{\tilde{D}}^{(x)}\right]$$

Associativity :

$\tilde{A}$ and

$\tilde{A}$ and $\tilde{B}$

$$T\left[T\left[\mu_{\tilde{A}}^{(x)}, \mu_{\tilde{B}}^{(x)}\right], \mu_{\tilde{C}}^{(x)}\right] = T\left[\mu_{\tilde{A}}^{(x)}, T\left[\mu_{\tilde{B}}^{(x)}, \mu_{\tilde{C}}^{(x)}\right]\right]$$

$\wedge \rightarrow$ Tnorm operator, T norm operator is also called "S_conorm".

S_norm :-

Let consider $S\,[0,1] \times [0,1] \rightarrow [0,1]$

Mapping the membership functions of two fuzzy sets $\tilde{A}$ and $\tilde{B}$ with membership function of S norm fuzzy sets $\tilde{A}$ and $\tilde{B}$ is defined as

$$S\left[\mu_{\tilde{A}}^{(x)}, \mu_{\tilde{B}}^{(x)}\right] = \mu_{\tilde{A} \cup \tilde{B}}^{(x)} = \mu_{\tilde{A}}^{(x)} \cup \mu_{\tilde{B}}^{(x)}.$$

the function of 'S' is an Union and must be satisfy the following 4 requirements.

1. **Maximum condition :**

$$S[1,1] = \max[1,1] = 1 \cup 1 = 1$$

$$S\left[\mu_{\tilde{A}}^{(x)}, 0\right] = S\left[0, \mu_{\tilde{A}}^{(x)}\right] = \mu_{\tilde{A}}^{(x)}\Big] = \mu_{\tilde{A}}^{(x)}$$

Commutativity:

$$S\left[\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)\right] = S\left[\mu_{\tilde{B}}(x), \mu_{\tilde{A}}(x)\right]$$

Non Decreasing:

if $\mu_{\tilde{A}}(x) \leq \mu_{\tilde{B}}(x)$ and

$\mu_{\tilde{C}}(x) \leq \mu_{\tilde{D}}(x)$ then,

Associativity:

$$S\left[S\left[\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)\right], \mu_{\tilde{C}}(x)\right] = S\left[\mu_{\tilde{A}}(x), S\left[\mu_{\tilde{B}}(x), \mu_{\tilde{C}}(x)\right]\right]$$

$\vee \rightarrow$ S norm operator.

$S \rightarrow$ norm is also called T-conorm

Tnorm operator:

1) Minimum :-

$$T\left[\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)\right] = \min\left[\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)\right] = \mu_{\tilde{A}}(x) \wedge \mu_{\tilde{B}}(x)$$

2. Algebric :

$$T\left[\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)\right] = \mu_{\tilde{A}}(x) \times \mu_{\tilde{B}}(x)$$

3. Bounded product:

$$T\left[\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)\right] = 0 \vee \left[\mu_{\tilde{A}}(x) + \mu_{\tilde{B}}(x) - 1\right]$$

Drastic product :

$$T[\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)] = \begin{cases} \mu_{\tilde{A}}(x) & \text{if } \mu_{\tilde{B}}(x) = 1 \\ \mu_{\tilde{B}}(x) & \text{if } \mu_{\tilde{A}}(x) = 1 \\ 0 & \text{if } \mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x) < 1 \end{cases}$$

S_norm operator :-

1. Minimum :-

$$S[\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)] = \max[\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)] = \mu_{\tilde{A}}(x) \vee \mu_{\tilde{B}}(x)$$

2. Algebric sum :

$$S[\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)] = \mu_{\tilde{A}}(x) + \mu_{\tilde{B}}(x) - (\mu_{\tilde{A}}(x) \times \mu_{\tilde{B}}(x))$$

3. Bounded sum :

$$S[\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)] = 1 \wedge (\mu_{\tilde{A}}(x) + \mu_{\tilde{B}}(x))$$

4. Drastic sum :

$$S[\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)] = \begin{cases} \mu_{\tilde{A}}(x) & \text{if } \mu_{\tilde{B}}(x) = 0 \\ \mu_{\tilde{B}}(x) & \text{if } \mu_{\tilde{A}}(x) = 0 \\ 1 & \text{if } \mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x) > 0. \end{cases}$$

31/08/2021

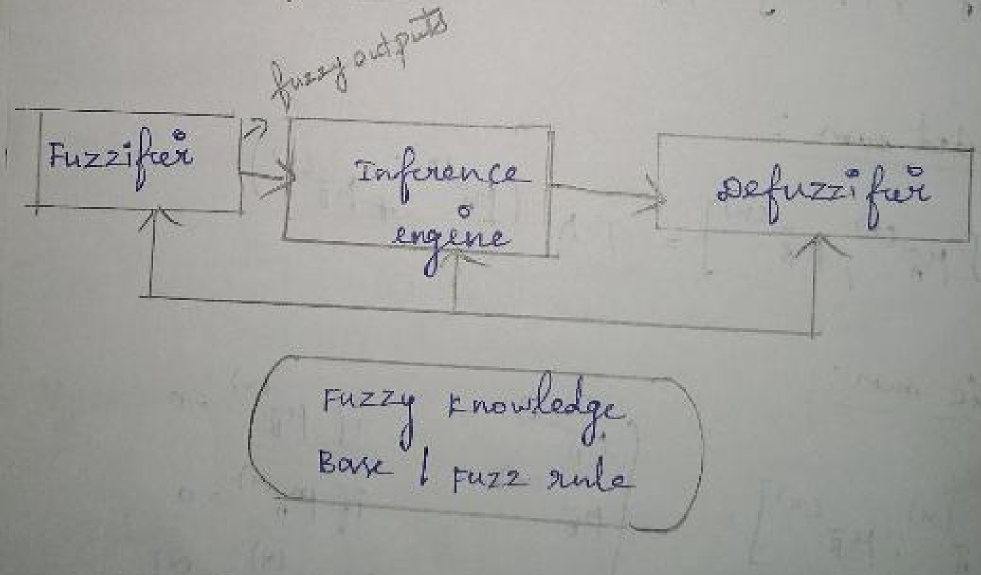
Fuzzy inference system :-

FIS Mapping from given i/p into an outside by using Fuzzy logic.

FIS is also known as "Fuzzy system, Fuzzy associative memory, Fuzzy export system, fuzzy logic controller, Fuzzy Model, Fuzzy rule based system."

FIS is a "Non-linear Mapping" that derives its "output based on fuzzy Reasoning" and set of "fuzzy if-then rule".

Architecture of FIS :



Fuzzifier gets "input as an either crisp value (or) Fuzzy value.

If input is "crisp value" then the fuzzifier convert into fuzzy value by using Membership function is called degree, this is called Fuzzification

If input is "Fuzzy value" then the Fuzzifier automatically transfer it "fuzzy value" to interference engine.

<u>Interfrence engine</u>:

Interference engine gets fuzzy value as an input from Fuzzifier and compute, with the help of "Fuzzy knowledge Base" (applying rules) and produce computed fuzzy value.

Inforence engine applying more than if - then rule and gets outputs. all the outputs are unioned (maximized) by using "Aggregator".

<u>Deffuzzifier</u>:

Defuzzifier gets input as a fuzzy value, and convert it to produce as a crisp value.

Fuzzy Reasoning :-

single Rule with single Antecedent

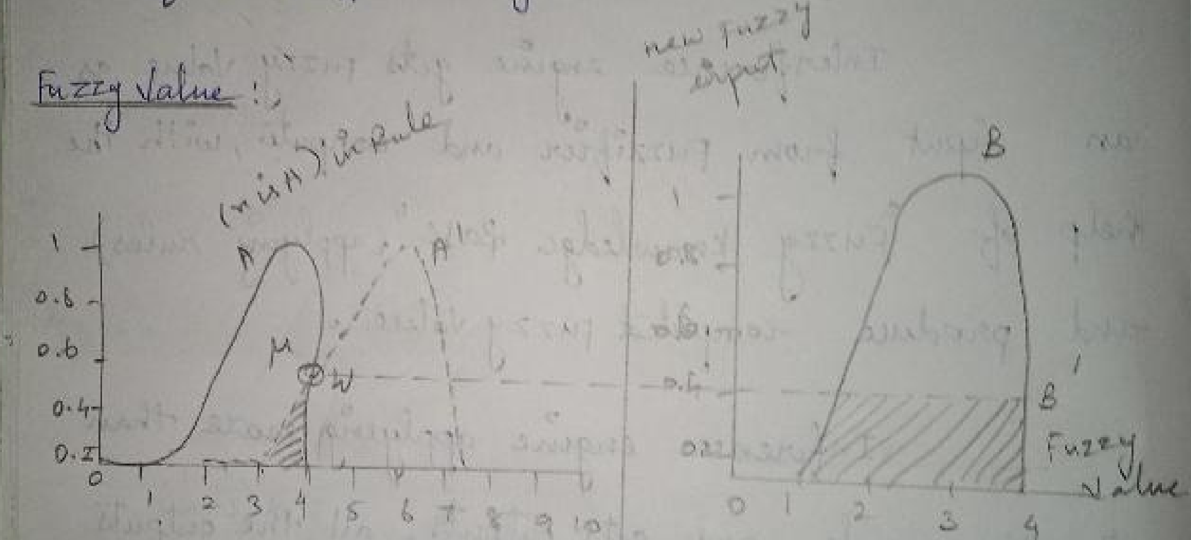
Single Rule with Multiple Antecedent

Multiple Rule with Multiple Antecedent.

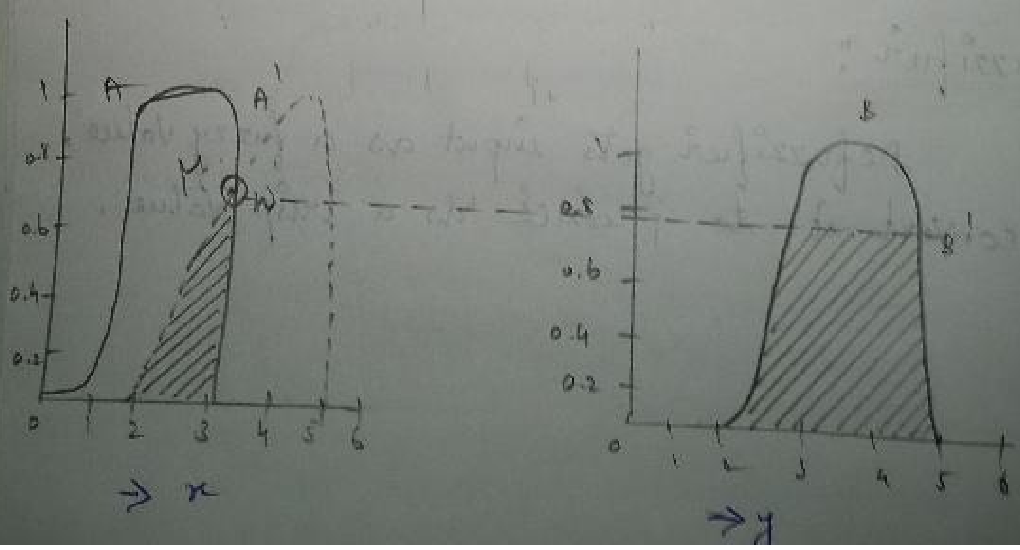
single rule with single Antecedent :

eg: if x is A then y is B

Fuzzy Value :



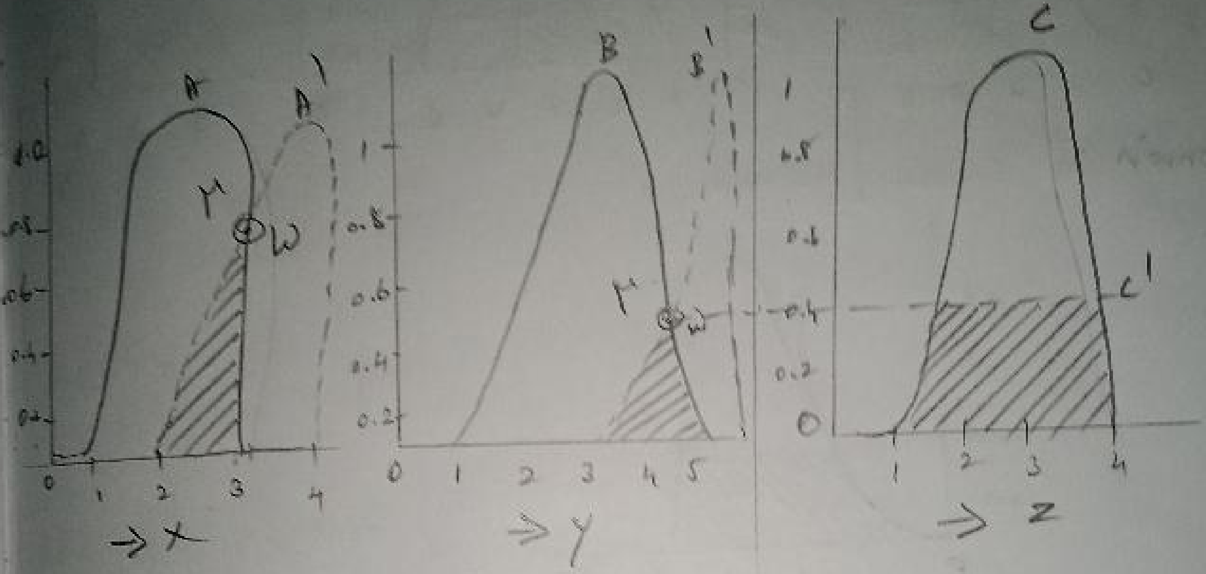
Crisp input :



Single rule with Multiple Antecedent :

connector

eg: if x is A (and | or) y is B then z is C

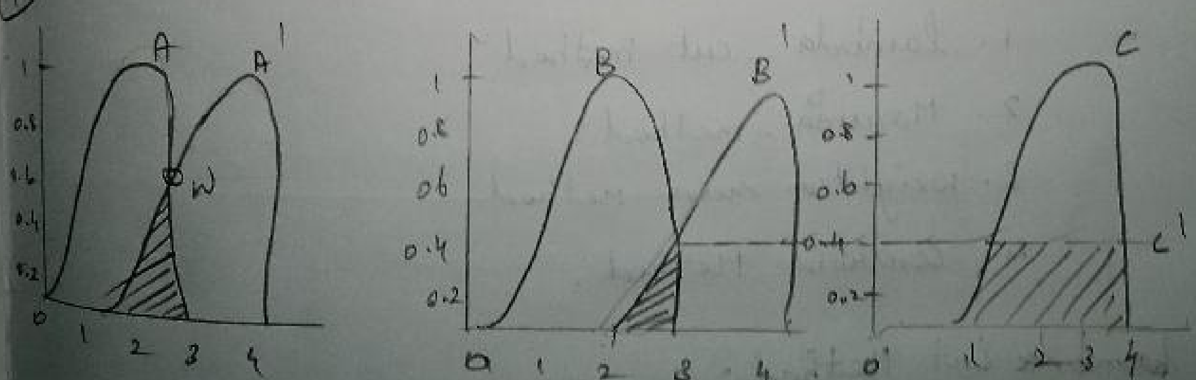


Multiple rule with multiple Antecedent :

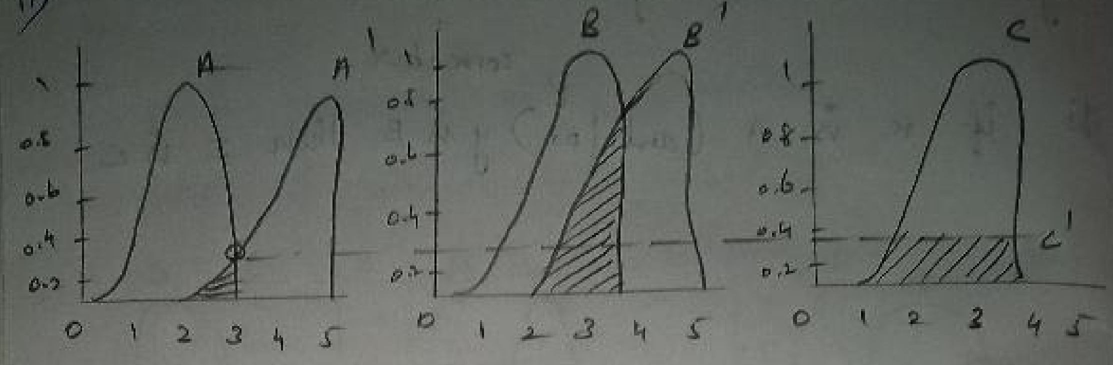
eg: i) if x is A and y is B then z is C

ii) if x_1 is A_1' and y_1 is B_1' then z_1 is C_1.

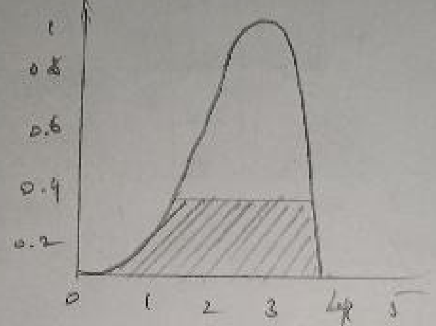
(i)



ii)



Union :



Defuzzification :-

convert Fuzzy value into crisp value.

Methods :

1. lambda _ cut method
2. Maxima _ method.
3. weighted sum method
4. centroid Method.

i) Lambda Cut Method :

$$R = \begin{bmatrix} 0.2 & 1 \\ 0.3 & 0.6 \end{bmatrix} \Rightarrow \text{Fuzzy Relation}$$

$\lambda = 1, 0.5, 0.1, 0 \Rightarrow$ lambda value.

compare every lambda value to relation.
if lambda value is ≥ relation value (0)
otherwise (1).

crisp value :

$$R_1 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \qquad R_{0.5} = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$$

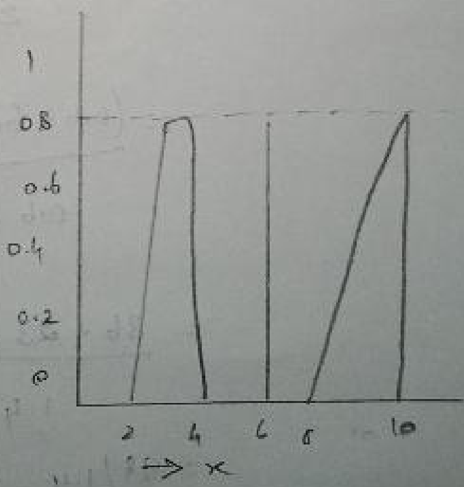
$$R_{0.1} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \qquad R_0 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

2) Maxima Method :

1. First of Maxima / smallest of maxima
2. Mean of Maxima / Medium of Maxima
3. Last of Maxima / Largest of Maxima.

crisp Value :

$$x^* = \frac{\sum x_i \in M^{x_i}}{|M|}$$



$M = \{x \mid \mu_{\tilde{A}}^{(x)} = \text{height of fuzzy set}\}$

$|M|$ = cardinality.

$x = 4, 6, 8$

$M = 4, 6, 8.$

First of Maxima = 4

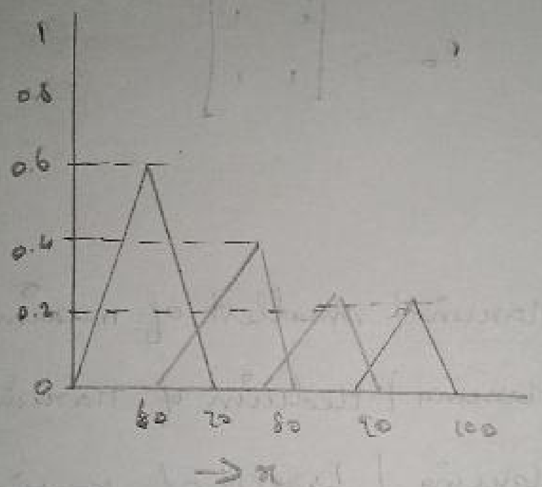
Last of Maxima = 8

Medium of Maxima $= \frac{4+6+8}{3}$

$= 18/3$

$= 6.$

Weighted average Method:



crisp Number $x^* = \frac{\sum \mu_A^{(x)} \cdot x}{\sum \mu_A^{(x)}}$

$= \frac{(0.6 \times 60) + (0.4 \times 70) + (0.2 \times 80) + (0.2 \times 90)}{0.6 + 0.4 + 0.2 + 0.2}$

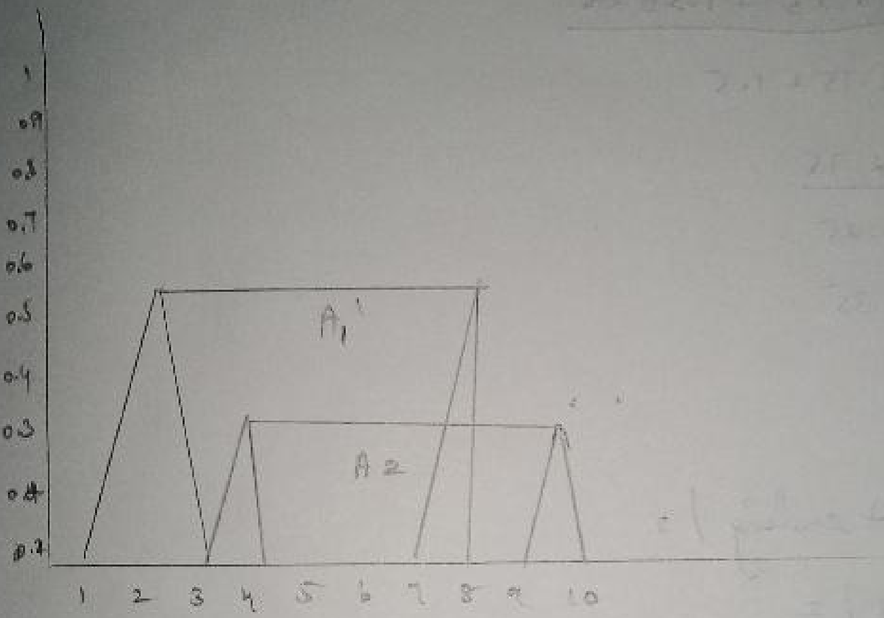
$= \frac{36 + 23 + 16 + 18}{1.4}$

$= 98/1.4 = 70$ //.

centroid Method :

center of sum :

$$X^* = \frac{\Sigma A^C}{\Sigma A} = \frac{\Sigma AC}{\Sigma A}$$



$$A_1 = \tfrac{1}{2}\left[(8-1)+(7-3)\right] \times 0.5 = 2.75$$

$$A_2 = \tfrac{1}{2}\left[(9-3)+(8-4)\right] \times 0.3 = 1.50$$

$$A_1 = \tfrac{1}{2}\,[7+4] \times 0.5$$

$$= \tfrac{1}{2}\,[11 \times 0.5]$$

$$= \tfrac{1}{2}\,[5.5]$$

$$= 2.75$$

$A_2 = \frac{1}{2}[6+4] \times 0.3$

$= \frac{1}{2}[10] \times 0.3$

$= \frac{1}{2}[3.0]$

$A_2 = 1.50$

$$x^* = \frac{2.75 \times 5 + 1.50 \times 6}{2.75 + 1.5}$$

$$= \frac{22.75}{4.25}$$

$= 5.35$.

<u>center</u> :-

starting + ending / 2

$\Rightarrow 1 + 8 / 2$

$\Rightarrow 9/2$

$\Rightarrow 4.5 = 5$

$\Rightarrow 3 + 9 / 2 = 12/2$

$= 6$